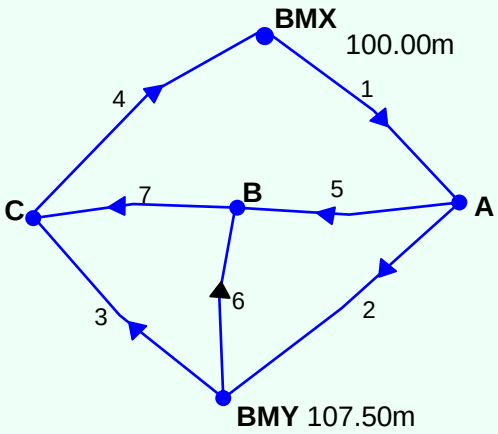
MATHEMATICS
Level Three

Least Squares

Consider a specific example of a survey level circuit, the sketch and survey data for which are shown below. In the problem it will be assumed that each observation is of equal weight (the concept of observation relative weight shall be considered later.) The arrow indicates the direction the observation was made, Thus BMX to A is line one and the observed difference in height is 5.10m (i.e. A is 5.10m higher than BMX).



Line	Diff in elevation
1	+5.10
2	+2.34
3	-1.25
4	-6.13
5	-0.68
6	-3.00
7	+1.70

Measurement equations (one for each measurement) are written to obtain the best estimate for the unknown elevations A, B and C. These equations include the random error associated with each measurement and, where applicable, the given elevations.

1) Write the seven equations needed to determine the heights of A, B and C. The first equation has been written for you to give you a start

   $A = BMX + 5.10 + v_1$ *where v allows for the error in the measurement*

2) Calculate and rearrange your seven equations so that only A, B, C and v_1 to v_7 appear in the equations. Ensure that the v^2 is on the left hand side. For example:

   $V_1^2 = (A-105.10)^2$ (for example)

The function F(v) is now written which expresses the least squares condition, i.e.,

$$F(v) = V_1^2 + V_2^2 + V_3^2 + \text{-------} + V_7^2 = min$$

3) Substitute the equations calculated in question two into the above equation

Taking the partial derivative of the above function with respect to each unknown (A, B C) and setting the results equal to zero there should be three equations, one for each unknown.

4) Write the three equations, $\frac{\partial f}{\partial A}, \frac{\partial f}{\partial B}, \frac{\partial f}{\partial C}$ and set them equal to zero.

5) Simplify these equations and solve them for A, B and C. The result will be the least squares estimate for the elevation of these three points.