MATHEMATICS

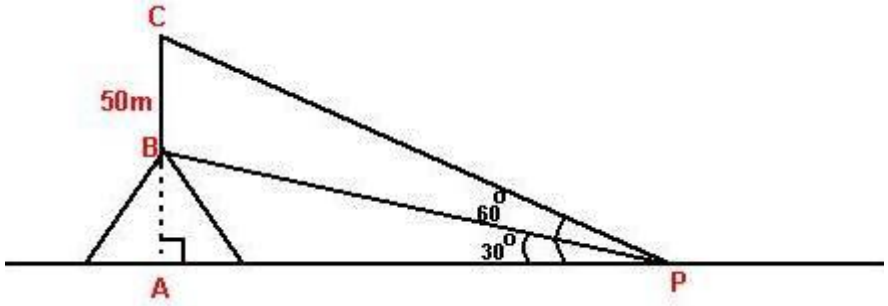
ANSWERS

Level Three

Angles and Angles

Solution 1:

A tower 50 meters high stands on the top of a hill. From a point P on the level ground, a man observes the top of the tower at an angle of elevation of 60^0 and the bottom of the tower at an angle of elevation of 30°. Find the height of the hill and the distance of P from the foot of the hill.



Step 1: AB denotes the height of the hill BC denotes the height of the tower
Given BC = 50 meters Given $\angle$ APC = 60° and $\angle$APB = 30°

Step 2: Let AB = x meters and AP = y meters

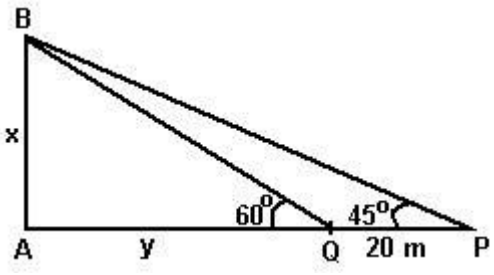
Step 3: In the triangle **APB**, $\angle$ ABP = 90° - 30° = 60°
Thus $\angle$ CBP = 120°

Now in triangle **PBC,** $\angle$ CPB = 60° - 30° = 30° and $\angle$ CBP = 120°
Thus $\angle$ BCP = 30°, i.e., this is an isosceles triangle. Using the properties of isosceles triangles we can conclude that BP = BC = 50 m

Step 4: Calculate AB and AP

AB (the height of the hill) = 50 Sin 30° = 25 m
AP = 50 Cos 30° = 43.30 m

2) On ground level, the angle of elevation of the top of a tower is 45°. Moving 20 m closer to the tower, the angle of elevation is 60°. What is the height of the tower?



Step 1: AB denotes the height of the tower.

P is the position at which the angle of elevation of the top of the tower is 45°.

Q is the position at which the angle of elevation of the top of the tower is 60°.

Given PQ = 20 meters

Step 2: Let AB = x meters and AQ = y meters

Step 3: In the right triangle APB,

$\text{Tan } 45° = \frac{AB}{AP}$ $\text{Tan } 45° = \frac{x}{AQ + PQ}$

$\text{Tan } 45° = \frac{x}{y + 20}$

$y + 20 = x$

$y = x - 20$ -------- (1)

In the right triangle AQB,

$\text{Tan } 60° = \frac{AB}{AQ}$ $\sqrt{3} = \frac{x}{y}$

$x = y\sqrt{3}$ ---------(2)

Step 4: Substitute y = x – 20 in Equation (2)

$x = (x-20)\sqrt{3}$ $x = x\sqrt{3} - 20\sqrt{3}$

$20\sqrt{3} = x\sqrt{3} - x$

$x = 47.321m$

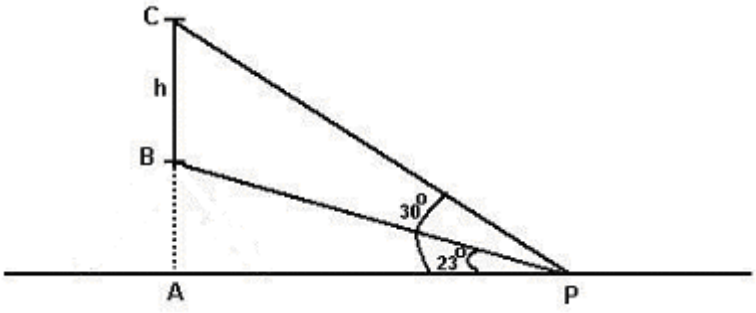
Therefore the height of the tower is 47.321m

3) A tower of height 'h' stands on the top of a cliff. An observer in a boat finds that

i) The angle of elevation to the top of the tower is 30° and the

ii) Angle of elevation of the top of the cliff is 23°.

Find the height of the cliff and the distance of the boat from the foot of the cliff.



Step 1: AB denotes the height of the cliff and BC denotes the height of the tower.
P denotes the boat. Given BC = h meters
$\angle$ APC = 30° and $\angle$APB = 23°

Step 2: Let AP = x meters and AB = y meters

Step 3: In right triangle BAP,

$$\text{Tan } 23° = \frac{AB}{AP} \qquad \text{Tan } 23° = \frac{y}{x}$$

$$y = x \text{ Tan } 23° \qquad x = \frac{y}{\text{Tan } 23°}$$

$$= y \text{ Cot } 23°$$

$$= y \text{ Tan } 67° \quad \text{[Since 23° and 67° are complementary angles]}$$

In the right triangle CAP,

$$\text{Tan } 30° = \frac{AC}{AP} \qquad \frac{1}{\sqrt{3}} = \frac{AB+BC}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{y+h}{x} \qquad x = \sqrt{3}(y+h) \quad \text{----------(2)}$$

Step 4: Substitute x = y Tan 67^0 in Equation (2)

$$y\text{Tan}67^\circ = \sqrt{3}(y+h)$$
$$y\text{Tan}67^\circ - \sqrt{3}y = \sqrt{3}h$$
$$y(\text{Tan}67^\circ - \sqrt{3}) = \sqrt{3}h$$

$$y = \frac{\sqrt{3}\,h}{\text{Tan } 67° - \sqrt{3}} \qquad \frac{1.732h}{2.3569 - 1.732}$$

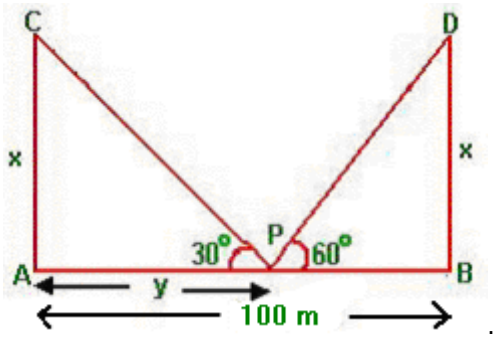
$$= \frac{1.732}{0.624}h \qquad = 2.775 \text{ h}$$

Then x = y Tan 67^0

= (2.775) (2.356) h **= 6.539 h**

4) Two pillars of equal height stand on either side of a roadway, which is 100 metres wide. At a point on the roadway between the pillars, the angles of elevation of the tops of the pillars are 60° and 30°.

Find the height of each pillar and define the position of the point on the roadway



Step 1: AC and BD are two pillars of equal height say, 'x' metres each.

AB = 100 metres, the distance between the two pillars.

Let P be situated on AB such that

AP = y, and PB = 100 – y and the angles $\angle$APC = 30° and $\angle$DPB = 60°

[Note: The angle of elevation is greater at a point of observation closer to an object than at a point of observation farther from it]

Step 2: In Δ APC,

$\tan 30° = \frac{x}{y} \Rightarrow x = y \tan 30°$

Also $\frac{x}{100 - y} = \tan 60°$

$$\Rightarrow \frac{y \tan 30^\circ}{100 - y} = \tan 60^\circ$$

$y \tan 30° = (100-y)\tan 60°$

$\frac{y}{\sqrt{3}} = 100\sqrt{3} - \sqrt{3}y$

$\frac{y}{\sqrt{3}} - \sqrt{3}y = 100\sqrt{3}$

$2.3094y = 100\sqrt{3}$

y = 75 metres and
x = 75/√3
= 43.3 metres