School *of* Business
Te Kura Pakihi

ISSN 1178-2293 (Online)

**University of Otago
Economics Discussion Papers
No. 1002**

May 2010

Capital Accumulation, Non-traded Goods and International Macroeconomic Dynamics with Heterogeneous Firms*

Daniel Farhat

Address for correspondence:

Daniel Farhat
Department of Economics
University of Otago
P O Box 56
Dunedin 9054
NEW ZEALAND
E-mail: dan.farhat@otago.ac.nz
Telephone: +(64) 3 479-8645
Fax: +(64) 3 479-8174

*Special thanks to Jang-Ting Guo, Roger Farmer, R. Robert Russell, Richard Suen, Victoria Umanskaya for their invaluable comments. All errors are my own. For additional comments or suggestions, I can be reached at dan.farhat@otago.ac.nz

Abstract

This paper examines international business cycle transmission within a two-country dynamic stochastic general equilibrium model featuring an endogenously determined trade pattern. In contrast to existing literature, this model distinguishes between non-traded final goods and traded inputs. The model incorporates capital into the production of final goods and shows that shocks to final goods production are important in replicating the empirical regularities of imports, exports, the real exchange rate and their relationship to GDP. Endogenously determined labour supply and high asset market frictions are incorporated into the model to improve the model's ability to replicate labour market statistics and international co-movement.
Keywords: International Real Business Cycles, Non-traded Final Goods, Imperfect Competition, International Trade.
JEL: F12, F21, F41.

1 Introduction

Many would agree that trade and financial linkages are important to the transmission of business cycles across countries. However, past international real business cycle (IRBC) models featuring trade in goods and assets have had difficulty generating realistic international co-movement[1]. Poor international co-movement tends to result from the free flow of assets across countries in particular. When one country experiences a positive productivity shock, asset trade leads to an inflow of productive capital from the rest of the world, strengthening the economic expansion at home but inducing recessions abroad. These models[2], which extend micro-founded business cycle frameworks to include international trade while taking the trade pattern as exogenously given, require asset market rigidities to produce appropriate business cycle transmission.

An emerging class[3] of IRBC models do just the opposite: adapt new[4] models of international trade to measure macroeconomic dynamics. In these frameworks, the trade pattern is endogenously determined and fluctuates with the business cycle. One such model, proposed by Ghironi and Melitz (2005) [GM], is capable of replicating a variety of empirical regularities, including deviations in purchasing power parity, persistent trade deficits, the Harrod-Balassa-Samuelson[5] effect and an environment in which only the most productive firms become exporters[6]. Calibrated results suggest that changes in the trade pattern over time enhance business cycle persistence and induce international co-movement. While the model produces an array of desirable results, it oversimplifies the structure of production by omitting capital.

When capital is omitted from the framework, "investment" is defined only as new firm construction. Investment in the data, however, includes plant and equipment purchases for both new firm creation and existing firm expansion. Without capital, comparing "investment" in the model and investment in the data is erroneous. In this paper, I will include capital accumulation into framework proposed by Ghironi and Melitz (2005) in such as way as to leave the underlying mechanism of trade undisturbed. By including capital in the production of non-traded final goods, I can improve the definition of "investment" as well as explore an alternative source of business cycle fluctuations: productivity shocks to final goods production[7].

I show that the model with capital can generate more realistic relationships between imports, exports, and GDP when business cycles are driven by the same process as in the original GM framework: shocks to the production of intermediate goods. The inclusion of capital improves the model's fit for these international variables, but does so at the expense of international co-movement[8]. When there is a positive productivity shock, changes in the capital stock prolong economic expansions and strengthen the flow of investment from abroad. Further, an increase in capital prices leads to an increase in the domestic price level and a fall in the exchange rate. The inflow of investment funds from abroad coupled with declines in the exchange rate worsen international correlations. This effect persists even when asset market frictions are present. Productivity shocks to intermediate goods production alone are not enough to replicate business cycle co-movement.

When shocks to the production of final goods are present, the model can produce positive correlation of output, consumption and investment. Further, the model can more closely match the data by generating increased volatility for imports, exports and the real exchange rate since shocks to final goods production directly affect the demand for intermediate inputs. Fluctuations in the exchange rate along with changes in the trade pattern in response to shocks to final goods production generate these improved results. Although helpful, productivity shocks to final goods production alone are not sufficient to generate high output volatility. A model with shocks to both intermediate and final goods production is proposed. The improved results suggest both sources of business cycle fluctuations are necessary to produce international co-movement adequately.

Because the original GM framework assumes inelastic labour supply, I further extend the model with capital by adding a developed labour market. As in Farhat (2009), output volatility more closely matches the data for relatively high elasticities of labour supply. In the presence of shocks to final goods production, however, employment becomes negatively correlated across countries. With lower

[1] As measured by international correlations of output, consumption, investment and employment.
[2] Mendoza (1991) and Backus, Kehoe, and Kydland (1992, 1994) are seminal in this strand of literature.
[3] Examples of models in this class include Head (2002); Cook (2002) and Ghironi and Melitz (2005).
[4] Such as Helpman and Krugman (1987).
[5] The Harrod-Balassa-Samuelson effect states that prices are generally higher in wealthier (or more productive) countries.
[6] Which is consistent with firm-level data.
[7] Shocks to final goods production behave as 'demand shocks' for intermediate goods in the model.
[8] As in the older strand of IRBC literature, trade in assets leads to capital inflow, inducing negative business cycle correlations between countries.

labour supply elasticities[9], output volatility is similar to the benchmark framework without capital and positive employment correlation is achieved.

In summary, including capital in the production of final goods induces improved relationships between imports, exports and GDP when business cycles are driven by shocks to intermediate goods production, but results in negative business cycle correlation. Shocks to final goods production, which act as demand shocks for intermediate inputs, is capable of generating positive international correlation of consumption, output and investment along with increased volatility of the international market variables. Adding fluctuations to employment increase output volatility, but may result in negative correlation of employment if labour supply elasticity assumed to be high.

The remainder of the chapter is as follows. I outline recent literature on international business cycle transmission and the set of stylized facts the model intends to reproduce. The model is then described and a series of numerical experiments are presented. I end with a short conclusion.

2 Literature Review and Stylized Facts

2.1 Literature Review

Starting in the mid-1980's, researchers such as Dellas (1986); Stockman and Svensson (1987) and Cantor and Mark (1988) began extending the newly developed real business cycle models of Kydland and Prescott (1982) and Long and Plosser (1983) to multiple countries (coined *International Real Business Cycle models* or *IRBC models*). The main empirical findings of these early studies suggested that the main sources of business cycle transmission were likely correlated productivity shocks[10], trade linkages (in inputs[11]) and international sales of assets[12]. Backus et al. (1992) and Mendoza (1991) applied calibration techniques to two-country DSGE models featuring complete financial markets (which facilitate international risk sharing), correlated productivity shocks, frictionless trade and capital adjustment costs to find how accurately a simulated model can reflect the real economy. Unfortunately their answer was *not much.*

Many inconsistencies between the simulated results and the real economy were derived by Backus, Kehoe and Kydland and persist in the face of several experiments. Unlike the real economy, simulation results often suggest consumption among countries has high correlation while output has negative or low correlation (dubbed the "consumption-output" or C-O anomaly). This result occurs from using financial markets to pool risk[13]. To a somewhat lesser extent, international correlations between investment, savings and employment were also difficult the match. Resolving the C-O anomaly, however, became the most sought after target for the next wave of research.

Continual advancements in the research program were made as economists tried to resolve the mystery. Some researchers attempt to augment the way financial markets are modeled to achieve better results. Baxter and Crucini (1995) and Kollmann (1996) include incomplete asset markets to try to retard the effect of risk pooling. Heathcote and Perri (2002) and Kose and Yi (2001, 2002, 2006) close financial markets completely and compare the results to those that occur in complete markets. Kehoe and Perri (2002) look not at exogenously constrained financial markets, but endogenously constrained financial markets by including imperfectly enforceable lending agreements. Olivero (2004) models non-competitive lenders which reduces the effects of risk-pooling via counter-cyclical mark-ups for loans. Some notion of asset market friction seems necessary in this class of models to prevent capital exodus and improve output correlation across countries. Other researchers have found, however, that changing non-financial components can improve international co-movement as well. Stockman and Tesar (1995) introduce a non-traded goods sector to allow for high correlation of traded goods, but not necessarily for non-traded goods.

The source of business cycle fluctuations is also important in matching international data with an IRBC model. Stockman and Tesar (1995) and Wen (2002) illustrate the importance of demand-side shocks to international co-movement by incorporating persistent demand shocks into an IRBC framework. The intuition is that business cycles driven by technology shocks, assuming perfect risk pooling, induce capital exodus out of less productive countries (lowering output correlation across countries) while demand shocks force the international market (trade in goods) to work as a medium for positive

[9] Fiorito and Zanella (2008) note that labour supply elasticities are empirically lower at the individual level than at the aggregate level.
[10] Dellas (1986).
[11] Dellas (1986); Canova and Dellas (1993).
[12] Cantor and Mark (1988); Stockman and Svensson (1987).
[13] Countries that receive positive productivity shocks also receive a flood of investment from the rest of the world. This leads to 'recessions' in other countries and capital is transmitted to the most productive nations via asset markets.

business cyclical synchronization[14] (improving output correlation across countries). However, when demand shocks are not persistent, investment is crowded out to increase consumption temporarily in the affected country, resulting in low or negative investment correlation across countries.

Although IRBC models featuring demand shocks and asset market frictions can replicate some of the empirical regularities of international co-movement, the vast majority of these models fail to explicitly focus on the functioning of the international market in transmitting business cycles. Focus on the dynamic properties imports, exports, the terms of trade (TOT) and the real exchange rate is explicitly done within a somewhat separate but related strand of research. Backus et al. (1994); Mendoza (1995); Zimmermann (1999) and Cuñat and Maffezzoli (2004) focus on constructing IRBC models that can replicate the data associated with these variables, but generally reach the same conclusions as the models that focus predominately on international co-movement: many refinements are required to the model to replicate the full gambit of stylized facts.

The source of the poor fit of IRBC models in general may be related to the 'microfoundations' that the models are build upon. All of the models described so far apply micro-founded macro models to the open economy. The evolution and determination of the trade pattern has been exogenous. A small but separate sect of IRBC literature has developed within the rubric of 'international trade', in which micro-founded trade models are adapted for macroeconomic analysis. These models, which are structured to reflect modern perceptions of international trade[15], endogenize firm entry then allow firms to choose whether or not to export. The trade pattern is thus endogenous and sources of business cycle transmission are twofold: through new firm construction and through changes in export decisions. Head (2002); Cook (2002); Alessandria and Choi (2004) and Ghironi and Melitz (2005) incorporate endogenous firm entry and export decisions in monopolistically competitive input markets and succeeds in reducing the C-O anomaly as well matching many features of international data[16].

Of these new models, the Ghironi and Melitz (2005) has shown to be particularly effective in studying international co-movement. The original Ghironi and Melitz model, however, is still quite underdeveloped when compared to the other branch of IRBC research and resent developments in closed-economy real business cycles. Labor is supplied inelastically to simplify the models already complex structure. The drawback of this feature is a lack of employment dynamics in the model. Since labor is used in the production of traded goods, to construct new firms and to pay export costs, changes in employment can dramatically affect the trade pattern. Further, the model distorts the definition of "investment" typically used in real business cycle models by omitting capital accumulation. Since investment exodus has been shown to be a central cause of low output correlation across countries, and since the presence of capital can prolong the effects of business cycle movements, incorporating capital into the model is certainly not a trivial exercise. This paper attempts to relax these structural assumptions and include a notion of demand shocks and asset market frictions, which have already been shown to be useful in replicating the stylized facts seen in the data. In the process, particular attention will be paid to the data associated with the medium of business cycle transmission: the international market.

2.2 Stylized Facts

When evaluating the success of an international real business cycle model, many researchers refer to the econometric analysis performed by Backus et al. (1992). This analysis looks at quarterly data from 1954q1 to 1983q7 for domestic (U.S.) business cycle statistics and transformed[17] quarterly data from 1970q1 to 1986q4 for international[18] business cycle correlations. Stock and Watson (2005) note that both domestic business cycle volatility and the correlation of business cycle movements between the U.S. and several European countries have changed since the 1980s. U.S. business cycles have become more moderate and less synchronized with Europe[19]. Because nearly 20 years of additional data are now available and the interrela-

[14]Building on this idea, researchers have also explored the value of indeterminacy and animal spirits in IRBC models (Guo and Sturzenegger (1998); Xaio (2002, 2003, 2004); Fukuda (2004)). These models are currently at the forefront of the IRBC literature.

[15]Specifically, 'new trade theory' as developed by Helpman and Krugman (1987). These trade models feature non-competitive markets incorporating intra-industry trade.

[16]such as persistent deviations in purchasing power parity, persistent trade deficits, the Harrod-Balassa-Samuelson effect and the Harberger-Larsen-Metzler effect

[17]Deflated quarterly data is transformed into comparable terms by comparing the 1985 annual average to the Penn World Table 1985 value.

[18]Business cycle correlations are between the U.S. and a "European Aggregate" which consists of Austria, Finland, France, Germany, Italy, Switzerland and the United Kingdom.

[19]According to Stock and Watson (2005), this is due to the reduction in "common shocks", i.e. oil price shocks and commodity price shocks

tionships between business cycle fluctuations have potentially changed, it is worthwhile to generate updated stylized facts before testing the fit of my model.

I organize the data on international co-movement into three categories: domestic business cycle statistics, international correlations and international market statistics. Table 1, which is available in the appendix, reports domestic business cycle statistics for the United States, 1957q1-2007q1. As is commonly found, consumption, investment and employment are pro-cyclical. Investment is 3.41 times more volatile than GDP, while both consumption and employment are less volatile than GDP (0.75 and 0.61 times as volatile respectively).

Table 2, also available in the appendix, reports international business cycle co-movement between the U.S. and Europe, 1970-2004[20]. The degree of business cycle co-movement, however, proves to be time-dependant. In the original analysis, Backus, Kehoe and Kydland find that output is more correlated across countries than consumption (with correlation coefficients 0.70 and 0.46 respectively). Many researchers have difficulty constructing a model that can replicate this relationship (the C-O anomaly). Table 2 confirms that this relationship is much weaker, as predicted by Stock and Watson (2005), when recent data is considered. Figure 1 shows how the correlation coefficients described in table 2 change over time by looking at 5-year and 10-year rolling window estimates. When looking at the 10-year rolling window for the Backus, Kehoe and Kydland sample (1970 - 1985), output correlation is significantly higher than consumption correlation. However, consumption is more correlated than output in later years (1985 - 2004). In general, we can say that output correlation and consumption correlation are "close" and generally positive.

A rich set of statistics pertaining to the international market are also available in the appendix. These measures include imports (IM), exports (EX), the trade balance as a fraction of output (Net Exports/Output = NX/Y) the terms of trade (TOT, measured as the ratio of import prices to export prices) and the real exchange rate index (Q) and are reported for the U.S. in table 3. The data suggests high volatility of quantities (imports and exports which are 3.31 and 3.74 times more volatile than GDP) as well as prices (the terms of trade and the real exchange rate, which are 1.69 and 3.37 times more volatile than GDP). Exports and imports are both pro-cyclical, net exports are counter-cyclical and prices are generally a-cyclical. Exports tend to be more volatile than imports, and less correlated with output. This set of data will be used to test how well the model captures the behavior of trade linkages, an important source of business cycle transmission across countries, over the business cycle.

3 The Model

The theoretical model is heavily influenced by Ghironi and Melitz (2005). I will briefly outline the parts of the original GM framework that I augment when adding capital accumulation[21]. The model considers two countries, home and foreign (which is denoted by *). Both countries are large and assumed to be structurally identical. That in mind, I construct the framework for one country (the home country) knowing that a symmetric framework exists for the foreign counterpart.

3.1 The Consumer's Problem

Denote the universe of intermediate input varieties as Ω. At any time, t, a subset $\Omega_t \subseteq \Omega$ are actually produced by firms. Ω_t contains both domestically produced and imported goods. Further, it need not be the case that $\Omega_t = \Omega_s$ for $t \neq s$ nor $\Omega_t = \Omega_t^*$. A composite intermediate good, M_t, is produced using Dixit-Stiglitz technology:

$$M_t = \left(\int_{\omega \in \Omega_t} m_t(\omega)^{(a-1)/a} d\omega \right)^{a/(a-1)}$$

where $m_t(\omega)$ denotes the quantity of variety ω used in the production of the composite, and a denotes the elasticity of substitution across varieties. The composite intermediate good is then combined with capital, K_t, to produce consumption using a simple Cobb-Douglas production process:

$$C_t = A_t M_t^b K_t^{1-b}$$

A_t denotes total factor productivity at time, t. At $A_t = b = 1$, the model reverts to the structure described by Ghironi and Melitz (2005).

It is straightforward to construct demand equations and price indices for m_t, M_t, K_t and C_t. Denote P_{Xt} as the price index for good X. I construct the following conditional demand equations by solving a straight-forward cost-minimization problem[22]:

$$\begin{aligned} m_t(\omega) &= M_t(P_t(\omega)/P_{Mt})^{-a} \\ M_t &= \frac{C_t}{A_t}\left(\frac{P_{Mt}}{P_{Kt}}\frac{1-b}{b}\right)^{b-1} \\ K_t &= \frac{C_t}{A_t}\left(\frac{P_{Mt}}{P_{Kt}}\frac{1-b}{b}\right)^{b} \end{aligned}$$

[20]Unlike Backus et al. (1992), the annual data from the Penn World Tables are considered directly.

[21]A detailed work-up of the model is available upon request.

[22]The production function for C_t is essentially a multi-step CES production function. First, I find the lowest-cost way to produce one unit of the intermediate index, $M_t^{'}$. Then, I solve a cost-minimization problem for C_t over $M_t^{'}$ and K_t.

I also construct the associated price indices:

$$P_{Mt} = \left(\int_{\omega \in \Omega_{Dt}} P_t(\omega)^{1-a} d\omega \right)^{1/(1-a)}$$

$$P_{Ct} = \frac{P_{Mt}^b P_{Kt}^{1-b}}{A_t(1-b)^{1-b} b^b}$$

Next, I turn to the household's problem faced by the representative agent. It is assumed that there exists a representative household that chooses to work, consume and save. The household is endowed with 1 unit of time that it can divide between labour, l_t, and leisure. labour earns the real wage, w_t. The household can choose to save by purchasing bonds, B_{t+1}, which each cost one unit of consumption, but yield $(1 + r_{t+1})$ units in the next period. Both domestic and foreign bonds (B_*) are available to the household and are subject to a quadratic transaction cost which is rebated, lump-sum (Γ_t), to the household. The household can also purchase shares of a mutual fund, x_{t+1}, which entitle the owner to a fraction of the profits of the producing firms. Arbitrage prices the shares at the firms expected discounted value ($N_{ht}\tilde{v}_t$ for new shares, $N_{dt}\tilde{v}_t$ for old shares). Households are obligated to hold bonds and stocks for only one period before they are resold in asset markets. Finally, the households have the option of saving real wealth in the form of savings accounts, S_{t+1}. Savings are used to construct capital in the next period and earn a real return of R_t[23]. The consumer's real period budget constraint is then given by:

$$\begin{aligned} &C_t + B_{t+1} + Q_t B_{*t+1} + \tfrac{n_1}{2}(B_{t+1}^2) + \\ &\tfrac{n_2}{2}(Q_t B_{*t+1}^2) + (N_{ht}\tilde{v}_t)x_{t+1} + S_{t+1} \\ &= \\ &w_t l_t + (1 + r_t)B_t + Q_t(1 + r_t^*)B_{*t} + \Gamma_t \\ &+ x_t(N_{dt}\tilde{v}_t + N_{dt}\tilde{d}_t) + R_t S_t \end{aligned} \tag{1}$$

where $n_2 \geq n_1 > 0$ are scale parameters on foreign and domestic bond adjustment costs, $\Gamma_t = \frac{n_1}{2}(B_{t+1}^2) + \frac{n_2}{2}(Q_t B_{*t+1}^2)$ in equilibrium and Q_t is the real exchange rate.

The instantaneous utility function for the agent is assumed to have the form:

$$U_t(C_t, l_t) = \log C_t - H \frac{l_t^{1+\lambda}}{1+\lambda}$$

The agent's maximization problem is then to choose a sequence:

$$\{C_s, l_s, B_{s+1}, B_{*s+1}, x_{s+1}, S_{s+1}\}_{s=t}^{\infty}$$

to maximize the discounted sum of expected lifetime utility, $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_t(C_t, l_t)$ subject to the budget constraint in equation (1).The first-order conditions for the consumer's problem generate three Euler equations and an equation that guides labour supply when $H \neq 0$. The Euler equation for domestic bonds is:

$$C_t^{-1}(1 + n_1 B_{t+1}) = (1 + r_{t+1})\beta E_t C_{t+1}^{-1}$$

Similarly, the Euler equation for foreign bonds is:

$$Q_t C_t^{-1}(1 + n_2 B_{*t+1}) = (1 + r_{t+1}^*)\beta E_t Q_{t+1} C_{t+1}^{-1}$$

The Euler equation for stocks is:

$$\tilde{v}_t = \beta(1-\delta)E_t(\frac{C_{t+1}}{C_t})^{-1}(\tilde{d}_{t+1} + \tilde{v}_{t+1})$$

Notice that repetitive forward substitution of the Euler equation for stocks generates the expected value of the firm's lifetime profit stream. Finally, the Euler equation for savings is:

$$C_t^{-1} = \beta E_t R_{t+1} C_{t+1}^{-1}$$

In the original Ghironi and Melitz (2005) framework, labour was supplied inelastically ($H = 0$, therefore $l_t = 1$).When endogenous labour supply is considered, $H = 1$ and the condition which guides labour supply is:

$$l_t = C_t^{-1/\lambda} w_t^{1/\lambda}$$

The parameter λ represents the elasticity of labour supply.

3.2 The Firm's Problem

I now consider the problem faced by a typical intermediate good producer in the home country. It is assumed that there exists an unbounded mass of firms in the economy that may begin production at any time. These firms are monopolistically competitive producers of unique varieties who produce for domestic and foreign markets separately [24]. Given the demand for their products derived in the previous section, the firm's:

Step 1: Decide whether or not to enter.
Step 2: Upon entry, choose how much output to produce for each market and what prices to set.
Step 3: Die or exit at the end of the period.

In practice, firms will first derive the solutions to step 2 and step 3 and form estimates on their future profit streams before choosing whether or not it's worth it to enter the industry. Prior to entry, the firm calculates average (expected) profit. Each firm j in the home country produces output according to the production function:

$$y_{jt} = Z_t z_j L_{jt}$$

[23] All savings instruments are denoted with the time subscript in which they yield a return.
[24] Markets are segmented – firm's practice third-degree price discrimination.

where y_{jt} denotes the quantity of output firm j produces, Z_t denotes an economy-wide technology variable, z_j is a productivity draw specific to firm j and $L_{jt}$is the quantity of labour firm j hires in the production of output. The firm will choose how much to produce for the domestic market (m_{djt}) and how much to produce for export ($m_{xjt} = m^*_{mjt}$). Exports are subject to an iceberg cost, $\tau \geq 1$. Thus, total output for the firm is given by $y_{jt} = m_{djt} + \tau m_{xjt}$.

The firm's expenses include a wage bill, a start-up cost paid in terms of labour, and an exporter fee (also paid in terms of labour). The wage bill is simply $w_t L_{jt}$. Start-up costs are a fixed cost paid once during the period the firm begins production. The "production" of entry is given by$F_{Et} = Z_t L_{Ejt}$ which suggests a total fixed cost of $w_t F_{Et}/Z_t$. Exporter fees are paid each period the firm chooses to export ($m_{xjt} > 0$). The production of the exporter fee is given as $F_{Xt} = Z_t L_{Xjt}$. Thus, the per-period exporter cost is $w_t F_{Xt}/Z_t$. Dropping the start-up cost, the firm's real per-period profit maximization problem is to choose domestic and exported output, c_{xjt}, c_{djt} to maximize profits, $\Pi_{jt} P_{Ct} = \frac{P_{djt}}{P_{Ct}} m_{djt} + \epsilon_t \frac{P_{xjt}}{P_{Ct}} m_{xjt} - \frac{w_t}{Z_t z_j}(m_{djt} + \tau m_{xjt}) - I_{xt} \frac{w_t F_{Xt}}{Z_t}$,where $I_{xt} = 1$ if $c_{xjt} > 0$, else $I_{xt} = 0$. ϵ_t denotes the nominal exchange rate. It is important to note that P_{xjt} is measured in terms of foreign currency. For any variable, X, we denote *real* prices as $\rho_X = P_X/P_C$ and substitute the demand functions from the previous section into the firm's problem (as is standard with monopolistic competition) to generate the first order conditions for the firm[25]:

$$m_{djt} = bC_t \rho_{Mt}^{a-1} \left(\frac{a}{a-1} \frac{w_t}{Z_t z_j}\right)^{-a}$$
$$m_{xjt} = bC_t^* \rho_{Mt}^{*a-1} \left(\frac{a}{a-1} \frac{w_t}{Z_t z_j} \frac{\tau}{Q_t}\right)^{-a}$$

where $Q_t = \epsilon_t P^*_{Ct}/P_{Ct}$ is the real exchange rate. For simplicity, it is assumed that $\epsilon_t = 1$. We then use the demand equations to find the optimal real prices charged by the firm:

$$\rho_{djt} = \frac{a}{a-1} \frac{w_t}{Z_t z_j}$$
$$\rho_{xjt} = \frac{a}{a-1} \frac{w_t}{Z_t z_j} \frac{\tau}{Q_t}$$

The prices suggest a constant markup over marginal cost ($w_t/Z_t z_j$).

Profits for the firm can be divided into "domestic production profits" and "export profits". They are given as:

$$d_{djt} = \frac{b}{a} C_t \rho_{Mt}^{a-1} \rho_{djt}^{1-a}$$
$$d_{xjt} = \begin{Bmatrix} \frac{b}{a} C_t^* \rho_{Mt}^{*a-1} Q_t \rho_{xjt}^{1-a} - \frac{w_t F_{Xt}}{Z_t} \text{ if } c_{xjt} > 0 \\ 0 \text{ if } c_{xjt} = 0 \end{Bmatrix}$$

For a continuum of potential firms, there exists some productivity draw, z_{xt}, such that the firm which receives that draw upon entering at time t earns zero profits whether it exports or not (the exporter cutoff). For this firm, $\frac{b}{a} C_t^* \rho_{Mt}^{*a-1} Q_t \rho_{xjt}^{1-a} = \frac{w_t F_{Xt}}{Z_t}$.

Knowing the profits for any given productivity draw, potential firms decide whether or not to enter. Their productivity draw is not assured prior to entry, so firms form an expectation of their profits (they calculate an "average" profit from entry). To do this, we employ the "special averages" developed by Melitz (2003). First, we suppose that the productivity draws follow a Pareto distribution with CDF $G(z) = 1 - (z_{\min}/z_j)^k$ and PDF $g(z) = k z_{\min}^k z_j^{-k-1}$ where k denotes the shape parameter of the distribution. Next, define:

$$\tilde{z}_d^{1-a} = \int_{z_{\min}}^{\infty} z_j^{1-a} g(z) dz$$
$$\tilde{z}_{xt}^{1-a} = \int_{z_{xt}}^{\infty} z_j^{1-a} \frac{g(z)}{1-G(z_{xt})} dz$$

Finally, substitute these definitions in constructing average prices: $\tilde{\rho}_{dt}^{1-a} = \int_{z_{\min}}^{\infty} \rho_{djt}^{1-a} g(z) dz = (\frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_d})^{1-a}$ and $\tilde{\rho}_{xt}^{1-a} = \int_{z_{xt}}^{\infty} \rho_{xjt}^{1-a} \frac{g(z)}{1-G(z_{xt})} dz = (\frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_{xt}} \frac{\tau}{Q_t})^{1-a}$. We define N_{dt} as the number of firms that produce for the domestic market and N_{xt} as the number of firms that produce for export. The model is recast in terms of average prices:

$$\tilde{\rho}_{dt} = \left(\frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_d}\right)$$
$$\tilde{\rho}_{xt} = \left(\frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_{xt}} \frac{\tau}{Q_t}\right)$$
$$\tilde{\rho}_{Mt} = (N_{dt} \tilde{\rho}_{dt}^{1-b} + N_{xt}^* \tilde{\rho}_{xt}^{*1-b})^{1/(1-b)}$$
$$\tilde{d}_{dt} = \frac{b}{a} C_t \tilde{\rho}_{Mt}^{a-1} \tilde{\rho}_{dt}^{1-a}$$
$$\tilde{d}_{xt} = \frac{b}{a} C_t^* \tilde{\rho}_{Mt}^{*a-1} Q_t \tilde{\rho}_{xt}^{1-a} - \frac{w_t F_{Xt}}{Z_t}$$

Completing the integral for the special productivity averages suggests that $\tilde{z}_D = (\frac{k}{k+1-a})^{1/(a-1)} z_{\min}$ and $\tilde{z}_{xt} = (\frac{k}{k+1-a})^{1/(a-1)} z_{xt}$ where $k > a-1$ for boundedness. Knowing that all existing firms produce for the domestic market, and a fraction of those become exporters, the expected per-period profit for a potential firm, on average, is:

$$\tilde{d}_t = \tilde{d}_{dt} + (1 - G(z_{xt})) \tilde{d}_{xt}$$

There are two important features guiding firm creation and destruction. The first is a lag in production. A firm that enters in period t starts producing at period $t+1$. The entering firm, however, is still *counted* as a firm in period t. The total number of firms that exist at period t, N_{ht}, is given by the number of producing firms that already exist, N_{dt}, plus the number of new firms, N_{et}. Second, firms are subject to an exogenous exit shock. The number of firms that "survive" to produce in period $t+1$ is given by $N_{dt+1} = (1-\delta) N_{ht} = (1-\delta)(N_{dt} + N_{et})$.

[25]It is assumed that firms do not observe the impact of price setting on the aggregate price level when solving the firm's maximization problem. Also, note that the definition of P_{Ct} implies $M_t = bC_t \rho_{Mt}^{-1}$.

Therefore, the expected value of the firm's lifetime profit stream, $\tilde{v}_t$, is given by:

$$\tilde{v}_t = E_t \sum_{s=t+1}^{\infty} [1-\delta]^{s-t} \left[\beta^{s-t}(\frac{C_s}{C_t})^{-1}\right] \tilde{d}_s$$

where $\left[\beta^{s-t}(\frac{C_s}{C_t})^{-1}\right]$ is the stochastic discount factor to be discussed shortly and $(1-\delta)$ is the firm's survival probability. Firms continue to enter as long as the discounted value of their profit stream exceeds the cost of entry. Therefore, the entry cutoff is determined by $\tilde{v}_t = w_t F_{Et}/Z_t$.

Finally, there are two important characteristics of the firm's problem with capital accumulation that are of note. First, the indexed price of intermediate goods (which equals the aggregate price of consumption in the model without capital, but not in the model with capital) affects the equilibrium quantity produced by any individual firm. Recall:

$$m_{djt} = bC_t\rho_{Mt}^{a-1}(\tfrac{a}{a-1}\tfrac{w_t}{Z_t z_j})^{-a}$$
$$m_{xjt} = bC_t^*\rho_{Mt}^{*a-1}(\tfrac{a}{a-1}\tfrac{w_t}{Z_t z_j}\tfrac{\tau}{Q_t})^{-a}$$

Second, the domestic and export profits are strongly related to the level of consumption. Recall:

$$d_{djt} = \tfrac{b}{a}C_t\rho_{Mt}^{a-1}\rho_{djt}^{1-a}$$
$$d_{xjt} = \left\{ \begin{array}{c} \tfrac{b}{a}C_t^*\rho_{Mt}^{*a-1}Q_t\rho_{xjt}^{1-a} - \tfrac{w_t F_{Xt}}{Z_t} \text{ if } c_{xjt} > 0 \\ 0 \text{ if } c_{xjt} = 0 \end{array} \right\}$$

Increasing the volatility of consumption or the volatility of the aggregate price of labour services will result in increased volatility of firm profits and thus more volatility in the decision to export. Changes in export decisions affect the size of the export sectors as well as the trade pattern, which influences the transmission of business cycle fluctuations across countries.

3.3 Market Clearing and Variable Definitions

The market clearing conditions and definitions needed to solve the model are available in the appendix. The central differences from the original Ghironi and Melitz (2005) model when capital accumulation is added to the framework are the existence of a market for capital, the redefinition of investment and the redefinition of the aggregate price index.

In the capital market, savings today become the foundation for tomorrow's capital stock ($S_{t+1} = K_{t+1}$). In equilibrium, the gross return to savings ($R_t S_t = R_t K_t$) should be equal to total payments to capital plus the return of non-depreciated capital ($\rho_{Kt}K_t + (1-\delta_K)K_t$). This suggests the equilibrium price of capital is given by:

$$\rho_{Kt} = R_t - 1 + \delta_K$$

Capital demand for the production of consumption in period t is given by:

$$K_t = \frac{C_t}{A_t}(\frac{\rho_{Mt}}{\rho_{Kt}}\frac{1-b}{b})^b$$

while supply of capital in the next period is determined by the Euler equation:

$$C_t^{-1} = \beta E_t R_{t+1} C_{t+1}^{-1}$$

Capital market clearing suggests $S_{t+1} - R_t S_t = K_{t+1} - (\rho_{Kt}K_t + (1-\delta_K)K_t) = K_{t+1} - (1-\delta_K)K_t - \rho_{Kt}K_t$. I impose mutual fund market clearing and lump-sum transfers of bond adjustment costs to generate the balance of payments condition:

$$\begin{aligned} & 2(1+r_{t+1})B_t + 2(1+r_{t+1}^*)Q_t B_{*t} \\ & = \\ & (C_t - Q_t C_t^*) + (N_{et}\tilde{v}_t - Q_t N_{et}^*\tilde{v}_t^*) \\ & +2(B_{t+1} + Q_t B_{*t+1}) - (w_t l_t - Q_t w_t^* l_t^*) \\ & -(N_{dt}\tilde{d}_{dt} - Q_t N_{dt}^*\tilde{d}_{dt}^*) - (N_{xt}\tilde{d}_{xt} - Q_t N_{xt}^*\tilde{d}_{xt}^*) \\ & +(K_{t+1} - Q_t K_{t+1}^*) - (1-\delta_K)(K_t - Q_t K_t^*) \\ & -(\rho_{Kt}K_t - Q_t\rho_{Kt}^* K_t^*) \end{aligned}$$

Investment is defined as expenditures on new firm entry plus the purchases of new capital:

$$I_t = N_{et}\tilde{v}_t + K_{t+1} - (1-\delta_K)K_t$$

Additional definitions are also required. Following Ghironi and Melitz (2005), income (GDP) is defined as:

$$Y_t = w_t l_t + \rho_{Kt}K_t + N_{dt}\tilde{d}_{dt}$$

Further, total imports is constructed using the average revenue foreign exporters earn from their sales abroad:

$$IM_t = bN_{xt}^* C_t \tilde{\rho}_{Mt}^{a-1}\tilde{\rho}_{xt}^{*1-a}$$

Total exports is constructed using the average revenue domestic exporters earn from their sales abroad:

$$EX_t = bQ_t N_{xt} C_t^* \tilde{\rho}_{Mt}^* \tilde{\rho}_{xt}^{1-a}$$

Terms of trade is defines as the ratio of import prices to export prices:

$$TOT_t = \frac{\tilde{\rho}_{xt}^*}{Q_t\tilde{\rho}_{xt}}$$

In the model, the price of consumption (P_{Ct}) is measured as a welfare-based price index following Feenstra (2003). It is thus important to transform this welfare-based index into one which closer matches the price index calculated in the data. To do so, we redefine the price index for aggregate consumption:

$$1 = \frac{[N_{dt}\tilde{\rho}_{dt}^{1-a} + N_{xt}^*\tilde{\rho}_{xt}^{*1-a}]^{b/(1-a)}\rho_{Kt}^{1-b}}{A_t(1-b)^{1-b}b^b}$$

If we assume that all prices are, on average, $\tilde{\rho} = \tilde{P}_t / P_{Ct}$, we construct:

$$\frac{P_{Ct}}{\tilde{P}_t} = \frac{[N_{dt} + N_{xt}^*]^{b/(1-a)}}{A_t(1-b)^{1-b}b^b}$$

Any variable measured in terms of real consumption, X_t, is adjusted to this index: $\tilde{X}_t = \frac{P_{Ct}X_t}{\tilde{P}_t}$. Further, since the real exchange rate is constructed using the welfare-based price indices ($Q_t = P_{Ct}^*/P_{Ct}$), we construct an adjusted real exchange rate:

$$\tilde{Q} = \tilde{P}_t^*/\tilde{P}_t = Q_t[\frac{A_t^*}{A_t}][\frac{N_{dt} + N_{xt}^*}{N_{dt}^* + N_{xt}}]^{b/(1-a)}$$

4 Numerical Experiments

Many of the model's parameter values are calibrated to match Ghironi and Melitz (2005) and are listed in table 4. The value of the depreciation rate of capital is a new parameter to the model and is set to 2.5% quarterly and 10% annually (a standard in the literature). The other model parameters, $(b, n_1, n_2, \lambda$, and the parameters of the shock processes, will be thought of as "free parameters" and will be calibrated under different experiments discussed shortly. The model is solved numerically using the brute force algorithm developed by Uhlig (1999)[26].

Simulations are performed in order to generate a set of statistics to compare to the data. In each experiment, a 200-period model is simulated 200 times by drawing a random vector of innovations. During each simulation, I make the appropriate price adjustments as described above, then apply the HP filter[27]. Summary statistics (volatility, correlation with output, etc.) are calculated for each simulation and I report the average statistics across simulations in the results tables.

4.1 The Benchmark Model

To match the original Ghironi and Melitz (2005) framework as closely as possible, I set $n_1 = 0.0025$, $b = 1$ and omit endogenous labour supply from the model. Since the inclusion of capital accumulation may lead to a highly persistent and extreme capital exodus if bond markets are fluid, I limit the amount of international asset flow by imposing high adjustment costs for foreign bonds ($n_2 = 1$). I calibrate the technology shocks to intermediate goods production as a near-unit-root processes with correlated innovations[28]:

$$\begin{aligned}
\begin{bmatrix} \hat{Z}_t \\ \hat{Z}_t^* \end{bmatrix} &= \begin{bmatrix} 0.99 & 0 \\ 0 & 0.99 \end{bmatrix} \begin{bmatrix} \hat{Z}_{t-1} \\ \hat{Z}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_{Zt} \\ \xi_{Zt}^* \end{bmatrix} \\
\sigma_{\xi_{Zt}} &= \sigma_{\xi_{Zt}^*} = \sigma_\xi = 0.00852 \\
\rho_{\xi_{Zt}\xi_{Zt}^*} &= \rho_{\xi_Z\xi_Z^*} = 0.258
\end{aligned}$$

with covariance of (ξ_{Zt}, ξ_{Zt}^*) 0.0019. Technology shocks are calibrated as such in all the following experiments. I set $A_t = 1$ for all t. Results from simulations are reported in column (2) of table 5. The model fails to predict high correlation of investment across countries and high volatility of imports, exports, the trade balance, the terms of trade and the real exchange rate. Further, the model over-predicts the correlation of exports, the terms of trade and the real exchange rate with GDP.

The failure to capture the appropriate investment dynamics is due to how investment is defined. In the model without capital, investment is only new firm construction. When there is a positive productivity shock to intermediate goods production in the home country, each individual firm in that country earns increased profits, leading to a surge of new firm construction and an increase in home's interest rate. The increase in home's interest rate induces foreign agents to adjust their investment portfolio in favor of home bonds (via the Euler relationship). Although high bond adjustment costs have limited the extent to which foreign investors substitute, there is still an exodus of investment dollars out of the foreign country. This causes firm attrition in the foreign economy and a net reduction in foreign real investment[29].

[26] The algorithm used in experimentation is described as follows. After calibrated parameters are chosen, I restructure the linearized equation system: $(E_t[FX_{t+1} + GX_t + HX_{t-1} + MZ_t] = 0; Z_t = NZ_{t-1} + e_t$, where X is a vector of variables, Z is a vector of stochastic processes guided by a VAR and e is a vector of innovations). Using Uhlig's brute force method of undetermined coefficients, I find a policy rule: $(X_t = PX_{t-1} + QZ_t)$. In each experiment, a 200-period model is simulated 200 times by drawing a random vector of innovations.

[27] Since the model exhibits a high degree of persistence, we take out the low-frequency trend using the HP filter.

[28] Backus et al. (1992) calibrate technology shocks as a VAR process with spill-overs:

$$\begin{aligned}
\begin{bmatrix} \hat{Z}_t \\ \hat{Z}_t^* \end{bmatrix} &= \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix} \begin{bmatrix} \hat{Z}_{t-1} \\ \hat{Z}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_{Zt} \\ \xi_{Zt}^* \end{bmatrix} \\
\sigma_{\xi_{Zt}} &= \sigma_{\xi_{Zt}^*} = \sigma_\xi = 0.00852 \\
\rho_{\xi_{Zt}\xi_{Zt}^*} &= \rho_{\xi_Z\xi_Z^*} = 0.258
\end{aligned}$$

Baxter and Crucini (1995) note that this specification is not statistically different from a near-unit-root process without spill-overs.

[29] The reduction in foreign investment is reduced after applying the real exchange rate to transform foreign denominations into domestic terms. The positive productivity shock to home intermediate good production leads to a fall in the price of

The failure of the model to appropriately reproduce features of the international market suggests that international trade in goods is failing to act as the appropriate mechanism for the transmission of business cycles across countries. Inspection of the impulse response functions for the endogenous variables of the model suggest the relationship between the international market statistics and relatively docile endogenous variables are responsible for the low volatilities of imports, exports, the trade balance and international prices. Aggregate productivity shocks in the benchmark model lead to increases in foreign consumption, the size of the domestic export sector, export prices and the real exchange rate. Each of these are less volatile than domestic income, which results in low volatility of exports relative to GDP. A similar story holds for imports. Since prices in the model are marked-up over marginal costs, and marginal costs in the model are less volatile than GDP, both the terms of trade and the real exchange rate are predicted to have low volatility.

4.2 Capital Accumulation

I now turn to the model with capital in the production of final consumption to improve the definition of investment. There are two important characteristics of the model worth noting when capital is used in the production of domestic consumables. First, since capital is not used to produce the traded intermediate inputs, the two countries are in effect trading labour services as in the original framework. Including capital in this fashion allows me to bypass a potentially complicated discussion of changes in relative factor intensities. Second, since the production of final consumption entails its own production process, I can introduce an additional source of business cycle fluctuations, A_t. While A_t acts as a technology shock to the domestic production of final consumption, it acts as an input demand shock to intermediate goods producers[30].

To proceed with experimentation, I first calibrate the parameter b so that the steady state capital-to-output ratio matches the average value for the US (approximately 3.10[31]). I then analyze the model with aggregate shocks to intermediate goods production only, followed by the model with shocks to the production of composite consumption. Both models fail to reproduce sufficiently volatile output, so as an aside I analyze a version of the model with endogenous labour supply.

4.2.1 Capital Accumulation and Shocks to the Production of Intermediate Goods

I start with a model whose only deviation from the original framework is the significance of capital in the production of final consumption. Labour is supplied inelastically, $A_t = 1$ for all t, and the underlying process driving business cycles is shocks to the production of intermediate inputs. As described above, the costs associated with purchasing foreign bonds is high ($n_2 = 1$) to limit the amount of investment exodus that can occur with open bond markets. The simulation results are reported in column (3) of table 5.

At first glance, the model with capital results in slightly improved volatilities of imports and exports as well as improved correlations between imports, exports and GDP. The model produces worse estimates of output and investment volatility and reduced correlation of consumption and investment across countries. Inspection of the impulse response functions link the worsening results of the simulation to changes in international prices once capital is incorporated into the model.

When there is a positive productivity shock to the production of input goods in the home country, consumption increases with an expansion of firm output. The production of intermediate goods becomes more profitable and new firms enter the market. More workers are demanded to expand output and to build new firms, leading to an increase in the domestic wage pushing up domestic income. The equilibrium price of capital ($\tilde{\rho}_{Kt}$) rises via the Euler equation for savings, leading to further increases in income. The home country experiences a state of expansion.

Home consumers demand more intermediate goods, leading to an increase in the demand for imports. In the foreign country, the export sector expands and

domestic goods and expansion of the foreign export sector (both of which push down the domestic price level in spite of rising import prices). Increases in the home export sector are not sufficient to outweigh increases in the prices of domestically produced goods in the foreign country; the foreign price level rises. Changes in aggregate prices brought about by the productivity shock cause the exchange rate to rise.

[30] Stockman and Tesar (1995) and Wen (2002) show that demand shocks are important to international business cycle synchronization. Although the formulation I adopt here does not impose demand shocks on behalf of consumers, demand shocks for intermediate inputs are potentially equally important.

[31] Annual data for the US, 1990-2007, from the Bureau of Economic Analysis for real total fixed assets plus consumer durables (Table 9.1) and real GDP (Table 1.1.6) are used to construct this average. MATLAB has difficulty numerically determining b in the model, so it is calibrated by hand. For quarterly simulations, $b = 0.88$ corresponds to a capital-output ratio of 3.09. For annual simulations, $b = 0.57$ corresponds to a capital-output ratio of 2.84. When labour supply is endogenous, $b = 0.60$ is used for annual calibrations which corresponds to a capital-output ratio of 2.77.

demand for labour increases, pushing up foreign wages and income. Although open asset markets lead to firm attrition and a reduction in labour demand for new firm construction, the model predicts wages to rise in equilibrium implying the effect of an expanding export sector on employment outweighs the effect of firm attrition. Capital prices in the foreign country are also higher via the Euler equations for savings and home bonds.

Although wages rise in both countries, the productivity shock in the home country pushes down the marginal cost of labour while the marginal cost of labour abroad rises. This results in a fall in the price of domestically produced goods in the home country and a rise in the price of goods produced abroad. An expansion of the export sector in both countries occurs to meet increased demand and export prices both at home and abroad rise. Changes in the number of exporting firms coupled with changes in domestic and export prices result in a fall in the intermediate good price index ($\tilde{\rho}_{Mt}, \tilde{\rho}^*_{Mt}$) for both countries. The fall in $\tilde{\rho}_{Mt}$ results primarily from a decline in domestic prices coupled with an increase in the number of foreign exporters. The fall in $\tilde{\rho}^*_{Mt}$ seems to result only from the increase in home's export sector and is of smaller magnitude than the change in $\tilde{\rho}^*_{Mt}$. The input price ratio ($\tilde{\rho}_{Kt}/\tilde{\rho}_{Mt}$) rises in both countries inducing households to use relatively more labour services to produce final consumption. In equilibrium, households actually reduce their capital holdings[32] in favor of new firm investment. Total investment in the home country is higher due to new firm construction although investment in capital has fallen. Investment in the foreign country, however, is lower due to a fall in both new firm construction and capital purchases.

The impulse response functions for a positive shock to domestic intermediate goods production show increases in both foreign and domestic consumption denoted in national terms. The simulation results, however, predict negative correlation when consumption is measured in comparable terms due to changes in the real exchange rate. In the model without capital, the real exchange rate increases when there is a positive productivity shock[33]. In the model with capital, changes in capital prices greatly influence changes in the real exchange rate. Percentage changes in the real exchange rate can be written as:

$$\hat{Q}_t = b(\hat{\rho}^*_{Mt} - \hat{\rho}_{Mt}) + (1-b)(\hat{\rho}^*_{Kt} - \hat{\rho}_{Kt})$$

Because declines in the price index for intermediate goods are severe in the home country, ($\hat{\rho}^*_{Mt} - \hat{\rho}_{Mt} > 0$), the real exchange rate is pushed up. Increases in capital prices are also severe in the home country ($\hat{\rho}^*_{Kt} - \hat{\rho}_{Kt} < 0$), which pushes the real exchange rate down. Changes in the price for capital in the home country is sufficiently strong ($\hat{\rho}_{Kt} > \frac{b}{(1-b)}(\hat{\rho}^*_{Mt} - \hat{\rho}_{Mt}) + \hat{\rho}^*_{Kt}$) to induce in a small fall in the real exchange rate when there is a positive productivity shock to intermediate goods production. The correlation between output and the real exchange rate is therefore negative in simulation, which corresponds more closely to the data. However when foreign variables have small positive changes, converting them to domestic terms using the reduced real exchange rate results in a negative movement. Consumption in the model, which increases in both countries, becomes negatively correlated after converting the foreign measure to comparable terms[34]. Output in the foreign country, which changes only slightly in real terms, declines after applying the exchange rate.

4.2.2 Capital Accumulation Shocks to the Production of Aggregate Consumption

By developing a production process for final goods that includes both capital and labour services, it is possible to extend the model by incorporating shocks to final goods production. Shocks of this nature can also be thought of as demand shocks for intermediate inputs. To analyze versions of the model with this feature, I will allow the final goods technology variables (A_t, A^*_t) to follow a VAR process. Since A_t is also related to technology, I use the same near-unit root shock process as was used for the shocks to the production of intermediate goods:

$$\begin{aligned}
\begin{bmatrix} \hat{A}_t \\ \hat{A}^*_t \end{bmatrix} &= \begin{bmatrix} 0.99 & 0 \\ 0 & 0.99 \end{bmatrix} \begin{bmatrix} \hat{A}_{t-1} \\ \hat{A}^*_{t-1} \end{bmatrix} + \begin{bmatrix} \xi_{At} \\ \xi^*_{At} \end{bmatrix} \\
\sigma_{\xi_{At}} &= \sigma_{\xi^*_{At}} = \sigma_{\xi_A} = 0.00852 \\
\rho_{\xi_{At}\xi^*_{At}} &= \rho_{\xi_A \xi^*_A} = 0.258
\end{aligned}$$

As an intermediate step, I allow only A_t to follow a stochastic process while $Z_t = 1$ for all t. The results for this calibration are reported in column

[32] Although less capital is purchased, investment in the model ($K_{t+1} - (1-\delta_K)K_t + N_{et}\tilde{v}_t$) still increases due to new firm construction.

[33] A positive productivity shock leads to a fall in the price for domestically produced goods and an increase in the variety of imported goods, which offsets increases in import prices. In the foreign country, only an increase in the number of imported goods offsets increases in the prices of all goods. Although prices in both countries are likely to decrease, prices in the home country would decrease more than the foreign country, leading to a rise in the real exchange rate. This conclusion is based on impulse response functions for $P_{Ct}/\tilde{P}_t$, which decline with a positive productivity shock, suggesting that changes in the number of firms are more influential than changes in the prices of intermediate goods.

[34] The home export sector expands as the real exchange rate falls, leading to an increase in foreign consumption. This increase in consumption, however, is not enough to overpower the fall in the exchange rate after converting foreign variables into comparable terms.

(4) of table 5. Although shocks to final goods production alone are not sufficient to capture all the dynamics of the real economy, the behavior of many economic variables are dramatically improved. Imposing shocks of this nature results in increased volatility of investment, increased international correlation, increased volatility of imports and exports, increased volatility of terms of trade, and increased volatility of the exchange rate index.

Investment volatility increases since input demand shocks have a more direct impact on capital than the productivity shocks of input producers. A positive productivity shock to the production of final goods will directly increase the marginal product of capital along with the marginal product of intermediate 'labour services'. The higher marginal product of capital increases the demand for capital in the current period which leads to an immediate increase in capital prices, since the current capital stock is predetermined. Further, since the shock is persistent, consumers expect future returns to savings to be higher and supply more savings to capital markets, expanding future capital holdings. The part of investment that is held as capital dramatically increases, pushing up investment volatility.

Inspection of the impulse response functions suggests that a positive productivity shock to final goods production results in firm attrition in the home country (N_{et} declines) because the return to capital makes savings relatively more profitable as an investment tool than new firm construction in the current period. Further, since a positive productivity shock to final goods production leads to an immediate expansion of consumption, there is higher demand for input products and thus increased demand for labour services in intermediate goods production. Intermediate producers expand production and demand more workers, causing the real wage to rise. With higher wages, it becomes more expensive to pay the fixed entry cost associated with starting a firm. Although the discounted value of existing firms ($\tilde{v}_t$) rises since each existing firm now earns more profits, this increase is more than offset by the decline in firm entry. The part of investment that is comprised by new firm construction ($N_{et}\tilde{v}_t$) decreases, but only slightly. There is an overall increase in investment resulting from capital purchases. Higher labour income, coupled with an increased price of capital, leads to an increase in aggregate income. In the home country, aggregate output, investment and consumption all increase.

Because capital earns a higher return, there is a movement away from issuing domestic bonds, purchasing foreign bonds and building new firms towards capital accumulation in the home country. A reduction in the supply of home bonds results in lower bond issuance and higher bond returns. A reduction in home's demand for foreign assets leads to a fall in foreign bond purchases and a reduction in the foreign interest rate. As the return to foreign bonds falls, foreign agents adjust their investment portfolios in favor of capital and new firm construction. There is a wave of firm entry in the foreign country coupled with rising capital prices, leading to higher foreign investment. As home's demand for imports expands, the pressure in the foreign export sector to expand production coupled with an increase in labour demand for new firm construction leads to higher foreign wages. Aggregate foreign income rises as does foreign consumption. The two countries exhibit synchronous business cycle fluctuations.

Final goods production shocks have a more direct effect on aggregate prices, resulting in a higher volatility of the exchange rate index. We can define the real exchange rate as:

$$Q_t = \frac{P^*_{Ct}}{P_{Ct}} = [\frac{A_t}{A^*_t}][\frac{P^{*b}_{Mt} P^{*(1-b)}_{Kt}}{P^b_{Mt} P^{(1-b)}_{Kt}}]$$

The effect of a movement in A_t on Q_t is strong and positive[35]. Sharp increases in the real exchange rate increase the profitability of exporting in the home country and a fall in the profitability of exporting in the foreign country. There is an expansion of the home export sector and a contraction of the foreign export sector. In addition, increases in consumption in both countries lead to higher demand for intermediate inputs and thus higher prices for both imported and exported goods. The volatility of the terms of trade rises in part due to an increase in intermediate good price volatility as well as increases in the volatility of the real exchange rate. Although intermediate goods prices rise, the expansion of the domestic export sector and the contraction of the foreign export sector induce home's exports to sharply increase while home's imports change negligibly. This, however, implies a pro-cyclical trade balance.

Changes in aggregate prices are not trivial. The factor used to transform welfare-based variables to variables that more closely match the data ($\frac{P_{Ct}}{\hat{P}_t}$) sharply declines in the home country in response

[35] This is confirmed in the impulse response function for Q_t for a 1 standard deviation shock to A_t. A positive productivity shock to final goods production results in an increase in the real interest rate, which suggests $\hat{A}_t > b(\hat{P}_{Mt} - \hat{P}^*_{Mt}) + (1-b)(\hat{P}_{Kt} - \hat{P}^*_{Kt})$.

[36] $\frac{P_{Ct}}{\hat{P}_t} = \frac{[N_{dt}+N^*_{xt}]^{b/(1-a)}}{A_t(1-b)^{1-b}b^b}$. A positive shock to A_t coupled with a net increase in $N_{dt} + N^*_{xt}$ lead to a sharp fall in $\frac{P_{Ct}}{\hat{P}_t}$.

to a positive shock to final goods production[36]. All home variables subject to the price adjustment are slightly reduced. Home country variables that exhibit small or marginal increases, such as consumption and imports, can even show declines after applying the price adjustment. Foreign variables subject to the price adjustment are also subject to changes in the real exchange rate[37]. Since the real exchange rate is rising, the negative effect of changes in the price adjustment factor is more than made up for. Foreign variables are seemingly more responsive to shocks in the home country than home variables due to changes in the exchange rate.

Timing is an important factor when connecting the intuition gleamed from the impulse response functions to the simulation results. From the impulse response functions, consumption and imports seem contemporaneously negatively correlated with GDP after applying the price adjustment while exports and investment seem contemporaneously positively correlated with GDP. Over time, after a positive productivity shock, GDP rises before returning to the steady state. Consumption and imports follow the same pattern, they rise then gradually return to the steady state. Imports and exports, however, continuously fall to the steady state after the initial shock. Consumption and imports are positively correlated with GDP when considering the entire transition period, while movements of investment and exports along with the trade balance become negatively correlated with GDP. These results are consistent with predictions made in the simulation results.

The central benefits of productivity shocks to final goods production are an increase in investment volatility, high international correlation of output, investment and consumption, high volatility of exports and imports, high volatility of international prices. This stochastic feature can not, however, capture correctly the volatility of output and consumption. Further, changes in aggregate prices, changes in the exchange rate and timing induce counter-cyclical investment and exports. Since productivity shocks to intermediate goods production had many successes on these dimensions, I now turn to the model with both sources of shocks.

The model in which both A_t and Z_t follow a near unit root process is reported in column (5) of table 5[38]. The model captures output and consumption volatility as well as the benchmark model, but also succeeds in generating highly correlated consumption, output and investment across countries, higher volatility of imports and exports, higher volatility of the real exchange rate and low correlation between international prices and GDP. These features are not captured by the model with capital accumulation and productivity shocks to intermediate goods production alone. The volatility of the terms of trade is improved, but still falls short of unity. The model over-predicts the volatility of investment and slightly under-predicts correlation between exports, imports and GDP as well as investment and GDP.

When capital is used in the production of non-traded final goods, the benchmark framework is unable to reproduce a fully functioning international market. The mechanism that correlates business cycles across countries becomes distorted when the model is driven by productivity shocks to intermediate goods production alone. When we subject final goods production to productivity shocks, we can improve the functioning of the international market. These sorts of productivity shocks can be interpreted as demand shocks for intermediate inputs. A model in which business cycles are driven by both sorts of shocks can capture a larger number of stylized facts more accurately while allowing us to represent investment in a manner that is consistent with the rest of the real business cycle literature.

4.2.3 Endogenous Labour Supply

As in Farhat (2009), I try to push the model further by incorporating endogenously determined labour supply. I calibrate the elasticity of labour supply to be consistent with empirical measurements at the macro level ($\lambda = 1$) and impose strong costs to adjusting foreign bond holdings ($n_2 = 1$). I allow for capital accumulation as in the previous sections[39]. The results from the model with intermediate goods shocks as the source of business cycle fluctuations, the model with shocks to final goods production and the model with both types of shocks are reported in columns (6), (7) and (8) in table 6 respectively. The incorporation of the labour-leisure decision increases output volatility and captures pro-cyclical employment and positive correlation of employment across countries. Further, some of the successes that were achieved in the previous sections (such as increased volatility of imports and exports, more correlated consumption, higher volatilities of international prices) can still be achieved.

When business cycles are driven by shocks to the production of intermediate goods, both the im-

[37] In addition to converting foreign welfare-based variables to measures that more closely match the data, the real exchange rate must also be used to make the variables comparable to home measures.

[38] Although there is underlying correlation between Z_t and Z_t^*, it is assumed that Z_t and A_t are uncorrelated.

[39] See Farhat (2009) for the model with endogenous labour supply without capital accumulation

pulse response functions and many of the simulation statistics are strikingly similar between the model with capital accumulation and endogenous labour supply (table 6) and the model with only capital accumulation (table 5). The model with a more complete labour market, however, can achieve a higher estimate of output volatility, can generate pro-cyclical employment and can reproduce positive correlation of employment across countries. In the model with endogenously determined labour supply, a positive productivity shock leads to firm entry as well as firm expansion, resulting in increased labour demand. Increased composite consumption from both firm entry (more variety) and expansion (more goods produced) in the home country compels households to supply *less* labour. The increase in labour demand resulting from the positive productivity shock offsets the decline in labour supply, resulting in more employment and higher wages in equilibrium. The boost in employment further increases aggregate output, leading to the higher output volatility seen in the results table.

As income and consumption rise, the demand for imported goods rises inducing an expansion of the foreign export sector. This boosts foreign labour demand. Increases in the return to home bonds leads to less firm construction in the foreign country (as investment portfolios adjust) which reduces labour demand. The contradicting effects of export expansion and firm attrition make deducing changes in labour demand difficult. Slight increases in the home country's export sector in response to the improved productivity of labour lead to mild increases in foreign consumption[40], causing a reduction in foreign labour supply. The model predicts an overall fall in foreign employment.

Although home employment rises and foreign employment falls contemporaneously, implying negative international correlation, the lagged response to a positive productivity shock matter in generating the positive employment correlation seen in the simulation results. After the initial positive shock, employment in the home country gradually falls back to the steady state. In the foreign country, employment continues to fall after the initial shock for several periods[41]. As both the foreign and domestic employment series move in the same direction for several periods (in addition to employment changes in the foreign country being quite small), the simulation statistics report a positive correlation. The analysis of output, consumption and investment correlations across countries, as well as the other variables reported after applying the appropriate price adjustments follow from the model with capital and inelastic labour supply described above.

When business cycles are driven by shocks to the production of final consumption, major differences between the model with only capital accumulation and the model with both capital accumulation and complete labour markets lie in the dynamic response to employment. A positive shock to final goods production in the home country leads to an immediate increase in the demand for intermediate goods. Employment increases, then gradually returns to its steady state. With shocks to final goods production, there is no prolonged increase in the quantity or variety of goods that are exported to the foreign country. Although foreign consumption immediately increases[42], it then smoothly returns to the steady state. Foreign labour supply immediately falls, then returns to its steady state, resulting in the dynamic negative correlation between domestic and foreign employment observed in the simulation results.

The model with both shock processes achieves the same successes as it had in the model with inelastic labour supply. A noticeable drawback is the negative international correlation of employment. The effect of final goods production shocks on the variety of products is dominate, resulting in a dynamic negative correlation of employment across countries.

When comparing international correlations between the model with developed labour markets (column (8) of table 6) and the model with inelastic labour supply (column (5) of table 5) we notice an inverted relationship between output volatility and business cycle synchronization. When we incorporate endogenous labour supply, any productivity shock results in a dramatic increase in output in the home country. This triggers a larger degree of investment exodus out of the foreign country and reduces the international correlations of consumption, output and investment. To test the implications of this result, I experiment with a lower labour supply elasticity[43] to mitigate changes in employment when shocks occur. The results are reported in columns (9), (10) and (11) of table 6 for the three different specifications of the shock

[40]This is due to changes in the variety of goods available to foreign customers.

[41]Since the home country is expanding, increased varieties and quantities of exports to the foreign country result in increasing foreign consumption over time which results in persistent declines in foreign labour supply. Firm attrition and expansion of the foreign export sector continue to keep changes in labour demand marginal. Employment falls and wages rise in the foreign country for several periods after the initial shock.

[42]This is due to increased investment in capital in response to the shock, as described in the previous section.

[43]$\lambda = 5$ corresponds to a labour supply elasticity of 0.2, which is a common estimate for men according to Fiorito and Zanella (2008)).

processes. The results for output volatility and the international correlations are as expected. Output volatility declines since labour supply is less responsive to economic shocks. The reduction in output volatility increases the international correlation of output, consumption, investment and employment since there is less investment exodus from the foreign country in response to a positive productivity shock. Reductions in labour supply elasticity increase the volatility of imports, exports and international prices relative to GDP since GDP volatility decreased. A better statistical fit for all the other endogenous variables save output volatility is achieved with a lower labour supply elasticity.

5 Conclusion

Without any notion of capital, the definition of "investment" in the original Ghironi and Melitz (2005) model is inconsistent with the real business cycle literature. Although new firm construction is an important component, changes in plant and equipment by existing firms is included in the statistical measure of investment. By including capital into the production of non-traded goods, more reasonable comparisons between the data and the model's results can be made.

At first, the inclusion of capital into the model improves the correlation between imports, exports and GDP when foreign bonds purchases are costly and economic fluctuations are driven by productivity shocks to intermediate goods production. The presence of capital, however, results in negative international correlations of output, investment and consumption and inadequately captures the volatility of the international market. Increases in the price of capital when there are positive productivity shocks leads to decreases in the real interest rate which induces statistical declines in foreign variables after transforming them into comparable terms.

When productivity shocks to final goods are incorporated into the model, dramatic improvements in international correlations along with higher volatilities of international market variables are achieved. Positive shocks to final goods production directly increase the exchange rate, making positive international correlations of output, consumption and investment statistically achievable. Since shocks to final goods production also acts as demand shocks for intermediate goods, there is increased volatility of trade measures and international prices. Further, the relationships between the exchange rate, terms of trade and GDP that are seen in the data are captured by the model. This suggests that productivity shocks to both final (non-traded) and intermediate (traded) goods are essential to reproducing the mechanism that transmits business cycles across countries. Although results are much *improved* when shocks to final goods production are added to the model, they are far from ideal. Improving the fit further by estimating and recalibrating the VAR processes for both shocks to intermediate and final goods production is left for future work.

When endogenous labour supply is incorporated into the model, increased output volatility along with pro-cyclical employment and positive correlation of employment across countries is achieved when business cycles are driven by shocks to intermediate goods production (productivity shocks to firms that demand labour). When shocks to final goods production are added, the model continues to produce improved international co-movement and replicates the behaviour of international market variables. To keep employment positively correlated across countries, low labour supply elasticities must be considered which results in reduced output volatility. Correcting this attribute of the model continues to be a mystery[44] and is left for future work.

6 Acknowledgements

Special thanks to Jang-Ting Guo, Roger Farmer, R. Robert Russell, Richard Suen, Victoria Umanskaya for their invaluable comments. All errors are my own. For additional comments or suggestions, I can be reached at dan.farhat@otago.ac.nz.

[44]Neither the preferences described by Greenwood, Hercowitz, and Huffman (1988), which produce labour supply equations without income effects, nor indivisible labour as described by Hansen (1985) seem to improve output volatility without distorting the other results.

References

Alessandria, G., & Choi, H. (2004). Export decisions and international business cycles [Working Paper]. , *August*.

Backus, D. K., Kehoe, P. J., & Kydland, F. E. (1992). International real business cycles. *The Journal of Political Economy*, *100*(4), 745-775.

Backus, D. K., Kehoe, P. J., & Kydland, F. E. (1994). Dynamics of the trade balance and the terms of trade: The j-curve? *The American Economic Review*, *84*(1), 84-103.

Baxter, M., & Crucini, M. J. (1995). Business cycles and the asset structure of foreign trade. *International Economic Review*, *36*(4), 821-854.

Bureau of Economic Analysis. (2009). National income and product accounts. Available from `http://www.bea.gov`

Canova, F., & Dellas, H. (1993). Trade interdependence and the international business cycle. *Journal of International Economics*, *34*(1-2), 23-47.

Cantor, R., & Mark, N. C. (1988). The international transmission of real business cycles. *International Economic Review*, *29*(3), 493-507.

Cook, D. (2002). Market entry and international propagation of business cycles. *Journal of International Economics*, *56*(1), 155-175.

Cuñat, A., & Maffezzoli, M. (2004). Hecksher-ohlin business cycles. *Review of Economic Dynamics*, *7*(3), 555-585.

Dellas, H. (1986). A real model of the world business cycle. *Journal of International Money and Finance*, *5*(3), 381-394.

Farhat, D. F. (2009). Endogenous labour supply, hetergenous firms and international business cycles. *University of Otago, Economics Discussion Papers*(0909).

Feenstra, R. C. (2003). A homothetic utility function for monopolistic competition models, without constant price elasticity. *Economics Letters*, *78*(1), 79-86.

Fiorito, R., & Zanella, G. (2008). Labor supply elasticities: Can micro be misleading for macro? *Department of Economics, University of Sienna Working Paper*, *547*.

Fukuda, S. (2004). Extraneous shocks and international linkage of business cycles in a two-country monetary model. *Journal of Economic Behavior and Organization*, *54*(3), 389-409.

Ghironi, F., & Melitz, M. J. (2005). International trade and macroeconomic dynamics with heterogeneous firms. *The Quarterly Journal of Economics*, *120*(3), 865-915.

Greenwood, J., Hercowitz, Z., & Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. *American Economic Review*, *78*(3), 402-17.

Guo, J.-T., & Sturzenegger, F. (1998). Crazy explanations of international business cycles. *International Economic Review*, *39*(1), 111-133.

Hansen, G. D. (1985). Indivisible labor and the business cycle. *Journal of Monetary Economics*, *16*(3), 309-327.

Head, A. C. (2002). Aggregate fluctuations with national and international returns to scale. *International Economic Review*, *43*(4), 1101-1125.

Heathcote, J., & Perri, F. (2002). Financial autarky and international business cycles. *Journal of Monetary Economics*, *49*(3), 601-627.

Helpman, E., & Krugman, P. (1987). *Market structure and foreign trade: Increasing returns, imperfect competition, and the international economy* (Vol. 1). The MIT Press.

Heston, A., Summers, R., & Aten, B. (2006). Penn world tables version 6.2. *Center for International Comparison of Production, Income and Prices. University of Pennsylvania*.

International Monetary Fund. (2006). International financial statistics. ESDS International, University of Manchester. Available from `http://www.imfstatistics.org/imf/`.

Kehoe, P. J., & Perri, F. (2002). International business cycles with endogenous incomplete markets. *Econometrica*, *70*(3), 907-928.

Kollmann, R. (1996). Incomplete asset markets and the cross-country consumption correlation puzzle. *Journal of Economic Dynamics and Control*, *20*(5), 945-961.

Kose, M. A., & Yi, K.-M. (2001). International trade and business cycles: Is vertical specialization the missing link? *American Economic Review*, *91*(2), 371-375.

Kose, M. A., & Yi, K.-M. (2002). The trade comovement problem in international macroeconomics [Staff Reports]. (155).

Kose, M. A., & Yi, K.-M. (2006). Can the standard international business cycle model explain the relation between trade and comovement? *Journal of International Economics*, *68*(2), 267-295.

Kydland, F. E., & Prescott, E. C. (1982). Time to build and aggregate fluctuations. *Econometrica*, *50*(6), 1345-70.

Long, J. B., & Plosser, C. I. (1983). Real business cycles. *Journal of Political Economy*, *91*(1), 39-69.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, *71*(6), 1695-1725.

Mendoza, E. G. (1991). Real business cycles in a small open economy. *American Economic Review*, *81*(4), 797-818.

Mendoza, E. G. (1995). The terms of trade, the real exchange rate, and economic fluctuations. *International Economic Review*, *36*(1), 101-37.

Olivero, M. P. (2004). Trade, non-competitive banking and the international transmission of business cycles [Mimeo].

Organization for Economic Cooperation and Development. (2009). Oecd.stat database. Available from `http://stats.oecd.org/`.

Stock, J. H., & Watson, M. W. (2005). Understanding changes in international business cycle dynamics. *Journal of the European Economic Association*, *3*(5), 968-1006.

Stockman, A. C., & Svensson, L. E. (1987). Capital flows, investment, and exchange rates. *Journal of Monetary Economics*, *19*(2), 171-201.

Stockman, A. C., & Tesar, L. L. (1995). Tastes and technology in a two-country model of the business cycle: Explaining international comovements. *The American Economic Review*, *85*(1), 168-185.

Uhlig, H. (1999). A toolkit for analyzing nonlinear dynamic stochastic models easily. In R. Marimon & A. Scott (Eds.), *Computational methods for the study of dynamic economies* (p. 30-61). Oxford University Press.

Wen, Y. (2002). Fickle consumers versus random technology: Explaining domestic and international comovements. *Mimeo, Cornell University*.

Xaio, W. (2002). Fickle consumers verusus random technology: Explaining domestic and international co-movements [Mimeo,Cornell University].

Xaio, W. (2003). Indeterminacy, demand shocks and international business cycles [Working Paper]. *Computing in Economics and Finance*, *47*.

Xaio, W. (2004). Can indeterminacy resolve the cross-country correlation puzzle? *Journal of Economic Dynamics and Control*, *28*, 2341-2366.

Zimmermann, C. (1999). International business cycles and exchange rates. *Review of International Economics*, *7*(4), 682-98.

A Appendix

Table 1:

U.S. Business Cycle Statistics (1957q1-2007q1)			
Volatility (Std. Dev.)		Correlation with GDP	
$\sigma_Y(\%)$	1.54	$\rho_{Y,Y}$	1.00
σ_C/σ_Y	0.75	$\rho_{C,Y}$	0.88
σ_I/σ_Y	3.41	$\rho_{I,Y}$	0.90
σ_L/σ_Y	0.61	$\rho_{L,Y}$	0.81

Table 1: U.S. quarterly data (1957q1-2007q1) for output (Y), consumption (C) and investment (I) is extracted from the International Financial Statistics maintained by the International Monetary fund. Data for investment includes gross fixed capital formation plus changes in inventories. Labor data is generated from civilian employment measures from the OECD.Stat database. Logs of deflated measures are de-trended using the HP filter.

Table 2:

U.S. - European Co-movement (1970-2007)	
Correlation	
$\rho_{Y,Y*}$	0.56
$\rho_{C,C*}$	0.45
$\rho_{I,I*}$	0.37
$\rho_{L,L*}$	0.39

Table 2: Annual data for output (Y), consumption (C) and investment (I) for the U.S. and a "European Aggregate" consisting of Austria, Finland, France, Germany, Italy, Switzerland and the United Kingdom are taken from the Penn World Tables maintained by Heston, Summers, and Aten (2006). Annual labor (L) data is taken from the OECD.Stat database and omits Finland due to missing data. Data availability for Germany restricts the sample to 1970-2004. Logs for deflated variables are de-trended using the HP filter.

Table 3:

U.S. International Market Statistics (1957q1-2007q1)			
Volatility (Std. Dev.)		Correlation with GDP	
σ_{IM}/σ_Y	3.31	$\rho_{IM,Y}$	0.6
σ_{EX}/σ_Y	3.74	$\rho_{EX,Y}$	0.28
$\sigma_{NX/Y}$	0.4	$\rho_{NX/Y,Y}$	-0.36
σ_{TOT}/σ_Y	1.69	$\rho_{TOT,Y}$	0.07
σ_Q/σ_Y	3.37	$\rho_{Q,Y}$	-0.18

Table 3: U.S. quarterly data (1957q1-2007q1) for imports (IM), exports (EX) the trade balance relative to output (NX/Y), the terms of trade (TOT) (index) and the real exchange rate (Q) (index) is extracted from the International Financial Statistics database maintained by the World Bank. Logs of deflated measures are de-trended using the HP filter, with exception to the trade balance which is de-trended in levels.

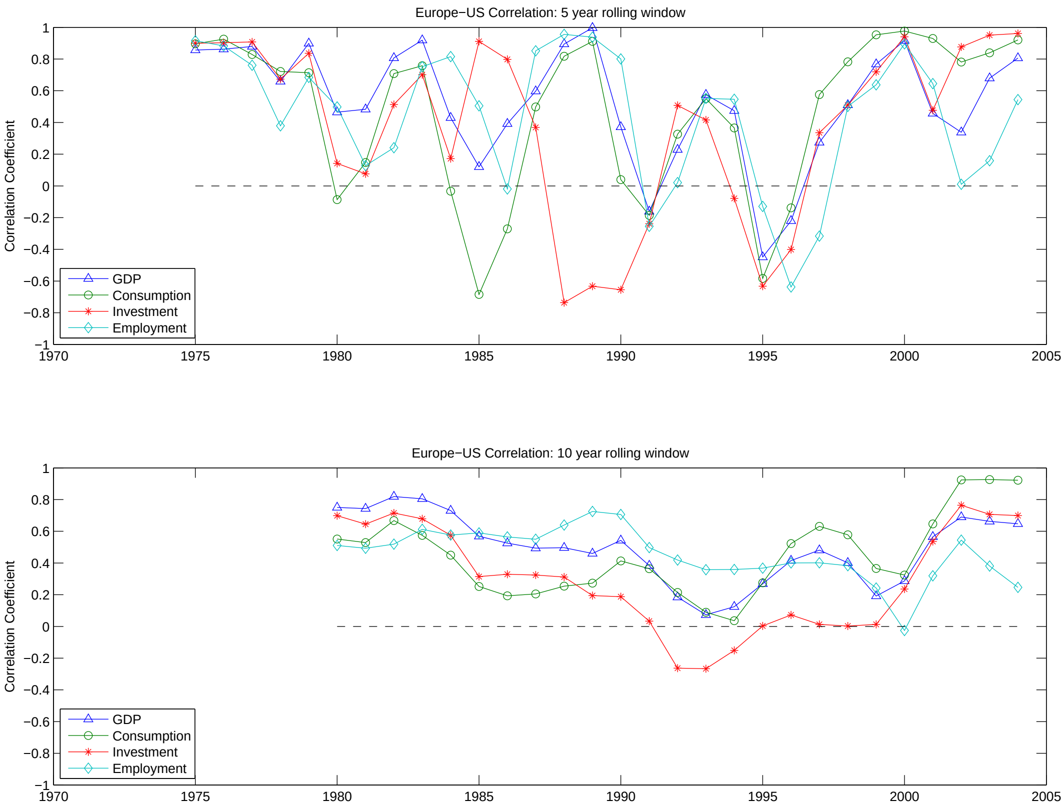

Figure 1: U.S. - European Co-movement (1970-2004) - Rolling Window

Figure 1: Annual data for output (Y), consumption (C) and investment (I) for the U.S. and a ”European Aggregate” consisting of Austria, Finland, France, Germany, Italy, Switzerland and the United Kingdom are taken from the Penn World Tables maintained by Heston et al. (2006). Annual labour (L) data is taken from the OECD.Stat database. Data availability for Germany restricts the sample to 1970-2004. Logs for deflated variables are de-trended using the HP filter. At any time, t, the figure plots the international correlations of GDP, consumption, and investment using data from the proceeding j years, where j=5 and 10 respectively.

Table 4: **Calibrated Parameters.**

Parameter		Value
β	Discount factor	0.99, 0.96
δ	Probability of death shock	0.025, 0.10
δ_k	Capital depreciation rate	0.025, 0.10
τ	Iceberg costs associated with trade	1.3
a	Elasticity of substitution between inputs	3.8
k	Shape parameter on Pareto distribution	3.4
$z_{\min}$	Lower bound on Pareto distribution	1
$\tilde{z}_D$	”Special” average productivity draw	$z_{\min}(\frac{k}{k+1-a})^{1/(a-1)}$
F_{Et}	Fixed cost of entry	1
F_{Xt}	Fixed exporter costs	$0.235(F_{Et})(\frac{1-\beta(1-\delta)}{\beta(1-\delta)})$

Table 5: **Simulation Results: Capital Accumulation**

	(1)	(2)	(3)	(4)	(5)
Domestic Statistics					
Volatility					
$\sigma_Y(\%)$	1.54	0.97	0.76	0.15	0.77
σ_C/σ_Y	0.75	0.61	0.50	1.78	0.57
σ_I/σ_Y	3.41	3.11	2.19	43.45	8.12
σ_L/σ_Y	0.61	—	—	—	—
Correlation with GDP					
$\rho_{C,Y}$	0.88	0.99	0.99	0.48	0.86
$\rho_{I,Y}$	0.90	0.98	0.59	-0.17	0.14
$\rho_{L,Y}$	0.81	—	—	—	—
International Co-movement					
Correlation					
ρ_{Y,Y_*}	0.56	0.28	-0.15	0.67	0.45
ρ_{C,C_*}	0.45	0.34	-0.27	0.31	0.29
ρ_{I,I_*}	0.37	0.07	-0.75	0.77	0.72
ρ_{L,L_*}	0.39	—	—	—	—
International Market Statistics					
Volatility					
σ_{IM}/σ_Y	3.31	0.58	0.68	8.87	1.70
σ_{EX}/σ_Y	3.74	0.57	0.71	11.73	2.18
$\sigma_{NX/Y}$	0.40	0.02	0.12	0.56	0.58
σ_{TOT}/σ_Y	1.69	0.12	0.21	4.22	0.77
σ_Q/σ_Y	3.37	0.10	0.14	3.19	0.58
Correlation with GDP					
$\rho_{IM,Y}$	0.60	0.82	0.79	0.14	0.31
$\rho_{EX,Y}$	0.28	0.80	0.40	0.10	0.13
$\rho_{NX/Y,Y}$	-0.36	-0.22	-0.34	-0.002	-0.06
$\rho_{TOT,Y}$	0.07	0.55	0.56	-0.03	0.15
$\rho_{Q,Y}$	-0.18	0.41	-0.55	0.17	-0.12

(1) Data
(2) Benchmark,($K_t = 0$), shocks to Z_t only
(3) Capital Accumulation, shocks to Z_t only
(4) Capital Accumulation, shocks to A_t only
(5) Capital Accumulation, shocks to Z_t and A_t

Table 6: **Simulation Results: Capital Accumulation with Endogenous Labour Supply.**

	(1)	(6)	(7)	(8)	(9)	(10)	(11)
Domestic Statistics							
Volatility							
$\sigma_Y(\%)$	1.54	1.05	0.20	1.08	0.83	0.15	0.86
σ_C/σ_Y	0.75	0.44	1.11	0.47	0.48	1.55	0.55
σ_I/σ_Y	3.41	2.36	30.86	6.32	2.26	40.81	7.48
σ_L/σ_Y	0.61	0.38	0.88	0.41	0.12	0.29	0.13
Correlation with GDP							
$\rho_{C,Y}$	0.88	0.98	-0.05	0.86	0.99	0.30	0.86
$\rho_{I,Y}$	0.90	0.60	0.12	0.28	0.60	-0.06	0.18
$\rho_{L,Y}$	0.81	0.98	0.69	0.94	0.99	0.36	0.91
International Co-movement							
Correlation							
ρ_{Y,Y_*}	0.56	-0.05	0.87	0.42	-0.11	0.72	0.46
ρ_{C,C_*}	0.45	-0.26	0.10	0.07	-0.32	0.21	0.18
ρ_{I,I_*}	0.37	-0.59	0.48	0.41	-0.73	0.69	0.64
ρ_{L,L_*}	0.39	0.33	-0.31	-0.12	0.42	0.01	0.14
International Market Statistics							
Volatility							
σ_{IM}/σ_Y	3.31	0.64	5.70	1.24	0.69	7.95	1.53
σ_{EX}/σ_Y	3.74	0.66	7.42	1.54	0.70	10.45	1.89
$\sigma_{NX/Y}$	0.40	0.17	0.53	0.57	0.13	0.54	0.56
σ_{TOT}/σ_Y	1.69	0.20	2.64	0.52	0.21	3.72	0.67
σ_Q/σ_Y	3.37	0.13	2.01	0.38	0.14	2.80	0.51
Correlation with GDP							
$\rho_{IM,Y}$	0.60	0.76	-0.50	0.28	0.78	-0.10	0.33
$\rho_{EX,Y}$	0.28	0.35	0.70	0.28	0.36	0.34	0.17
$\rho_{NX/Y,Y}$	-0.36	-0.32	0.62	0.03	-0.35	0.24	-0.05
$\rho_{TOT,Y}$	0.07	0.56	-0.62	0.07	0.57	-0.27	0.14
$\rho_{Q,Y}$	-0.18	-0.54	0.68	-0.04	-0.54	0.38	-0.06

(1) Data
(6) $\lambda = 1$, shocks to Z_t only
(7) $\lambda = 1$, shocks to A_t only
(8) $\lambda = 1$, shocks to Z_t and A_t
(9) $\lambda = 5$, shocks to Z_t
(10) $\lambda = 5$, shocks to A_t
(11) $\lambda = 5$, shocks to Z_t and A_t

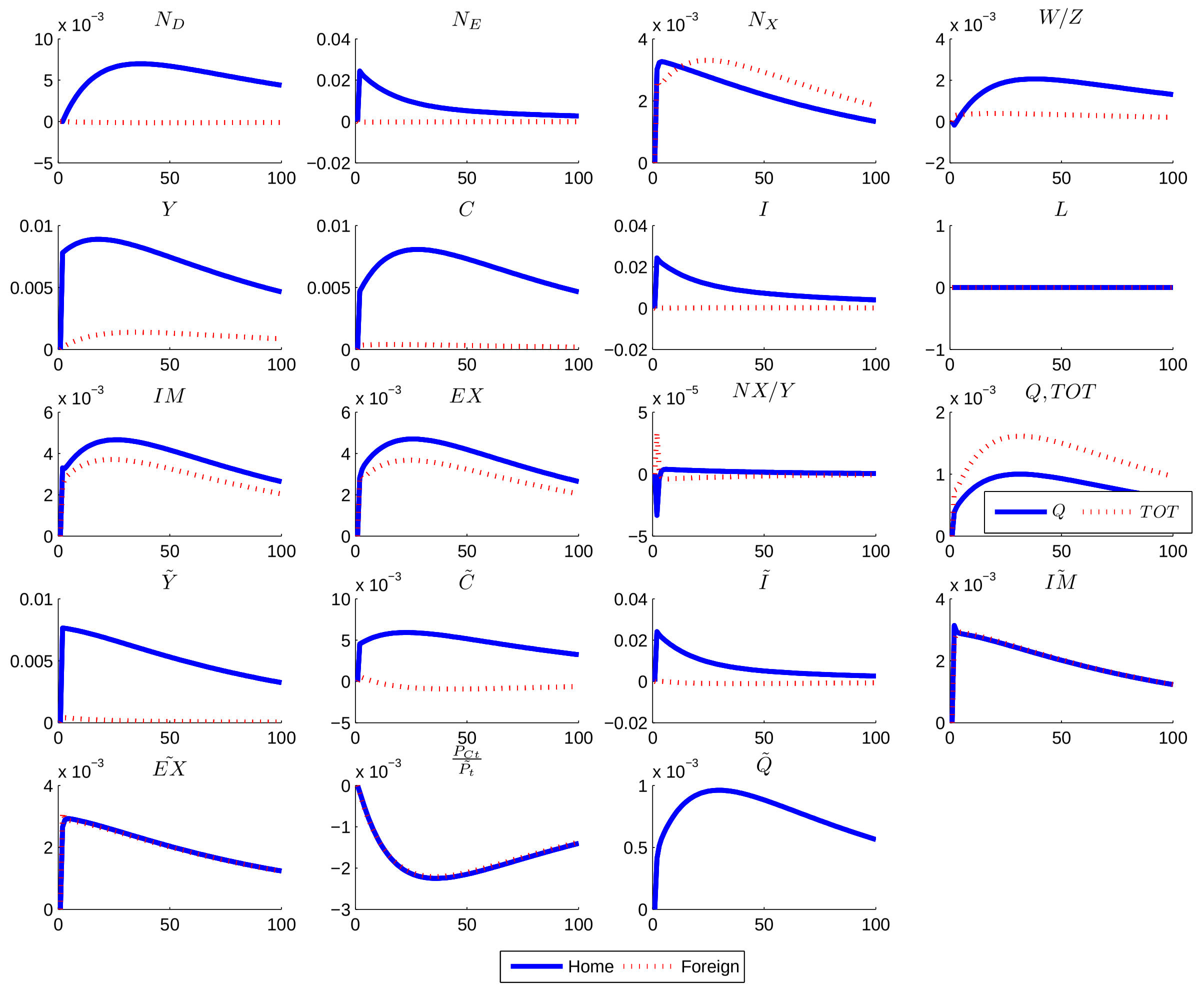

Figure 2: Impulse Response Functions - Benchmark Model ($n_2 = 1$)

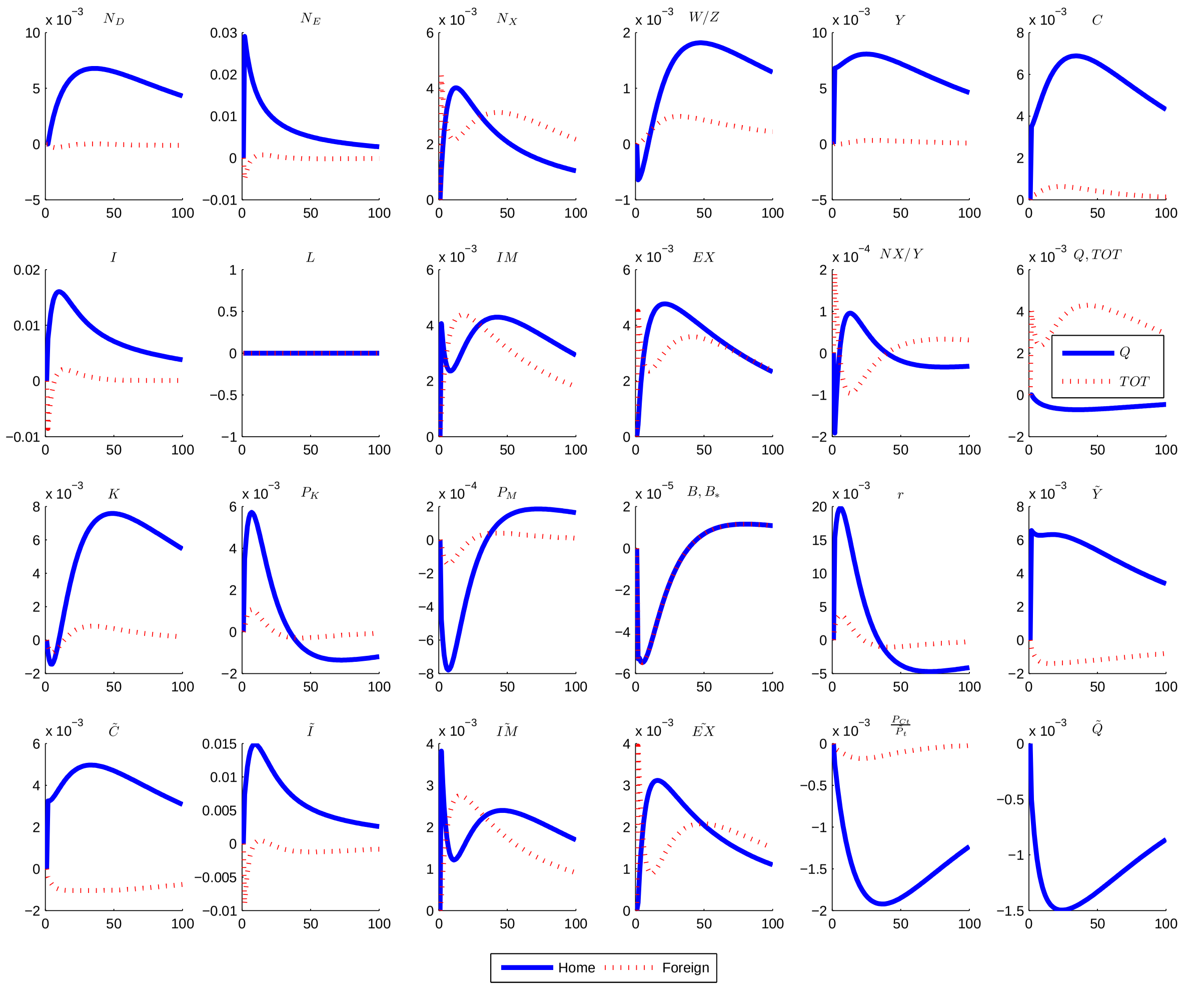

Figure 3: Impulse Response Functions - Capital Accumulation, Inelastic Labour Supply, Shocks to Intermediate Goods Production

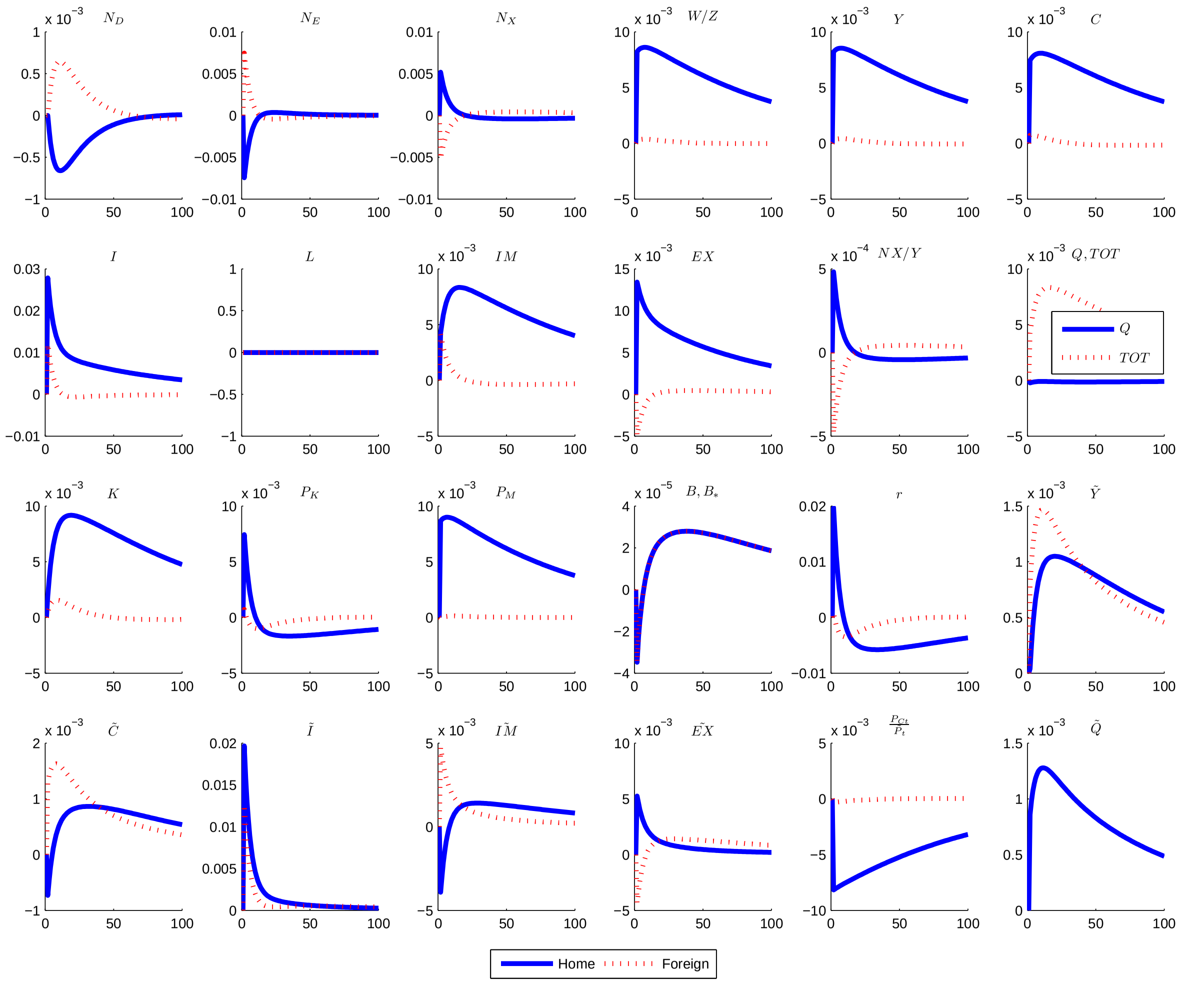

Figure 4: Impulse Response Functions - Capital Accumulation, Inelastic Labour Supply, Shocks to Final Goods Production

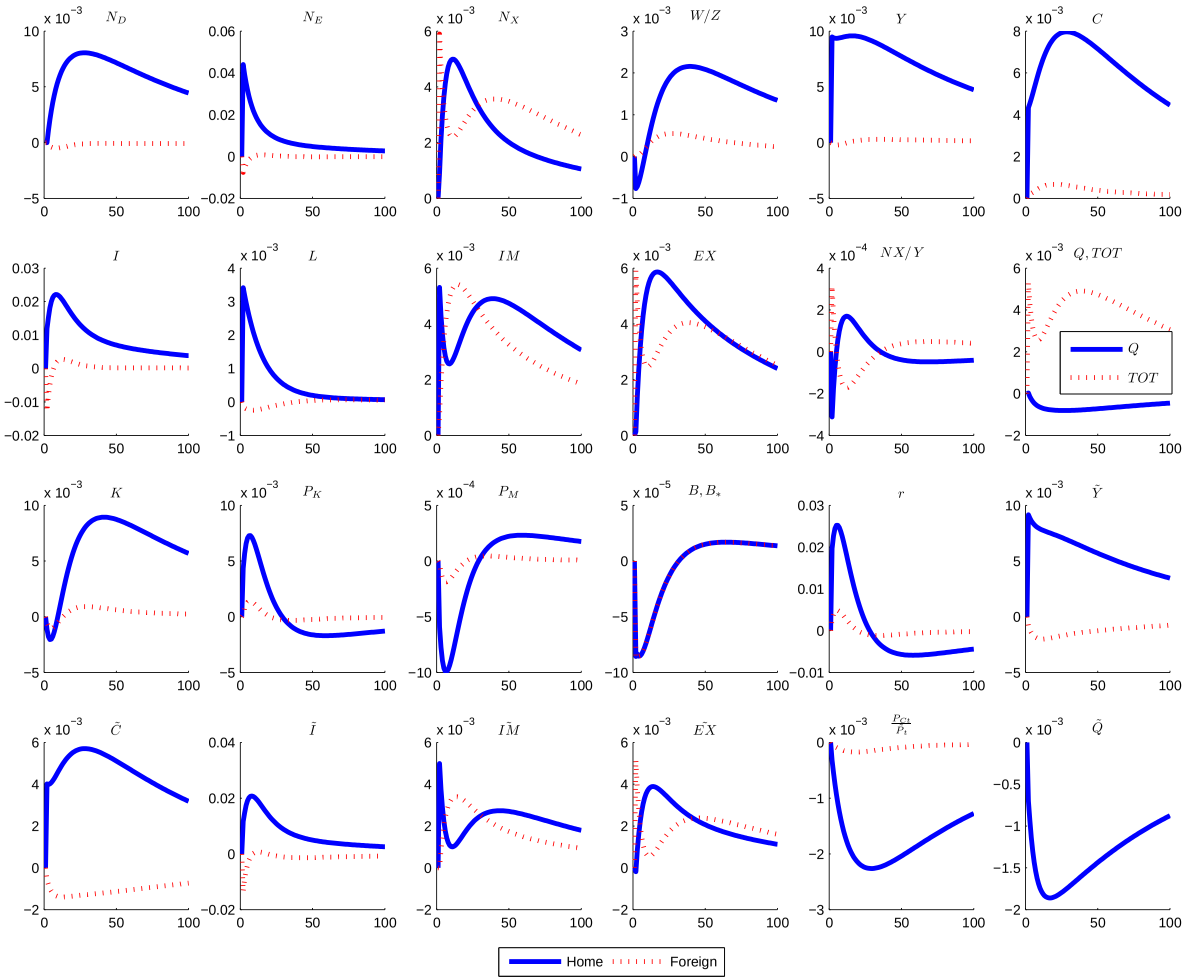

Figure 5: Impulse Response Functions - Capital Accumulation, Endogenous Labour Supply ($\lambda = 1$), Shocks to Intermediate Goods Production

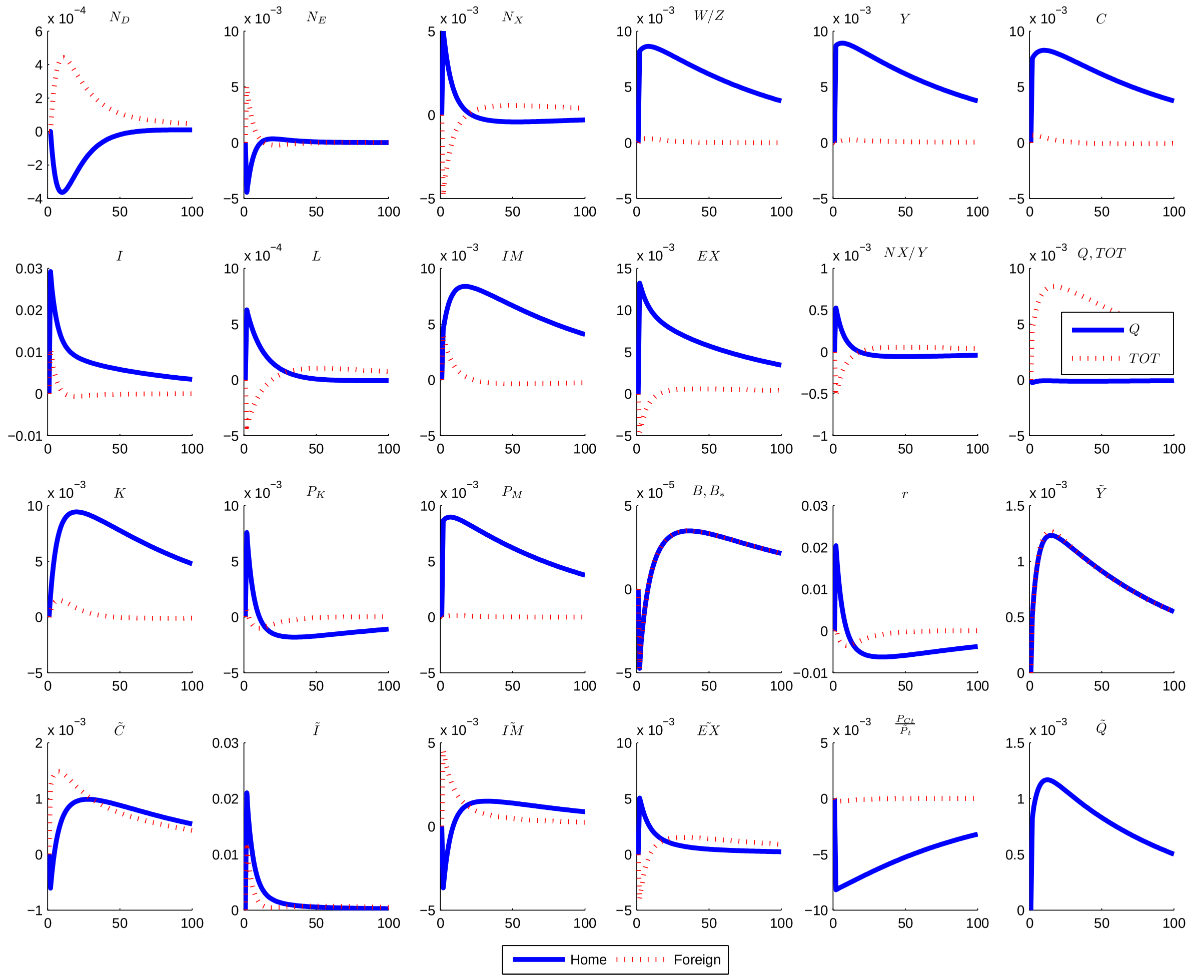

Figure 6: Impulse Response Functions - Capital Accumulation, Endogenous Labour Supply ($\lambda = 1$), Shocks to Final Goods Production

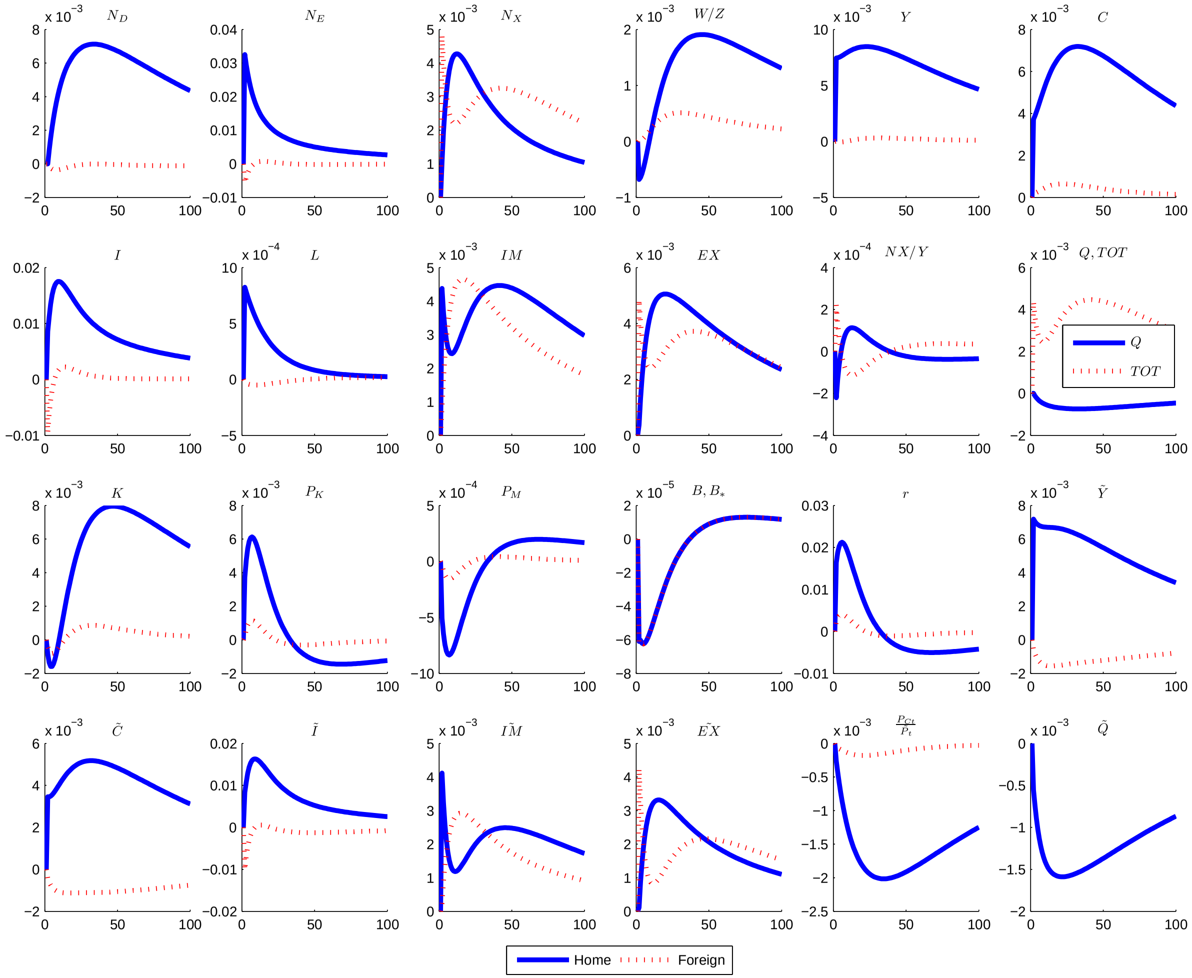

Figure 7: Impulse Response Functions - Capital Accumulation, Endogenous Labour Supply ($\lambda = 5$), Shocks to Intermediate Goods Production

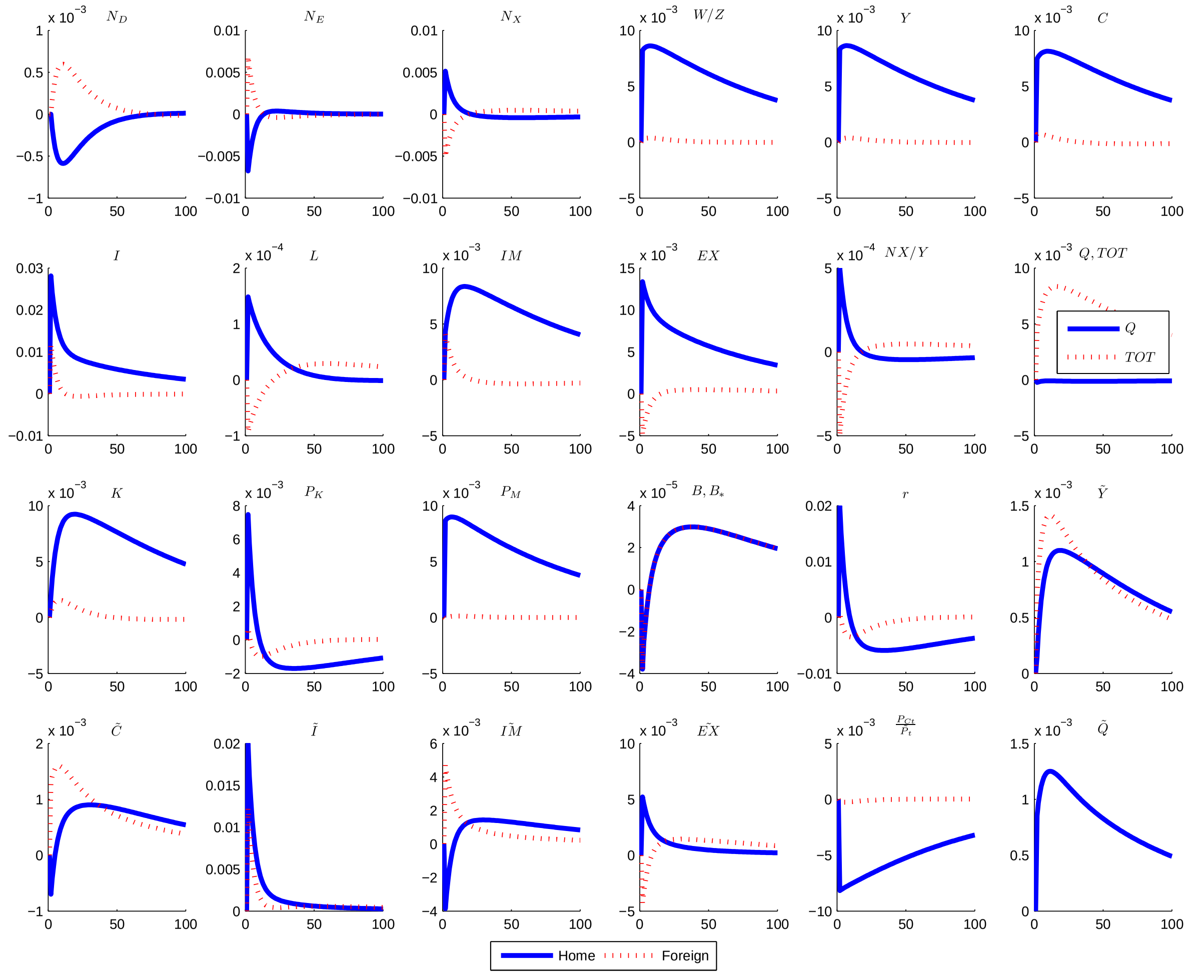

Figure 8: Impulse Response Functions - Capital Accumulation, Endogenous Labour Supply ($\lambda = 5$), Shocks to Final Goods Production