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Finite sample critical values for flexible Fourier form Lagrange-Multiplier and Dickey-Fuller unit root tests

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Enders and Lee (2012a, b) present two unit root tests that use Gallant's (1981) flexible Fourier form to model structural breaks of unknown number, form and timing in a variable's deterministic trend. The distribution of the *t* statistic generated by these Fourier-form unit root tests depends on a parameter, *k*. Before now, finite-sample critical values for these tests have been available only for $T \geq 100$ and integer values of *k*. We present critical values suitable for non-integer values of *k* and a wider range of *T*. These prove to be a non-monotonic function of *k* and quite sensitive to *T* when the number of lagged terms required to control for serial correlation are selected by Hall's (1994) general-to-specific method. In addition, we highlight an error in the critical values Enders and Lee (2012a) provide of the *F* statistic for assessing whether a nonlinear trend exists. We also find the *F* statistic's distribution to be sensitive to whether or not only integer values of *k* are considered by the unit root test and to whether or not lagged terms are included in the test equation.

JEL classifications: C12, C15, C22

Keywords: unit root test, critical values, structural breaks, flexible Fourier form

1. Introduction

Building on the work of Becker et al. (2004, 2006), Enders and Lee (2012a, b) (hereafter, E&L) extend the standard Lagrange-Multiplier (LM) and Dickey-Fuller (DF) unit root tests by using a variant of Gallant's (1981) flexible Fourier form to model the deterministic component of a variable's data-generating process (DGP). These Fourier-form LM and DF tests (hereafter referred to as the FLM and FDF tests, respectively) have been widely applied.[1] Their popularity likely stems from the fact that, among other advantages, they do not require the number, form or timing of any structural breaks in the DGP to be known beforehand.

When a variable's deterministic component is modelled under each test, the value of an important parameter, k, is generated. The distribution of each test's t statistic (*viz.* τ_{LM} and τ_{DF}) depends on the value taken by k. Helpfully, E&L provide the critical values of τ_{LM} and τ_{DF} for the integer values of k between 1 and 5, and a variety of sample sizes (*viz*., T = 100, 200, 500, 2500). E&L also provide critical values of the F statistic used to determine whether the deterministic trend is nonlinear. If the null of linearity cannot be rejected, then the FLM (or FDF) test equation is overspecified and the standard LM (or DF) test should be more powerful. Nevertheless, researchers face some potentially serious problems when using either test.

First, it is not uncommon, especially when dealing with annual data, for significantly fewer than 100 observations to be available. While the critical values presented by E&L are relatively insensitive to T, how they behave when $T < 100$ is unknown.

Second, limiting k to integer values places a potentially unwanted restriction when estimating the form of a variable's deterministic component. Some studies avoid this restriction by allowing k to take non-integer values (e.g., Christopoulos and Leon-Ledesma, 2011), but it is not known whether accurate critical values for non-integer values of k can be obtained by, say, interpolating those provided by E&L. Nor is it known if the F statistic's distribution (which does not depend on the value of k per se) is affected by whether or not k is limited to taking integer values.

Third, both tests can be run with the range of values for k expanded to include those between zero and one. In some cases this could enable a variable's DGP to be modelled more accurately than would be possible when k is limited to values greater than one. The critical values of τ_{LM} and τ_{DF} when $k < 1$ are also unknown.

[1] Google Scholar lists over 850 studies, 400 of which are also listed by Web of Science. This does not include a number of pre-2012 studies citing working-paper versions of E&L's papers.

Finally, E&L recommend including sufficient lagged terms in their test equations to control for serial correlation. It is common practice in empirical work to employ a data-driven process to determine the optimal number of lags (e.g., the general-to-specific method recommended by Hall, 1994). However, the published critical values have been constructed under the assumption that no lagged terms are required. Hence, the impact of pretesting to select the optimal number of lags on the tests' critical values is unknown.

Researchers can, of course, estimate their own critical values using a Monte Carlo simulation method tailored to their sample size, the values taken by k and their method of lag selection. However, as every individual variable could have a different value of k, and this value could change should more observations become available or revisions are made to the original data, this can be onerous when the dataset contains a large number of variables. Furthermore, it is an open question whether it is even necessary to go to the trouble of simulating critical values in the first place.

Therefore, the purpose of this paper is to present a comprehensive set of critical values for both the FLM test and the FDF test. Each set will cover all practical values of k (in 0.1 increments) for a range of sample sizes commonly encountered in applied economics (i.e., T = 40, 60 and 80, in addition to the larger values considered by E&L). Moreover, separate sets of values are provided for the zero-lag case and for when a data-driven lag-length selection method is employed.[2]

The remainder of the paper is structured as follows: Section 2 contains an outline of the FLM and FDF tests. The method used to estimate the distributions of τ_{LM} and τ_{DF}, and the results obtained are presented in Section 3. Section 4 presents the critical values of the F statistic generated by the test for a nonlinear trend, highlighting an error in the values presented by E&L (2012a). Section 5 summarises and concludes the paper.

[2] E&L's RATS code for generating critical values has not been used in this study, as it is does not accommodate a data-driven lag-length selection method. Instead, all critical values presented here have been generated using code written for the SHAZAM econometrics package (examples of which are shown in the Appendix). Being able to compare E&L's values with at least some of those presented below therefore provides some insurance against coding errors.

2. The FLM and FDF unit root tests

The FLM and FDF tests both approximate the deterministic component of a time series (y) as a single-frequency Fourier function:

$$y_t = \alpha_0 + \gamma t + \alpha_k \sin(2\pi kt/T) + \beta_k \cos(2\pi kt/T) + e_t \quad (1)$$

$$e_t = \rho e_{t-1} + u_t \quad (2)$$

where k is the single frequency selected, $\pi = 3.1415926\ldots$, T is the number of observations, and α_k and β_k measure the amplitude and displacement of the deterministic component's sinusoidal element.

To test the null hypothesis that y has a unit root (i.e., $\rho = 1$) against the alternative ($\rho < 1$), the FLM test employs the Lagrange Multiplier, or 'Score', principle by imposing the null on equation (1) and estimating it after taking first-differences:

$$\Delta y_t = \delta_0 + \delta_1 \Delta\sin(2\pi kt/T) + \delta_2 \Delta\cos(2\pi kt/T) + u_t \quad (3)$$

The coefficient estimates obtained ($\tilde{\delta}_0, \tilde{\delta}_1, \tilde{\delta}_2$) are then used to construct the following detrended series:

$$\tilde{S}_t = y_t - \tilde{\psi} - \tilde{\delta}_0 t - \tilde{\delta}_1 \sin(2\pi kt/T) - \tilde{\delta}_2 \cos(2\pi kt/T), \quad t = 2,\ldots, T \quad (4)$$

where $\tilde{\psi} = y_1 - \tilde{\delta}_0 - \tilde{\delta}_1 \sin(2\pi k/T) - \tilde{\delta}_2 \cos(2\pi k/T)$, and y_1 is the initial observation of y_t. Once augmented with j lags of $\Delta\tilde{S}$ to control for serial correlation, the FLM test equation takes the following form:

$$\Delta y_t = \varphi \tilde{S}_{t-1} + \eta_0 + \eta_1 \Delta\sin(2\pi kt/T) + \eta_2 \Delta\cos(2\pi kt/T) + \textstyle\sum_{i=1}^{j} \lambda_i \Delta\tilde{S}_{t-i} + v_t \quad (5)$$

The unit-root hypothesis is assessed using the t-statistic for the estimate of φ (τ_{LM}). E&L (2012a) show that the asymptotic distribution of τ_{LM} depends on k, but is invariant to all other parameters within the DGP.

The FDF test, on the other hand, dispenses with the pre-test detrending of y and instead estimates the following (augmented) test equation:[3, 4]

$$\Delta y_t = \varphi y_{t-1} + \mu_0 + \mu_1 \Delta\sin(2\pi kt/T) + \mu_2 \Delta\cos(2\pi kt/T) + \mu_3 t + \textstyle\sum_{i=1}^{j} \pi_i \Delta y_{t-i} + v_t \quad (6)$$

In common with the FLM test, the unit-root hypothesis is assessed using the t-statistic for the estimate of φ, denoted τ_{DF}, which also has an asymptotic distribution that depends only on k.

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[3] E&L (2012b) note that detrending can significantly reduce the power of a test when y has a large initial value. However, E&L (2012a) report that their LM version of the test generally has better size and power properties than the DF version.

[4] E&L (2012b) also present a version of the FDF test without a linear trend component (i.e., $\mu_3 = 0$). However, because the existence of a linear trend cannot be safely excluded *a priori* for most economic time series, the usefulness of the no-trend test is arguably limited and so it will not be considered here. Nevertheless, it seems reasonable to assume that the properties of the no-trend test's critical values (i.e., their degree of sensitivity to the value of T, or to the presence or absence of lagged terms, etc.) are similar to those of the with-trend variant.

As the value of k is unknown *a priori*, both tests select it using a data-driven grid-search method. Specifically, the relevant test equation is estimated for all potential values of k and the value minimizing the sum of squared residuals ($\hat{k}$) is selected.

E&L suggest that $\hat{k}$ be chosen from the integer values 1 through 5. Their upper limit on k can be justified on the grounds that it implies there are ten slope and/or intercept changes in the variable's deterministic trend. This is more likely to signify a cyclical pattern in the DGP of most economic variables rather than a series of true structural changes. For example, if y were a measure of national income, then $\hat{k} = 5$ could arise when the trend growth rate was stable but there had been a quintet of booms and recessions over the sample period.

However, the lower limit imposed on k is harder to justify. Values in the range $0 < k < 1$ could characterize a DGP with a steadily increasing (or diminishing) slope coefficient (which other unit root tests might approximate with two or more contiguous line segments). Therefore, excluding these values could limit the usefulness of the FLM and FDF tests.

Limiting k to integer values further restricts each test by forcing the trigonometric component of y's deterministic trend to end at its starting position relative to its linear component. Some researchers, therefore, have followed Christopoulos and Leon-Ledesma (2011) by selecting k from the range [0.1, 0.2 … 4.9, 5.0].

Even with the limits E&L place on k, their FLM and FDF tests offer several advantages over other unit root tests that try to account for structural breaks. First, neither test requires the number of breaks or their location to be specified beforehand. Nor is it necessary to specify whether breaks are expected to arise in the deterministic trend's intercept, slope, or both. In many cases this information cannot be known with any confidence and so the non-Fourier-form tests have some risk of generating misleading results because they have been inadvertently over- or underspecified.

A second, related advantage of the FLM and FDF tests is that their number of estimated parameters is unrelated to the number of structural breaks. Therefore, they avoid the loss of power that can arise in tests that require a dummy variable for each element of every possible break.

The non-Fourier-form unit root tests also assume that any structural breaks are instantaneous (i.e., sharp breaks) and hence the variable's deterministic trend can be modelled as a series of linear segments. The FLM and FDF tests, on the other hand, can accommodate structural changes that evolve over multiple time periods (i.e., smooth breaks). Moreover, even though the sharp-break case is not explicitly nested within equations (5) and (6), E&L (2012a) show that the power of the FLM test, at least, is comparable to that of other tests when sharp breaks are present.

A further advantage of the FLM and FDF tests is that they do not assume the only alternative to a deterministic trend with (sharp) breaks is a strictly linear one. As mentioned above, they can accommodate the possibility of a smoothly-curving deterministic trend. This feature can be useful, for example, when testing the income convergence hypothesis, as an economy may go through a lengthy transitional phase of catching up with the reference economy (e.g., due to post-WW2 reconstruction) before settling on its steady-state relative growth path (Christopoulos and Leon-Ledesma, 2011).

A final point worth noting is that the FLM and FDF tests both explicitly consider the possibility that y has no structural breaks and follows a strictly linear trend. This is done by means of an F-test of the sinusoidal terms in equations (5) and (6). If they prove to be jointly insignificant, then the test equation is overspecified and the standard LM (or DF) unit root test is likely to be more powerful. The distribution of this F statistic, $\mathrm{F}(\hat{k})$, is also nonstandard, as y is assumed to have a unit root under the null hypothesis, but it does not depend on the value of k. However, it is not known whether the distribution of $\mathrm{F}(\hat{k})$ is affected by whether or not k is limited to taking an integer value. Therefore, we estimate critical values under both scenarios.

3. Critical values of τ_{LM} and τ_{DF}

3.1 Without lagged terms to control for serial correlation

As mentioned in Section 1, E&L provide the 10%, 5% and 1% critical values of τ_{LM} and τ_{DF} for integer values of k between 1 and 5, when T = 100, 200, 500 and 2500. Their values were generated using a Monte Carlo simulation that involved 100,000 replications of each test (under the assumption that j = 0) for each pairing of k and T. We employ the same method to derive the critical values of τ_{LM} and τ_{DF} for all values of k within the range [0.1, 0.2 … 4.9, 5.0], when T = 40, 60 and 80, in addition to the larger sample sizes. The values obtained for τ_{LM} are presented in Tables 1–3 and those for τ_{DF} are presented in Tables 4–6. Each table shows E&L's original critical values in parentheses. Reassuringly, these almost exactly match our corresponding value.

Three characteristics of each test statistic's distribution can be identified from Tables 1–6. First, their relative insensitivity to the value of T extends to very small sample sizes. More specifically, the magnitude of the 10% critical value for each test when T = 40 (see Tables 1 and 4) is either the same (for moderate values of k) or only slightly larger (for small values of k) than that for $T = 2500$. For relatively large values of k, the magnitude of the 10% critical value is actually slightly *positively*-related to T.

This pattern is less evident among the 5% and 1% critical values, but it is only at relatively low values of k that these noticeably increase in magnitude when the sample size is small – especially in the case of τ_{LM}. However, even here the practical implications are relatively minor. For example, had a researcher relied on E&L's (2012a) critical values for $T = 100$ when testing a variable with 40 observations, the true significance level of τ_{LM} would have been understated by no more than two percentage points.

Second, the critical values for non-integer values of $k > 1$ can be reasonably accurately estimated by interpolation. However, as illustrated by Figure 1, which plots each test statistic's 10%, 5% and 1% critical values against k (for $T = 100$), this is truer for τ_{DF} than for τ_{LM}. To some extent the critical values of τ_{LM} change in a series of steps. More specifically, they are relatively stable around the integer values of k, but become more sensitive near the midpoint of some inter-integer ranges. This implies that the use of interpolated critical values will generally result in the strength of evidence against the null hypothesis being overstated. However, this is also a minor problem in practice. For example, when $T = 100$, the difference between the actual 10% critical value of τ_{LM} (shown in Table 1) and its interpolated value is greatest when $k = 1.2$. However, the true significance level of the interpolated value is only a little greater at 12.2%.

Finally, Figure 1 also reveals that the critical values of both τ_{LM} and τ_{DF} are not monotonic functions of k. Instead, their magnitude peaks at or around $k = 1$ and then gradually diminishes as k approaches zero. This is not what one might have anticipated from the values provided by E&L, as their magnitude consistently rises as k falls and in fact does so most strongly between $k = 2$ and $k = 1$.

3.2 When lagged terms are selected by a general-to-specific method

The critical values presented in Tables 1–6 were derived under the assumption that no lagged terms were needed to control for serial correlation when estimating equations (5) and (6) (i.e., $j = 0$). It is, however, normal practice to augment these test equations with a relatively large number of lagged terms (j_{max}) and then employ a testing process that lets the data determine the optimal lag length. To assess the effect of such pretesting on the distributions of τ_{LM} and τ_{DF}, a second set of critical values is constructed.

Here it is assumed that, for each and every replication, Hall's (1994) general-to-specific method is used to determine the optimal lag length. More specifically, a general version of the test equation is initially estimated with j_{max} set equal to the integer value of the square-root of T. If the highest lagged term is statistically insignificant, j is progressively reduced until a statistically significant term is obtained. The τ_{LM} (or τ_{DF}) statistic is then taken from this specific version of the test equation. As before, 100,000 replications of each test are conducted for each pairing of k and T. The critical values of τ_{LM} (τ_{DF}) obtained are presented in Tables 7–9 (Tables 10–12).

When compared with the first set of critical values, the corresponding second set are often, but not always, noticeably greater in magnitude. This is the case, for example, for both tests when $k \leq 2$ and $T \leq 500$. It is also true at higher values of k for some smaller values of T. However, in relatively large samples ($T \geq 200$), the relative magnitude of the two sets of critical values reverses between $k = 2$ and $k = 3$.

The sensitivity of these critical values to the use of a lag-length selection method and the influence of the values of k and T is more clearly illustrated by Figures 2 and 3. These plot – for τ_{LM} and τ_{DF}, respectively – the p-value implied by the first set of critical values (with $j = 0$) according to the corresponding distribution estimated using Hall's (1994) general-to-specific lag selection method. Clearly, there are many instances where reliance on the first set of critical values would materially overstate the strength of evidence against the null of a unit root when τ_{LM} or τ_{DF} have been estimated using a lag-length selection method.

The second set of critical values also differs from the first in being more sensitive to the value of T. This can be most clearly seen in Figures 4 (for τ_{LM}) and 5 (for τ_{DF}). Panel (a) in each figure plots the 10% critical value (for $j = 0$) against k for each value of T, whereas panel (b) plots the corresponding critical value obtained when the general-to-specific method is used to determine j. The lines plotted in Figures 4(a) and 5(a) are noticeably more closely stacked than those shown in Figures 4(b) and 5(b).

4. Critical values of $\mathbf{F}(\hat{k})$

As explained in more detail in the Appendix, there is an error in E&L's (2012a) RATS code for computing the critical values of $F(\hat{k})$ for the FLM test. As can be seen in Table 13, correcting this error results in substantially smaller critical values. For example, the corrected 1% critical value is actually smaller than E&L's (2012a) 10% critical value for all values of T. Hence, reliance on the original set of values creates a significant risk of preferring the FLM test over the standard LM test when the former is overspecified and potentially less powerful than the latter.

In general terms, the (corrected) critical values of $F(\hat{k})$ under the FLM test are not especially sensitive to the value of T. It is only between $T = 60$ and $T = 40$ that a noteworthy jump in their values occurs and even then the practical implications of this are noteworthy, but not dramatic. For example, should a researcher use the 10% critical value for $T = 100$ for a variable with only 40 observations, its true significance level would be a shade under 16%.

However, Panel (b) of Table 13 shows that allowing k to take non-integer values when estimating the $F(\hat{k})$ statistic does result in materially larger critical values. Specifically, the significance level of the critical values derived when k is limited to integer values at least doubles when the test is run without that restriction. This effect becomes a little less pronounced as T increases, but nonetheless remains sizeable for $T = 2500$.

Relaxing the assumption that $j = 0$ by selecting its value using a general-to-specific method further increases the critical values of $F(\hat{k})$, as can be seen in Panel (c) of Table 13. However, for $T \geq 500$ the increase is generally trivial. Moreover, when $T = 100$ or 200, the 10% critical values are largely unaffected. Therefore, it is only for relatively small sample sizes that the use of lag selection method has an important practical impact on the distribution of $F(\hat{k})$.

Table 14 contains the critical values of $F(\hat{k})$ in the context of the FDF test. E&L's (2012b) original critical values were correctly derived and, as can be seen in Panel (a), closely match our own. Generally speaking, the same patterns observed in the critical values of the FLM test's $F(\hat{k})$ statistic are also seen in the context of the FDF test. The only difference of note is that the distribution of the $F(\hat{k})$ statistic under the FDF test is somewhat more sensitive to the use of a lag selection process than is the case for the FLM test.

5. Summary and conclusions

The critical values of the t statistic generated by E&L's FLM and FDF unit root tests are known to depend on a parameter (k) generated as part of the testing process. Before now, however, only the values for integer values of k have been publicly available. In addition, even if the critical values for non-integer values of k could be accurately interpolated from these published values (which had not been established prior to this study), there has been a potentially useful range of values (i.e., $0 < k < 1$) with no publicly available critical values. Furthermore, while previously published critical values show little sensitivity to the sample size (when $T \geq 100$), whether this is also true for smaller samples has not been known until now. Another unknown has been the impact of pretesting to determine the optimal number of lags when controlling for serial correlation, as the published critical values had been derived under the assumption that lagged terms were unnecessary. Finally, the impact of non-integer values of k, small sample sizes and lag-length selection methods on the F statistic used to determine whether the FLM or FDF test is overspecified has not been previously considered. This study fills all of these gaps.

To begin with, two sets of critical values for each test's t statistic are presented for a broad range of values of both k and T. The first set assumes that no lagged terms are needed to control for serial correlation (i.e., $j = 0$), whereas the second employs Hall's (1994) general-to-specific method for enabling the data to determine the optimal number of lagged terms.

We find that, when $j = 0$ is assumed, the 10% critical values of the t statistic for both tests are relatively insensitive to the value of T, even for samples as small as 40 observations. The 5% and 1% critical values for relatively small values of k are more sensitive to changes in the sample size, but this only becomes noticeable when $T < 60$. Even then, the practical consequences of this are minor; each test's significance level is understated by two percentage points at most when the critical values for $T = 100$ are used to assess the t statistic for a variable with 40 observations.

We also find that simple interpolation is a reasonably accurate way to estimate the critical values of the t statistic for non-integer values of $k > 1$ from those provided by E&L for integer values of k. Strictly speaking, both tests' critical values are not linearly related to k, especially in the case of the FLM test. However, the relationship is very nearly so for relatively large values of k and sufficiently close to linear otherwise that interpolated values overstate the test statistic's significance level by approximately two percentage points at most.

Interestingly, the t statistic's critical values under both tests turn out to be non-monotonic functions of k, something that could not have been anticipated from E&L's own critical values. Specifically, although their magnitude increases as the value of k falls across a wide range of values, it reaches a peak at or around $k = 1$ and then diminishes as k tends to zero.

Another key finding is that each t statistic's critical values noticeably change when a general-to-specific method is employed to determine the optimal number of lagged terms in the test equation. Specifically, the critical values markedly increase in magnitude, unless both T and k take relatively large values. In addition, these critical values are more sensitive to the value of T than is those derived under the assumption that $j = 0$. The difference is such that relying on the E&L's set of critical values when a lag selection process has been used would materially overstate the unit root test's significance level in many instances. Therefore, given it is common practice to use a data-driven lag selection method to control for serial correlation, we strongly recommend that the critical values presented in Tables 7–9 (for the FLM test) and Table 10–12 (for the FDF test) be used, unless the sample size is very large (i.e., more than 500 observations).

Finally, aside from correcting an error in E&L's (2012a) original critical values for the F($\hat{k}$) statistic, we find that allowing k to take non-integer values materially affects the distribution of F($\hat{k}$) across all T under both tests. In addition, allowing the data to determine the optimal number of lagged terms in the test equation further increases the critical values of F($\hat{k}$), unless the sample size is reasonably large. This further underscores the need for researchers applying Fourier-form unit root tests to use critical values that are tailored to their sample size and testing method.

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Table 1: 10% critical values of τ_{LM} (j = 0, by assumption)

k	T = 40	T = 60	T = 80	T = 100	T = 200	T = 500	T = 2500
0.1	−3.79	−3.76	−3.73	−3.71	−3.70	−3.68	−3.66
0.2	−3.79	−3.76	−3.74	−3.72	−3.69	−3.68	−3.67
0.3	−3.81	−3.77	−3.74	−3.73	−3.71	−3.69	−3.67
0.4	−3.82	−3.77	−3.76	−3.74	−3.71	−3.70	−3.68
0.5	−3.84	−3.78	−3.76	−3.76	−3.73	−3.72	−3.69
0.6	−3.85	−3.81	−3.78	−3.77	−3.74	−3.73	−3.70
0.7	−3.88	−3.82	−3.8	−3.78	−3.76	−3.74	−3.71
0.8	−3.89	−3.84	−3.82	−3.80	−3.78	−3.76	−3.73
0.9	−3.91	−3.85	−3.82	−3.82	−3.79	−3.77	−3.74
1.0	**−3.93**	**−3.87**	**−3.83**	**−3.82**	**−3.80**	**−3.77**	**−3.74**
				(−3.82)	**(−3.79)**	**(−3.78)**	**(−3.77)**
1.1	−3.92	−3.86	−3.83	−3.82	−3.79	−3.76	−3.74
1.2	−3.88	−3.83	−3.81	−3.79	−3.76	−3.72	−3.71
1.3	−3.82	−3.77	−3.74	−3.73	−3.70	−3.67	−3.65
1.4	−3.72	−3.66	−3.65	−3.63	−3.61	−3.59	−3.57
1.5	−3.60	−3.56	−3.53	−3.52	−3.50	−3.48	−3.47
1.6	−3.47	−3.43	−3.42	−3.41	−3.39	−3.37	−3.37
1.7	−3.36	−3.33	−3.33	−3.32	−3.31	−3.30	−3.29
1.8	−3.29	−3.28	−3.27	−3.27	−3.25	−3.26	−3.25
1.9	−3.25	−3.24	−3.23	−3.24	−3.22	−3.23	−3.22
2.0	**−3.23**	**−3.23**	**−3.23**	**−3.22**	**−3.21**	**−3.22**	**−3.22**
				(−3.23)	**(−3.23)**	**(−3.22)**	**(−3.22)**
2.1	−3.23	−3.23	−3.22	−3.23	−3.22	−3.22	−3.23
2.2	−3.23	−3.23	−3.23	−3.23	−3.22	−3.22	−3.21
2.3	−3.22	−3.22	−3.21	−3.21	−3.21	−3.20	−3.21
2.4	−3.20	−3.19	−3.18	−3.19	−3.17	−3.17	−3.18
2.5	−3.13	−3.14	−3.14	−3.14	−3.14	−3.12	−3.14
2.6	−3.09	−3.09	−3.08	−3.09	−3.08	−3.07	−3.09
2.7	−3.02	−3.03	−3.04	−3.03	−3.03	−3.03	−3.05
2.8	−2.98	−2.99	−2.99	−2.99	−3.00	−3.00	−3.01
2.9	−2.95	−2.96	−2.98	−2.97	−2.98	−2.98	−2.99
3.0	**−2.92**	**−2.95**	**−2.96**	**−2.97**	**−2.97**	**−2.97**	**−2.99**
				(−2.96)	**(−2.98)**	**(−2.98)**	**(−2.98)**
3.1	−2.93	−2.94	−2.97	−2.97	−2.97	−2.97	−2.99
3.2	−2.93	−2.95	−2.96	−2.97	−2.98	−2.98	−2.99
3.3	−2.93	−2.96	−2.96	−2.96	−2.98	−2.98	−2.99
3.4	−2.93	−2.94	−2.95	−2.97	−2.98	−2.98	−2.98
3.5	−2.92	−2.93	−2.95	−2.94	−2.95	−2.96	−2.97
3.6	−2.90	−2.91	−2.92	−2.93	−2.93	−2.94	−2.94
3.7	−2.86	−2.89	−2.90	−2.90	−2.91	−2.91	−2.93
3.8	−2.83	−2.86	−2.86	−2.88	−2.88	−2.89	−2.90
3.9	−2.82	−2.84	−2.86	−2.86	−2.87	−2.88	−2.89
4.0	**−2.80**	**−2.83**	**−2.84**	**−2.85**	**−2.87**	**−2.87**	**−2.88**
				(−2.86)	**(−2.88)**	**(−2.88)**	**(−2.88)**
4.1	−2.79	−2.83	−2.85	−2.86	−2.87	−2.87	−2.89
4.2	−2.80	−2.83	−2.85	−2.86	−2.88	−2.88	−2.89
4.3	−2.80	−2.84	−2.86	−2.86	−2.89	−2.88	−2.89
4.4	−2.82	−2.84	−2.86	−2.86	−2.88	−2.88	−2.89
4.5	−2.80	−2.83	−2.85	−2.86	−2.87	−2.88	−2.88
4.6	−2.79	−2.82	−2.84	−2.84	−2.86	−2.85	−2.86
4.7	−2.79	−2.81	−2.83	−2.84	−2.85	−2.85	−2.85
4.8	−2.77	−2.80	−2.81	−2.82	−2.83	−2.83	−2.84
4.9	−2.76	−2.78	−2.81	−2.82	−2.82	−2.83	−2.84
5.0	**−2.74**	**−2.77**	**−2.80**	**−2.80**	**−2.82**	**−2.82**	**−2.83**
				(−2.81)	**(−2.83)**	**(−2.83)**	**(−2.83)**

Notes: Enders and Lee's (2012a, Table 1) values are shown in parentheses.

Table 2: 5% critical values of τ_{LM} ($j = 0$, by assumption)

k	$T = 40$	$T = 60$	$T = 80$	$T = 100$	$T = 200$	$T = 500$	$T = 2500$
0.1	−4.13	−4.06	−4.02	−3.99	−3.97	−3.95	−3.93
0.2	−4.11	−4.07	−4.03	−4.01	−3.97	−3.95	−3.94
0.3	−4.14	−4.08	−4.04	−4.02	−3.98	−3.96	−3.93
0.4	−4.15	−4.08	−4.06	−4.04	−3.99	−3.97	−3.95
0.5	−4.17	−4.09	−4.06	−4.06	−4.01	−3.99	−3.96
0.6	−4.18	−4.12	−4.08	−4.06	−4.01	−3.99	−3.98
0.7	−4.21	−4.13	−4.10	−4.07	−4.04	−4.02	−3.98
0.8	−4.22	−4.14	−4.12	−4.09	−4.05	−4.02	−4.00
0.9	−4.24	−4.17	−4.12	−4.11	−4.07	−4.04	−4.01
1.0	**−4.26**	**−4.17**	**−4.13**	**−4.11**	**−4.08**	**−4.04**	**−4.01**
				(−4.10)	**(−4.07)**	**(−4.05)**	**(−4.03)**
1.1	−4.24	−4.17	−4.13	−4.10	−4.07	−4.04	−4.01
1.2	−4.21	−4.13	−4.10	−4.08	−4.03	−3.99	−3.98
1.3	−4.16	−4.07	−4.04	−4.02	−3.98	−3.94	−3.92
1.4	−4.05	−3.98	−3.96	−3.94	−3.89	−3.86	−3.85
1.5	−3.95	−3.88	−3.84	−3.83	−3.80	−3.77	−3.77
1.6	−3.82	−3.77	−3.74	−3.74	−3.70	−3.69	−3.67
1.7	−3.72	−3.68	−3.67	−3.65	−3.63	−3.61	−3.60
1.8	−3.66	−3.63	−3.62	−3.61	−3.58	−3.57	−3.57
1.9	−3.63	−3.61	−3.57	−3.57	−3.56	−3.55	−3.54
2.0	**−3.62**	**−3.58**	**−3.58**	**−3.56**	**−3.55**	**−3.54**	**−3.54**
				(−3.57)	**(−3.55)**	**(−3.54)**	**(−3.54)**
2.1	−3.61	−3.60	−3.57	−3.58	−3.56	−3.54	−3.55
2.2	−3.61	−3.58	−3.58	−3.57	−3.56	−3.54	−3.53
2.3	−3.60	−3.58	−3.56	−3.55	−3.53	−3.53	−3.52
2.4	−3.57	−3.54	−3.52	−3.51	−3.50	−3.49	−3.51
2.5	−3.51	−3.49	−3.49	−3.47	−3.46	−3.44	−3.45
2.6	−3.47	−3.44	−3.42	−3.42	−3.40	−3.38	−3.41
2.7	−3.39	−3.38	−3.38	−3.36	−3.37	−3.34	−3.37
2.8	−3.34	−3.34	−3.33	−3.33	−3.32	−3.32	−3.33
2.9	−3.31	−3.32	−3.32	−3.32	−3.32	−3.31	−3.31
3.0	**−3.30**	**−3.30**	**−3.32**	**−3.31**	**−3.29**	**−3.30**	**−3.31**
				(−3.31)	**(−3.30)**	**(−3.31)**	**(−3.30)**
3.1	−3.30	−3.29	−3.32	−3.31	−3.30	−3.30	−3.31
3.2	−3.31	−3.30	−3.30	−3.31	−3.31	−3.31	−3.32
3.3	−3.30	−3.33	−3.31	−3.30	−3.31	−3.30	−3.31
3.4	−3.29	−3.28	−3.30	−3.30	−3.30	−3.29	−3.29
3.5	−3.28	−3.27	−3.28	−3.27	−3.27	−3.26	−3.28
3.6	−3.25	−3.25	−3.25	−3.26	−3.25	−3.26	−3.26
3.7	−3.21	−3.22	−3.23	−3.23	−3.23	−3.22	−3.23
3.8	−3.18	−3.19	−3.19	−3.20	−3.19	−3.20	−3.20
3.9	−3.17	−3.17	−3.18	−3.19	−3.19	−3.19	−3.20
4.0	**−3.15**	**−3.16**	**−3.17**	**−3.18**	**−3.18**	**−3.17**	**−3.19**
				(−3.18)	**(−3.18)**	**(−3.19)**	**(−3.19)**
4.1	−3.14	−3.17	−3.18	−3.19	−3.19	−3.19	−3.19
4.2	−3.15	−3.16	−3.17	−3.19	−3.20	−3.19	−3.20
4.3	−3.14	−3.17	−3.18	−3.18	−3.20	−3.19	−3.20
4.4	−3.16	−3.18	−3.18	−3.18	−3.20	−3.19	−3.19
4.5	−3.14	−3.16	−3.18	−3.18	−3.17	−3.18	−3.18
4.6	−3.13	−3.15	−3.16	−3.16	−3.17	−3.16	−3.16
4.7	−3.12	−3.13	−3.15	−3.16	−3.16	−3.15	−3.15
4.8	−3.09	−3.12	−3.14	−3.13	−3.14	−3.13	−3.14
4.9	−3.07	−3.10	−3.12	−3.13	−3.13	−3.12	−3.14
5.0	**−3.07**	**−3.09**	**−3.12**	**−3.12**	**−3.12**	**−3.13**	**−3.13**
				(−3.11)	**(−3.12)**	**(−3.14)**	**(−3.13)**

Notes: As for Table 1.

Table 3: 1% critical values of τ_{LM} (j = 0, by assumption)

k	T = 40	T = 60	T = 80	T = 100	T = 200	T = 500	T = 2500
0.1	−4.81	−4.70	−4.63	−4.59	−4.54	−4.50	−4.45
0.2	−4.81	−4.69	−4.63	−4.60	−4.51	−4.49	−4.47
0.3	−4.82	−4.71	−4.64	−4.60	−4.53	−4.49	−4.46
0.4	−4.83	−4.71	−4.68	−4.62	−4.54	−4.50	−4.49
0.5	−4.85	−4.71	−4.68	−4.64	−4.54	−4.52	−4.49
0.6	−4.89	−4.77	−4.68	−4.65	−4.56	−4.53	−4.51
0.7	−4.89	−4.76	−4.69	−4.66	−4.57	−4.54	−4.51
0.8	−4.92	−4.78	−4.72	−4.65	−4.58	−4.55	−4.52
0.9	−4.93	−4.80	−4.71	−4.69	−4.61	−4.56	−4.52
1.0	**−4.94**	**−4.81**	**−4.72**	**−4.69**	**−4.64**	**−4.58**	**−4.53**
				(−4.69)	**(−4.61)**	**(−4.57)**	**(−4.56)**
1.1	−4.92	−4.81	−4.72	−4.69	−4.62	−4.56	−4.54
1.2	−4.89	−4.76	−4.72	−4.66	−4.56	−4.54	−4.51
1.3	−4.84	−4.71	−4.61	−4.62	−4.52	−4.49	−4.45
1.4	−4.75	−4.64	−4.57	−4.52	−4.47	−4.40	−4.39
1.5	−4.64	−4.53	−4.46	−4.42	−4.38	−4.32	−4.33
1.6	−4.57	−4.42	−4.38	−4.36	−4.30	−4.28	−4.25
1.7	−4.47	−4.35	−4.33	−4.29	−4.24	−4.20	−4.19
1.8	−4.42	−4.32	−4.29	−4.25	−4.21	−4.18	−4.15
1.9	−4.36	−4.30	−4.26	−4.23	−4.19	−4.17	−4.13
2.0	**−4.36**	**−4.28**	**−4.25**	**−4.20**	**−4.16**	**−4.16**	**−4.15**
				(−4.25)	**(−4.18)**	**(−4.13)**	**(−4.15)**
2.1	−4.37	−4.29	−4.24	−4.24	−4.19	−4.16	−4.15
2.2	−4.36	−4.28	−4.25	−4.24	−4.19	−4.14	−4.14
2.3	−4.35	−4.27	−4.25	−4.21	−4.17	−4.12	−4.13
2.4	−4.36	−4.24	−4.19	−4.18	−4.13	−4.09	−4.11
2.5	−4.26	−4.20	−4.13	−4.11	−4.08	−4.05	−4.06
2.6	−4.23	−4.13	−4.09	−4.09	−4.03	−4.00	−4.02
2.7	−4.15	−4.06	−4.06	−4.01	−4.00	−3.95	−3.97
2.8	−4.09	−4.05	−4.01	−4.00	−3.96	−3.94	−3.95
2.9	−4.08	−4.01	−4.00	−3.99	−3.97	−3.95	−3.94
3.0	**−4.05**	**−4.02**	**−4.00**	**−3.99**	**−3.94**	**−3.92**	**−3.93**
				(−3.98)	**(−3.94)**	**(−3.94)**	**(−3.94)**
3.1	−4.07	−4.02	−4.00	−4.00	−3.95	−3.92	−3.94
3.2	−4.08	−4.00	−4.00	−3.99	−3.95	−3.94	−3.93
3.3	−4.06	−4.02	−4.01	−3.97	−3.94	−3.92	−3.92
3.4	−4.05	−3.98	−3.97	−3.94	−3.94	−3.91	−3.91
3.5	−4.04	−3.95	−3.95	−3.92	−3.90	−3.87	−3.89
3.6	−3.99	−3.96	−3.93	−3.90	−3.87	−3.88	−3.88
3.7	−3.93	−3.89	−3.90	−3.87	−3.86	−3.83	−3.83
3.8	−3.93	−3.88	−3.86	−3.86	−3.82	−3.83	−3.82
3.9	−3.90	−3.87	−3.86	−3.83	−3.81	−3.79	−3.79
4.0	**−3.87**	**−3.85**	**−3.86**	**−3.83**	**−3.81**	**−3.77**	**−3.79**
				(−3.85)	**(−3.80)**	**(−3.81)**	**(−3.80)**
4.1	−3.88	−3.85	−3.85	−3.83	−3.81	−3.79	−3.80
4.2	−3.89	−3.84	−3.83	−3.83	−3.81	−3.80	−3.81
4.3	−3.90	−3.87	−3.84	−3.82	−3.82	−3.79	−3.80
4.4	−3.89	−3.85	−3.85	−3.85	−3.82	−3.80	−3.78
4.5	−3.85	−3.84	−3.81	−3.84	−3.82	−3.79	−3.78
4.6	−3.86	−3.80	−3.81	−3.80	−3.78	−3.75	−3.74
4.7	−3.83	−3.80	−3.80	−3.78	−3.77	−3.76	−3.74
4.8	−3.81	−3.76	−3.79	−3.77	−3.76	−3.71	−3.73
4.9	−3.76	−3.75	−3.76	−3.76	−3.75	−3.71	−3.74
5.0	**−3.75**	**−3.76**	**−3.75**	**−3.74**	**−3.73**	**−3.71**	**−3.72**
				(−3.75)	**(−3.73)**	**(−3.75)**	**(−3.74)**

Notes: As for Table 1.

Table 4: 10% critical values of τ_{DF} ($j = 0$, by assumption)

k	$T = 40$	$T = 60$	$T = 80$	$T = 100$	$T = 200$	$T = 500$	$T = 2500$
0.1	−4.04	−4.00	−3.98	−3.96	−3.94	−3.93	−3.91
0.2	−4.05	−4.00	−3.98	−3.97	−3.94	−3.93	−3.91
0.3	−4.06	−4.01	−3.99	−3.98	−3.95	−3.93	−3.91
0.4	−4.06	−4.02	−3.99	−3.98	−3.96	−3.95	−3.92
0.5	−4.06	−4.04	−3.99	−3.99	−3.96	−3.96	−3.92
0.6	−4.09	−4.03	−4.02	−4.00	−3.97	−3.95	−3.94
0.7	−4.10	−4.04	−4.02	−4.02	−3.99	−3.96	−3.95
0.8	−4.11	−4.07	−4.04	−4.03	−4.00	−3.98	−3.96
0.9	−4.13	−4.08	−4.05	−4.04	−4.01	−4.00	−3.97
1.0	**−4.15**	**−4.09**	**−4.07**	**−4.05**	**−4.02**	**−4.00**	**−3.98**
				(−4.05)	**(−4.02)**	**(−4.01)**	**(−4.00)**
1.1	−4.15	−4.10	−4.07	−4.06	−4.03	−4.01	−3.99
1.2	−4.17	−4.10	−4.08	−4.05	−4.03	−4.01	−3.99
1.3	−4.15	−4.09	−4.06	−4.05	−4.01	−4.00	−3.97
1.4	−4.12	−4.07	−4.03	−4.02	−3.98	−3.97	−3.95
1.5	−4.08	−4.02	−3.99	−3.99	−3.94	−3.93	−3.91
1.6	−4.02	−3.97	−3.94	−3.93	−3.90	−3.88	−3.86
1.7	−3.96	−3.91	−3.88	−3.88	−3.85	−3.83	−3.81
1.8	−3.90	−3.85	−3.83	−3.81	−3.79	−3.78	−3.76
1.9	−3.83	−3.79	−3.77	−3.77	−3.75	−3.72	−3.72
2.0	**−3.77**	**−3.73**	**−3.70**	**−3.71**	**−3.68**	**−3.67**	**−3.67**
				(−3.71)	**(−3.69)**	**(−3.67)**	**(−3.67)**
2.1	−3.70	−3.68	−3.67	−3.66	−3.63	−3.63	−3.62
2.2	−3.67	−3.63	−3.63	−3.63	−3.60	−3.60	−3.59
2.3	−3.63	−3.61	−3.60	−3.60	−3.58	−3.57	−3.58
2.4	−3.59	−3.59	−3.58	−3.57	−3.56	−3.56	−3.55
2.5	−3.57	−3.55	−3.56	−3.55	−3.54	−3.53	−3.54
2.6	−3.55	−3.53	−3.53	−3.52	−3.52	−3.52	−3.52
2.7	−3.53	−3.51	−3.51	−3.50	−3.50	−3.49	−3.50
2.8	−3.50	−3.48	−3.49	−3.48	−3.47	−3.48	−3.47
2.9	−3.46	−3.46	−3.45	−3.46	−3.45	−3.45	−3.45
3.0	**−3.44**	**−3.43**	**−3.44**	**−3.44**	**−3.43**	**−3.43**	**−3.44**
				(−3.44)	**(−3.43)**	**(−3.43)**	**(−3.43)**
3.1	−3.41	−3.41	−3.41	−3.41	−3.42	−3.41	−3.41
3.2	−3.38	−3.38	−3.39	−3.39	−3.38	−3.39	−3.40
3.3	−3.35	−3.36	−3.37	−3.36	−3.38	−3.37	−3.38
3.4	−3.33	−3.34	−3.35	−3.36	−3.36	−3.36	−3.37
3.5	−3.32	−3.34	−3.34	−3.34	−3.35	−3.35	−3.36
3.6	−3.31	−3.33	−3.34	−3.34	−3.34	−3.34	−3.35
3.7	−3.30	−3.32	−3.32	−3.32	−3.33	−3.33	−3.34
3.8	−3.29	−3.31	−3.31	−3.31	−3.32	−3.32	−3.33
3.9	−3.28	−3.30	−3.31	−3.31	−3.32	−3.32	−3.32
4.0	**−3.26**	**−3.28**	**−3.30**	**−3.30**	**−3.31**	**−3.30**	**−3.31**
				(−3.29)	**(−3.31)**	**(−3.30)**	**(−3.30)**
4.1	−3.26	−3.27	−3.29	−3.29	−3.30	−3.30	−3.29
4.2	−3.24	−3.26	−3.26	−3.27	−3.28	−3.29	−3.29
4.3	−3.23	−3.25	−3.26	−3.27	−3.27	−3.27	−3.28
4.4	−3.21	−3.24	−3.25	−3.25	−3.26	−3.27	−3.27
4.5	−3.19	−3.24	−3.25	−3.26	−3.26	−3.27	−3.27
4.6	−3.19	−3.23	−3.25	−3.24	−3.26	−3.26	−3.26
4.7	−3.19	−3.22	−3.23	−3.24	−3.25	−3.26	−3.26
4.8	−3.19	−3.23	−3.23	−3.24	−3.24	−3.25	−3.24
4.9	−3.18	−3.21	−3.22	−3.23	−3.24	−3.25	−3.25
5.0	**−3.19**	**−3.21**	**−3.22**	**−3.23**	**−3.24**	**−3.24**	**−3.24**
				(−3.22)	**(−3.24)**	**(−3.24)**	**(−3.24)**

Notes: Enders and Lee's (2012b, Table 1a) values are shown in parentheses.

Table 5: 5% critical values of τ_{DF} (j = 0, by assumption)

k	$T = 40$	$T = 60$	$T = 80$	$T = 100$	$T = 200$	$T = 500$	$T = 2500$
0.1	−4.40	−4.32	−4.29	−4.26	−4.23	−4.22	−4.19
0.2	−4.40	−4.33	−4.29	−4.27	−4.24	−4.21	−4.18
0.3	−4.41	−4.33	−4.29	−4.28	−4.23	−4.21	−4.20
0.4	−4.42	−4.34	−4.30	−4.29	−4.24	−4.22	−4.19
0.5	−4.42	−4.36	−4.31	−4.29	−4.25	−4.23	−4.21
0.6	−4.43	−4.35	−4.32	−4.30	−4.26	−4.23	−4.22
0.7	−4.45	−4.37	−4.34	−4.32	−4.27	−4.25	−4.22
0.8	−4.46	−4.40	−4.35	−4.33	−4.28	−4.26	−4.24
0.9	−4.48	−4.40	−4.36	−4.34	−4.30	−4.28	−4.25
1.0	**−4.50**	**−4.42**	**−4.38**	**−4.35**	**−4.32**	**−4.28**	**−4.25**
				(−4.35)	**(−4.31)**	**(−4.29)**	**(−4.27)**
1.1	−4.51	−4.42	−4.38	−4.35	−4.31	−4.29	−4.27
1.2	−4.52	−4.42	−4.38	−4.36	−4.32	−4.29	−4.27
1.3	−4.50	−4.41	−4.37	−4.36	−4.30	−4.28	−4.25
1.4	−4.48	−4.39	−4.35	−4.32	−4.28	−4.25	−4.23
1.5	−4.44	−4.35	−4.32	−4.30	−4.24	−4.22	−4.20
1.6	−4.39	−4.31	−4.26	−4.25	−4.21	−4.17	−4.15
1.7	−4.33	−4.25	−4.21	−4.20	−4.16	−4.12	−4.11
1.8	−4.27	−4.21	−4.17	−4.14	−4.10	−4.09	−4.06
1.9	−4.21	−4.13	−4.11	−4.10	−4.06	−4.04	−4.03
2.0	**−4.15**	**−4.09**	**−4.05**	**−4.04**	**−4.00**	**−3.98**	**−3.99**
				(−4.05)	**(−4.01)**	**(−3.99)**	**(−3.98)**
2.1	−4.08	−4.05	−4.01	−4.00	−3.96	−3.95	−3.94
2.2	−4.05	−3.99	−3.97	−3.96	−3.93	−3.93	−3.91
2.3	−4.02	−3.98	−3.95	−3.94	−3.91	−3.89	−3.89
2.4	−3.98	−3.96	−3.93	−3.92	−3.89	−3.88	−3.87
2.5	−3.96	−3.92	−3.90	−3.90	−3.87	−3.86	−3.86
2.6	−3.94	−3.91	−3.90	−3.87	−3.86	−3.84	−3.84
2.7	−3.92	−3.89	−3.87	−3.85	−3.82	−3.81	−3.82
2.8	−3.90	−3.85	−3.84	−3.82	−3.80	−3.81	−3.80
2.9	−3.86	−3.84	−3.81	−3.81	−3.79	−3.77	−3.77
3.0	**−3.83**	**−3.80**	**−3.79**	**−3.77**	**−3.77**	**−3.75**	**−3.77**
				(−3.78)	**(−3.77)**	**(−3.76)**	**(−3.75)**
3.1	−3.80	−3.78	−3.76	−3.75	−3.75	−3.74	−3.73
3.2	−3.76	−3.75	−3.74	−3.73	−3.71	−3.71	−3.72
3.3	−3.73	−3.73	−3.73	−3.71	−3.70	−3.69	−3.70
3.4	−3.71	−3.71	−3.70	−3.70	−3.69	−3.69	−3.69
3.5	−3.71	−3.70	−3.69	−3.69	−3.68	−3.67	−3.68
3.6	−3.69	−3.69	−3.68	−3.68	−3.68	−3.66	−3.67
3.7	−3.68	−3.69	−3.66	−3.67	−3.65	−3.65	−3.65
3.8	−3.66	−3.67	−3.66	−3.65	−3.65	−3.65	−3.65
3.9	−3.67	−3.65	−3.65	−3.65	−3.64	−3.64	−3.64
4.0	**−3.64**	**−3.63**	**−3.64**	**−3.64**	**−3.63**	**−3.62**	**−3.63**
				(−3.65)	**(−3.63)**	**(−3.64)**	**(−3.63)**
4.1	−3.63	−3.62	−3.63	−3.63	−3.62	−3.61	−3.61
4.2	−3.61	−3.61	−3.60	−3.61	−3.61	−3.61	−3.60
4.3	−3.59	−3.60	−3.60	−3.60	−3.60	−3.58	−3.60
4.4	−3.58	−3.59	−3.59	−3.58	−3.58	−3.58	−3.59
4.5	−3.56	−3.58	−3.58	−3.59	−3.58	−3.58	−3.57
4.6	−3.56	−3.57	−3.57	−3.57	−3.57	−3.57	−3.56
4.7	−3.56	−3.56	−3.56	−3.57	−3.57	−3.57	−3.57
4.8	−3.56	−3.57	−3.57	−3.57	−3.57	−3.55	−3.55
4.9	−3.54	−3.57	−3.55	−3.56	−3.56	−3.56	−3.55
5.0	**−3.55**	**−3.54**	**−3.55**	**−3.55**	**−3.56**	**−3.55**	**−3.54**
				(−3.56)	**(−3.56)**	**(−3.56)**	**(−3.55)**

Notes: As for Table 4.

Table 6: 1% critical values of τ_{DF} (j = 0, by assumption)

k	T = 40	T = 60	T = 80	T = 100	T = 200	T = 500	T = 2500
0.1	−5.13	−4.97	−4.92	−4.87	−4.80	−4.78	−4.74
0.2	−5.12	−4.97	−4.92	−4.88	−4.81	−4.75	−4.71
0.3	−5.17	−4.98	−4.90	−4.87	−4.81	−4.77	−4.72
0.4	−5.15	−4.99	−4.95	−4.89	−4.80	−4.76	−4.74
0.5	−5.14	−5.02	−4.93	−4.89	−4.80	−4.78	−4.74
0.6	−5.16	−5.00	−4.93	−4.89	−4.82	−4.77	−4.76
0.7	−5.18	−5.02	−4.96	−4.92	−4.83	−4.80	−4.76
0.8	−5.16	−5.05	−4.97	−4.94	−4.85	−4.80	−4.78
0.9	−5.19	−5.05	−4.99	−4.94	−4.87	−4.81	−4.78
1.0	**−5.22**	**−5.08**	**−4.99**	**−4.94**	**−4.88**	**−4.84**	**−4.79**
				(−4.95)	**(−4.87)**	**(−4.81)**	**(−4.80)**
1.1	−5.23	−5.09	−5.00	−4.96	−4.87	−4.82	−4.78
1.2	−5.26	−5.08	−4.98	−4.96	−4.88	−4.83	−4.80
1.3	−5.26	−5.08	−4.98	−4.96	−4.88	−4.81	−4.79
1.4	−5.23	−5.06	−4.97	−4.91	−4.86	−4.81	−4.77
1.5	−5.17	−4.99	−4.96	−4.90	−4.82	−4.76	−4.74
1.6	−5.15	−4.99	−4.90	−4.87	−4.81	−4.74	−4.70
1.7	−5.09	−4.93	−4.86	−4.85	−4.75	−4.70	−4.67
1.8	−5.02	−4.89	−4.82	−4.78	−4.71	−4.66	−4.61
1.9	−4.98	−4.80	−4.74	−4.74	−4.67	−4.62	−4.61
2.0	**−4.91**	**−4.78**	**−4.71**	**−4.68**	**−4.63**	**−4.57**	**−4.56**
				(−4.69)	**(−4.62)**	**(−4.57)**	**(−4.58)**
2.1	−4.86	−4.75	−4.66	−4.64	−4.58	−4.55	−4.52
2.2	−4.80	−4.69	−4.66	−4.61	−4.55	−4.53	−4.49
2.3	−4.81	−4.68	−4.63	−4.59	−4.53	−4.50	−4.48
2.4	−4.76	−4.66	−4.61	−4.59	−4.54	−4.48	−4.49
2.5	−4.74	−4.66	−4.59	−4.56	−4.51	−4.47	−4.46
2.6	−4.73	−4.62	−4.58	−4.54	−4.50	−4.46	−4.44
2.7	−4.69	−4.60	−4.57	−4.52	−4.47	−4.43	−4.42
2.8	−4.67	−4.56	−4.53	−4.46	−4.43	−4.43	−4.42
2.9	−4.63	−4.57	−4.49	−4.48	−4.41	−4.39	−4.38
3.0	**−4.61**	**−4.52**	**−4.47**	**−4.45**	**−4.40**	**−4.36**	**−4.39**
				(−4.45)	**(−4.38)**	**(−4.38)**	**(−4.38)**
3.1	−4.59	−4.53	−4.47	−4.42	−4.40	−4.34	−4.34
3.2	−4.56	−4.48	−4.42	−4.41	−4.35	−4.32	−4.33
3.3	−4.52	−4.43	−4.40	−4.38	−4.35	−4.31	−4.32
3.4	−4.51	−4.43	−4.38	−4.39	−4.33	−4.30	−4.29
3.5	−4.49	−4.41	−4.38	−4.35	−4.31	−4.29	−4.30
3.6	−4.46	−4.40	−4.36	−4.37	−4.32	−4.30	−4.28
3.7	−4.47	−4.38	−4.36	−4.33	−4.29	−4.27	−4.27
3.8	−4.43	−4.40	−4.35	−4.31	−4.30	−4.25	−4.26
3.9	−4.43	−4.36	−4.33	−4.32	−4.29	−4.26	−4.24
4.0	**−4.41**	**−4.32**	**−4.31**	**−4.30**	**−4.26**	**−4.22**	**−4.23**
				(−4.29)	**(−4.27)**	**(−4.25)**	**(−4.24)**
4.1	−4.39	−4.34	−4.31	−4.29	−4.24	−4.23	−4.22
4.2	−4.38	−4.32	−4.29	−4.27	−4.23	−4.23	−4.20
4.3	−4.34	−4.34	−4.28	−4.26	−4.22	−4.19	−4.21
4.4	−4.33	−4.30	−4.28	−4.24	−4.21	−4.21	−4.20
4.5	−4.32	−4.28	−4.24	−4.25	−4.21	−4.21	−4.18
4.6	−4.30	−4.26	−4.23	−4.21	−4.21	−4.18	−4.16
4.7	−4.32	−4.25	−4.25	−4.23	−4.21	−4.18	−4.15
4.8	−4.31	−4.25	−4.27	−4.23	−4.20	−4.17	−4.15
4.9	−4.29	−4.26	−4.23	−4.22	−4.20	−4.16	−4.17
5.0	**−4.29**	**−4.22**	**−4.20**	**−4.20**	**−4.18**	**−4.17**	**−4.15**
				(−4.20)	**(−4.18)**	**(−4.18)**	**(−4.16)**

Notes: As for Table 4.

Table 7: 10% critical values of τ_{LM} (*j* selected by Hall's, 1994, general-to-specific method)

k	*T = 40*	*T = 60*	*T = 80*	*T = 100*	*T = 200*	*T = 500*	*T = 2500*
0.1	−4.27	−4.14	−4.08	−4.02	−3.89	−3.76	−3.66
0.2	−4.27	−4.14	−4.08	−4.01	−3.89	−3.76	−3.66
0.3	−4.29	−4.17	−4.08	−4.02	−3.90	−3.78	−3.67
0.4	−4.30	−4.18	−4.10	−4.04	−3.91	−3.80	−3.68
0.5	−4.31	−4.19	−4.12	−4.06	−3.92	−3.81	−3.70
0.6	−4.34	−4.21	−4.13	−4.08	−3.95	−3.82	−3.71
0.7	−4.37	−4.22	−4.15	−4.10	−3.97	−3.85	−3.73
0.8	−4.39	−4.26	−4.18	−4.12	−3.99	−3.86	−3.74
0.9	−4.42	−4.29	−4.20	−4.15	−4.01	−3.87	−3.76
1.0	**−4.45**	**−4.31**	**−4.22**	**−4.18**	**−4.02**	**−3.89**	**−3.77**
1.1	−4.45	−4.31	−4.23	−4.17	−4.03	−3.89	−3.76
1.2	−4.43	−4.31	−4.21	−4.15	−4.00	−3.86	−3.73
1.3	−4.40	−4.27	−4.16	−4.11	−3.95	−3.80	−3.67
1.4	−4.32	−4.17	−4.07	−4.00	−3.85	−3.70	−3.59
1.5	−4.19	−4.04	−3.94	−3.88	−3.72	−3.58	−3.48
1.6	−4.03	−3.89	−3.80	−3.74	−3.59	−3.45	−3.37
1.7	−3.86	−3.72	−3.65	−3.59	−3.46	−3.36	−3.29
1.8	−3.70	−3.59	−3.51	−3.46	−3.35	−3.27	−3.22
1.9	−3.55	−3.45	−3.40	−3.36	−3.28	−3.22	−3.19
2.0	**−3.44**	**−3.36**	**−3.32**	**−3.30**	**−3.24**	**−3.21**	**−3.18**
2.1	−3.35	−3.29	−3.27	−3.26	−3.23	−3.19	−3.18
2.2	−3.30	−3.27	−3.25	−3.24	−3.22	−3.19	−3.17
2.3	−3.25	−3.22	−3.22	−3.22	−3.20	−3.17	−3.17
2.4	−3.24	−3.20	−3.22	−3.21	−3.17	−3.15	−3.14
2.5	−3.22	−3.18	−3.18	−3.17	−3.13	−3.10	−3.10
2.6	−3.19	−3.16	−3.15	−3.13	−3.08	−3.03	−3.03
2.7	−3.17	−3.11	−3.08	−3.06	−3.01	−2.97	−2.99
2.8	−3.13	−3.06	−3.02	−3.00	−2.95	−2.91	−2.94
2.9	−3.11	−3.03	−2.97	−2.94	−2.89	−2.87	−2.92
3.0	**−3.07**	**−2.98**	**−2.93**	**−2.90**	**−2.85**	**−2.85**	**−2.92**
3.1	−3.05	−2.95	−2.88	−2.86	−2.84	−2.85	−2.90
3.2	−3.02	−2.93	−2.87	−2.84	−2.82	−2.84	−2.91
3.3	−3.01	−2.91	−2.86	−2.83	−2.82	−2.85	−2.90
3.4	−3.01	−2.91	−2.86	−2.83	−2.82	−2.85	−2.92
3.5	−3.00	−2.91	−2.87	−2.84	−2.82	−2.85	−2.89
3.6	−3.01	−2.92	−2.88	−2.85	−2.81	−2.82	−2.86
3.7	−3.02	−2.93	−2.89	−2.85	−2.80	−2.80	−2.84
3.8	−3.04	−2.95	−2.88	−2.84	−2.79	−2.77	−2.82
3.9	−3.05	−2.96	−2.89	−2.83	−2.77	−2.75	−2.80
4.0	**−3.05**	**−2.95**	**−2.88**	**−2.83**	**−2.74**	**−2.73**	**−2.79**
4.1	−3.07	−2.97	−2.88	−2.83	−2.73	−2.71	−2.79
4.2	−3.07	−2.96	−2.87	−2.82	−2.72	−2.72	−2.78
4.3	−3.07	−2.97	−2.86	−2.81	−2.73	−2.72	−2.78
4.4	−3.07	−2.96	−2.87	−2.81	−2.73	−2.73	−2.79
4.5	−3.07	−2.96	−2.87	−2.82	−2.74	−2.72	−2.79
4.6	−3.06	−2.97	−2.88	−2.83	−2.74	−2.72	−2.77
4.7	−3.07	−2.97	−2.89	−2.83	−2.75	−2.73	−2.76
4.8	−3.07	−2.97	−2.91	−2.84	−2.74	−2.71	−2.74
4.9	−3.07	−2.99	−2.90	−2.86	−2.74	−2.69	−2.74
5.0	**−3.08**	**−2.99**	**−2.92**	**−2.85**	**−2.73**	**−2.69**	**−2.73**

Table 8: 5% critical values of τ_{LM} (j selected by Hall's, 1994, general-to-specific method)

k	$T = 40$	$T = 60$	$T = 80$	$T = 100$	$T = 200$	$T = 500$	$T = 2500$
0.1	−4.62	−4.48	−4.40	−4.34	−4.20	−4.05	−3.93
0.2	−4.64	−4.47	−4.39	−4.33	−4.20	−4.05	−3.94
0.3	−4.65	−4.49	−4.39	−4.35	−4.20	−4.07	−3.95
0.4	−4.65	−4.51	−4.42	−4.36	−4.21	−4.09	−3.96
0.5	−4.66	−4.51	−4.43	−4.38	−4.22	−4.09	−3.97
0.6	−4.70	−4.53	−4.45	−4.39	−4.25	−4.12	−3.99
0.7	−4.74	−4.56	−4.48	−4.41	−4.27	−4.14	−4.00
0.8	−4.74	−4.59	−4.50	−4.43	−4.30	−4.16	−4.01
0.9	−4.77	−4.60	−4.52	−4.46	−4.31	−4.16	−4.03
1.0	**−4.79**	**−4.63**	**−4.54**	**−4.48**	**−4.32**	**−4.18**	**−4.04**
1.1	−4.80	−4.65	−4.54	−4.49	−4.33	−4.17	−4.04
1.2	−4.79	−4.63	−4.52	−4.46	−4.30	−4.15	−4.00
1.3	−4.75	−4.60	−4.47	−4.42	−4.26	−4.10	−3.94
1.4	−4.69	−4.52	−4.41	−4.33	−4.17	−4.01	−3.87
1.5	−4.57	−4.40	−4.28	−4.23	−4.05	−3.90	−3.77
1.6	−4.43	−4.28	−4.16	−4.09	−3.93	−3.79	−3.68
1.7	−4.28	−4.12	−4.03	−3.98	−3.82	−3.71	−3.60
1.8	−4.14	−4.01	−3.93	−3.86	−3.73	−3.63	−3.55
1.9	−4.00	−3.89	−3.83	−3.79	−3.68	−3.58	−3.52
2.0	**−3.92**	**−3.81**	**−3.76**	**−3.71**	**−3.64**	**−3.58**	**−3.51**
2.1	−3.84	−3.76	−3.73	−3.71	−3.64	−3.56	−3.51
2.2	−3.80	−3.74	−3.71	−3.68	−3.62	−3.56	−3.51
2.3	−3.75	−3.70	−3.66	−3.68	−3.60	−3.54	−3.50
2.4	−3.73	−3.67	−3.67	−3.65	−3.57	−3.51	−3.47
2.5	−3.72	−3.66	−3.61	−3.61	−3.53	−3.46	−3.42
2.6	−3.67	−3.61	−3.60	−3.55	−3.47	−3.39	−3.36
2.7	−3.64	−3.54	−3.50	−3.47	−3.41	−3.33	−3.31
2.8	−3.57	−3.48	−3.44	−3.41	−3.34	−3.28	−3.27
2.9	−3.54	−3.44	−3.38	−3.34	−3.27	−3.23	−3.26
3.0	**−3.50**	**−3.40**	**−3.33**	**−3.29**	**−3.24**	**−3.21**	**−3.25**
3.1	−3.46	−3.34	−3.28	−3.25	−3.22	−3.21	−3.24
3.2	−3.43	−3.32	−3.25	−3.21	−3.20	−3.19	−3.24
3.3	−3.40	−3.29	−3.24	−3.22	−3.19	−3.20	−3.24
3.4	−3.40	−3.29	−3.24	−3.21	−3.18	−3.20	−3.24
3.5	−3.40	−3.28	−3.25	−3.20	−3.18	−3.19	−3.21
3.6	−3.40	−3.29	−3.24	−3.22	−3.18	−3.15	−3.18
3.7	−3.40	−3.31	−3.25	−3.21	−3.15	−3.13	−3.15
3.8	−3.43	−3.32	−3.23	−3.19	−3.13	−3.10	−3.13
3.9	−3.42	−3.32	−3.24	−3.18	−3.10	−3.07	−3.10
4.0	**−3.45**	**−3.31**	**−3.23**	**−3.17**	**−3.07**	**−3.05**	**−3.11**
4.1	−3.45	−3.33	−3.22	−3.16	−3.06	−3.03	−3.11
4.2	−3.46	−3.31	−3.21	−3.15	−3.04	−3.05	−3.09
4.3	−3.45	−3.31	−3.19	−3.14	−3.05	−3.03	−3.08
4.4	−3.45	−3.32	−3.20	−3.14	−3.05	−3.04	−3.09
4.5	−3.45	−3.31	−3.20	−3.14	−3.06	−3.03	−3.09
4.6	−3.44	−3.32	−3.22	−3.15	−3.06	−3.04	−3.06
4.7	−3.45	−3.32	−3.22	−3.16	−3.06	−3.04	−3.06
4.8	−3.45	−3.33	−3.24	−3.16	−3.06	−3.01	−3.03
4.9	−3.46	−3.34	−3.24	−3.19	−3.05	−3.00	−3.03
5.0	**−3.46**	**−3.35**	**−3.26**	**−3.18**	**−3.04**	**−2.99**	**−3.02**

Table 9: 1% critical values of τ_{LM} (j selected by Hall's, 1994, general-to-specific method)

k	$T = 40$	$T = 60$	$T = 80$	$T = 100$	$T = 200$	$T = 500$	$T = 2500$
0.1	−5.36	−5.15	−5.01	−4.95	−4.79	−4.62	−4.48
0.2	−5.37	−5.13	−5.02	−4.95	−4.78	−4.62	−4.48
0.3	−5.39	−5.14	−5.02	−4.96	−4.78	−4.65	−4.49
0.4	−5.38	−5.15	−5.04	−4.97	−4.80	−4.66	−4.49
0.5	−5.41	−5.15	−5.06	−5.00	−4.81	−4.65	−4.51
0.6	−5.42	−5.19	−5.08	−5.00	−4.84	−4.68	−4.53
0.7	−5.48	−5.22	−5.09	−5.01	−4.83	−4.69	−4.55
0.8	−5.47	−5.22	−5.11	−5.06	−4.88	−4.71	−4.54
0.9	−5.49	−5.24	−5.15	−5.06	−4.89	−4.72	−4.56
1.0	**−5.54**	**−5.29**	**−5.14**	**−5.08**	**−4.90**	**−4.74**	**−4.58**
1.1	−5.55	−5.29	−5.15	−5.10	−4.90	−4.73	−4.58
1.2	−5.52	−5.29	−5.13	−5.06	−4.87	−4.72	−4.54
1.3	−5.51	−5.23	−5.09	−5.03	−4.85	−4.67	−4.49
1.4	−5.45	−5.17	−5.05	−4.94	−4.79	−4.60	−4.44
1.5	−5.33	−5.09	−4.93	−4.88	−4.67	−4.51	−4.33
1.6	−5.22	−4.98	−4.85	−4.78	−4.60	−4.42	−4.27
1.7	−5.07	−4.85	−4.77	−4.67	−4.51	−4.35	−4.19
1.8	−4.96	−4.76	−4.66	−4.57	−4.44	−4.30	−4.17
1.9	−4.85	−4.70	−4.60	−4.55	−4.39	−4.27	−4.14
2.0	**−4.81**	**−4.62**	**−4.55**	**−4.49**	**−4.37**	**−4.25**	**−4.13**
2.1	−4.72	−4.58	−4.53	−4.49	−4.38	−4.25	−4.14
2.2	−4.71	−4.61	−4.52	−4.48	−4.35	−4.24	−4.13
2.3	−4.69	−4.56	−4.46	−4.45	−4.33	−4.20	−4.12
2.4	−4.65	−4.52	−4.47	−4.44	−4.28	−4.17	−4.09
2.5	−4.65	−4.52	−4.42	−4.40	−4.28	−4.13	−4.05
2.6	−4.57	−4.45	−4.39	−4.32	−4.21	−4.07	−3.98
2.7	−4.53	−4.41	−4.30	−4.26	−4.15	−4.02	−3.96
2.8	−4.47	−4.30	−4.27	−4.21	−4.09	−3.97	−3.92
2.9	−4.41	−4.29	−4.18	−4.15	−4.05	−3.94	−3.91
3.0	**−4.37**	**−4.25**	**−4.13**	**−4.09**	**−4.01**	**−3.93**	**−3.89**
3.1	−4.31	−4.15	−4.09	−4.06	−3.97	−3.92	−3.92
3.2	−4.28	−4.12	−4.04	−4.02	−3.99	−3.90	−3.89
3.3	−4.25	−4.10	−4.07	−4.01	−3.96	−3.90	−3.88
3.4	−4.24	−4.07	−4.03	−4.01	−3.95	−3.91	−3.88
3.5	−4.24	−4.06	−4.02	−3.99	−3.93	−3.88	−3.85
3.6	−4.23	−4.05	−4.02	−3.99	−3.91	−3.82	−3.82
3.7	−4.22	−4.08	−4.00	−3.95	−3.88	−3.81	−3.78
3.8	−4.23	−4.10	−3.98	−3.92	−3.83	−3.77	−3.76
3.9	−4.24	−4.05	−3.96	−3.91	−3.81	−3.73	−3.73
4.0	**−4.26**	**−4.06**	**−3.95**	**−3.87**	**−3.76**	**−3.72**	**−3.73**
4.1	−4.25	−4.07	−3.94	−3.84	−3.75	−3.69	−3.73
4.2	−4.24	−4.03	−3.91	−3.83	−3.74	−3.71	−3.71
4.3	−4.22	−4.03	−3.90	−3.85	−3.72	−3.70	−3.70
4.4	−4.22	−4.03	−3.88	−3.82	−3.73	−3.70	−3.70
4.5	−4.21	−4.03	−3.90	−3.83	−3.73	−3.68	−3.72
4.6	−4.22	−4.01	−3.91	−3.83	−3.74	−3.69	−3.67
4.7	−4.21	−4.03	−3.90	−3.81	−3.74	−3.66	−3.66
4.8	−4.23	−4.03	−3.91	−3.82	−3.70	−3.64	−3.63
4.9	−4.24	−4.04	−3.93	−3.84	−3.69	−3.63	−3.62
5.0	**−4.24**	**−4.07**	**−3.92**	**−3.85**	**−3.67**	**−3.60**	**−3.62**

Table 10: 10% critical values of τ_{DF} (*j* selected by Hall's, 1994, general-to-specific method)

k	*T* = 40	*T* = 60	*T* = 80	*T* = 100	*T* = 200	*T* = 500	*T* = 2500
0.1	−4.64	−4.50	−4.40	−4.34	−4.18	−4.05	−3.93
0.2	−4.64	−4.50	−4.42	−4.35	−4.19	−4.06	−3.92
0.3	−4.66	−4.51	−4.42	−4.35	−4.20	−4.06	−3.93
0.4	−4.66	−4.53	−4.43	−4.36	−4.21	−4.06	−3.93
0.5	−4.67	−4.52	−4.45	−4.37	−4.22	−4.07	−3.95
0.6	−4.70	−4.54	−4.45	−4.39	−4.24	−4.09	−3.96
0.7	−4.69	−4.55	−4.47	−4.40	−4.26	−4.10	−3.97
0.8	−4.72	−4.58	−4.48	−4.42	−4.27	−4.11	−3.98
0.9	−4.74	−4.59	−4.51	−4.45	−4.29	−4.13	−3.99
1.0	**−4.77**	**−4.62**	**−4.52**	**−4.45**	**−4.30**	**−4.16**	**−4.01**
1.1	−4.78	−4.63	−4.54	−4.49	−4.32	−4.15	−4.02
1.2	−4.80	−4.65	−4.56	−4.48	−4.32	−4.17	−4.01
1.3	−4.81	−4.66	−4.56	−4.49	−4.32	−4.15	−4.01
1.4	−4.80	−4.65	−4.55	−4.47	−4.30	−4.12	−4.00
1.5	−4.79	−4.63	−4.52	−4.45	−4.27	−4.08	−3.95
1.6	−4.75	−4.58	−4.48	−4.41	−4.21	−4.04	−3.90
1.7	−4.71	−4.53	−4.41	−4.33	−4.15	−3.98	−3.85
1.8	−4.63	−4.46	−4.34	−4.27	−4.08	−3.93	−3.79
1.9	−4.55	−4.38	−4.26	−4.19	−4.02	−3.87	−3.74
2.0	**−4.47**	**−4.29**	**−4.18**	**−4.11**	**−3.94**	**−3.80**	**−3.69**
2.1	−4.38	−4.20	−4.09	−4.03	−3.87	−3.74	−3.64
2.2	−4.27	−4.12	−4.00	−3.95	−3.79	−3.68	−3.59
2.3	−4.15	−4.01	−3.92	−3.87	−3.74	−3.64	−3.57
2.4	−4.07	−3.93	−3.85	−3.81	−3.69	−3.60	−3.54
2.5	−3.98	−3.86	−3.79	−3.75	−3.66	−3.57	−3.53
2.6	−3.90	−3.79	−3.73	−3.70	−3.62	−3.53	−3.51
2.7	−3.85	−3.75	−3.69	−3.65	−3.58	−3.52	−3.48
2.8	−3.80	−3.70	−3.65	−3.62	−3.54	−3.49	−3.45
2.9	−3.76	−3.66	−3.62	−3.57	−3.52	−3.45	−3.43
3.0	**−3.72**	**−3.64**	**−3.59**	**−3.55**	**−3.48**	**−3.43**	**−3.41**
3.1	−3.70	−3.61	−3.55	−3.52	−3.44	−3.40	−3.38
3.2	−3.69	−3.58	−3.51	−3.47	−3.41	−3.36	−3.36
3.3	−3.67	−3.56	−3.49	−3.45	−3.37	−3.34	−3.33
3.4	−3.65	−3.54	−3.45	−3.41	−3.34	−3.31	−3.31
3.5	−3.63	−3.51	−3.43	−3.38	−3.32	−1.87	−3.29
3.6	−3.61	−3.49	−3.40	−3.36	−3.29	−3.27	−3.28
3.7	−3.61	−3.48	−3.39	−3.35	−3.28	−3.26	−3.28
3.8	−3.59	−3.47	−3.39	−3.33	−3.28	−3.24	−3.26
3.9	−3.58	−3.46	−3.36	−3.33	−3.27	−3.24	−3.26
4.0	**−3.59**	**−3.47**	**−3.38**	**−3.33**	**−3.25**	**−3.23**	**−3.25**
4.1	−3.59	−3.47	−3.37	−3.33	−3.25	−3.21	−3.23
4.2	−3.59	−3.47	−3.37	−3.32	−3.23	−3.20	−3.22
4.3	−3.61	−3.48	−3.37	−3.31	−3.23	−3.18	−3.21
4.4	−3.61	−3.48	−3.37	−3.32	−3.21	−3.18	−3.21
4.5	−3.61	−3.48	−3.36	−3.32	−3.20	−3.17	−3.18
4.6	−3.61	−3.49	−3.37	−3.30	−3.20	−3.16	−3.18
4.7	−3.61	−3.48	−3.37	−3.31	−3.18	−3.16	−3.18
4.8	−3.62	−3.49	−3.37	−3.30	−3.17	−3.15	−3.18
4.9	−3.61	−3.48	−3.37	−3.30	−3.19	−3.13	−3.17
5.0	**−3.62**	**−3.49**	**−3.38**	**−3.32**	**−3.19**	**−3.14**	**−3.16**

Table 11: 5% critical values of τ_{DF} (*j* selected by Hall's, 1994, general-to-specific method)

k	*T* = 40	*T* = 60	*T* = 80	*T* = 100	*T* = 200	*T* = 500	*T* = 2500
0.1	−5.01	−4.83	−4.74	−4.66	−4.49	−4.35	−4.21
0.2	−5.01	−4.83	−4.75	−4.66	−4.49	−4.35	−4.21
0.3	−5.03	−4.85	−4.74	−4.66	−4.51	−4.36	−4.22
0.4	−5.02	−4.86	−4.75	−4.68	−4.53	−4.36	−4.21
0.5	−5.04	−4.85	−4.77	−4.70	−4.53	−4.37	−4.23
0.6	−5.06	−4.87	−4.76	−4.70	−4.54	−4.38	−4.24
0.7	−5.06	−4.88	−4.79	−4.72	−4.56	−4.40	−4.25
0.8	−5.09	−4.91	−4.80	−4.74	−4.57	−4.41	−4.26
0.9	−5.10	−4.93	−4.82	−4.76	−4.59	−4.42	−4.27
1.0	**−5.13**	**−4.95**	**−4.84**	**−4.76**	**−4.60**	**−4.44**	**−4.29**
1.1	−5.15	−4.96	−4.87	−4.79	−4.62	−4.45	−4.30
1.2	−5.16	−4.98	−4.88	−4.79	−4.62	−4.46	−4.28
1.3	−5.18	−4.97	−4.88	−4.80	−4.62	−4.44	−4.29
1.4	−5.17	−4.99	−4.87	−4.79	−4.61	−4.42	−4.28
1.5	−5.16	−4.96	−4.85	−4.77	−4.59	−4.39	−4.23
1.6	−5.13	−4.94	−4.81	−4.73	−4.53	−4.34	−4.20
1.7	−5.09	−4.87	−4.76	−4.66	−4.47	−4.30	−4.15
1.8	−5.02	−4.83	−4.69	−4.62	−4.42	−4.25	−4.10
1.9	−4.96	−4.77	−4.63	−4.54	−4.36	−4.19	−4.05
2.0	**−4.89**	**−4.68**	**−4.56**	**−4.48**	**−4.29**	**−4.13**	**−3.99**
2.1	−4.80	−4.59	−4.48	−4.41	−4.23	−4.07	−3.96
2.2	−4.71	−4.51	−4.40	−4.34	−4.17	−4.03	−3.92
2.3	−4.60	−4.44	−4.33	−4.28	−4.12	−3.99	−3.90
2.4	−4.53	−4.37	−4.28	−4.22	−4.08	−3.96	−3.87
2.5	−4.45	−4.29	−4.22	−4.16	−4.04	−3.93	−3.86
2.6	−4.38	−4.23	−4.17	−4.13	−4.02	−3.90	−3.84
2.7	−4.33	−4.20	−4.13	−4.08	−3.97	−3.88	−3.81
2.8	−4.27	−4.14	−4.08	−4.05	−3.94	−3.86	−3.78
2.9	−4.23	−4.10	−4.05	−4.00	−3.91	−3.82	−3.76
3.0	**−4.20**	**−4.08**	**−4.02**	**−3.97**	**−3.87**	**−3.79**	**−3.74**
3.1	−4.17	−4.06	−3.98	−3.94	−3.83	−3.76	−3.71
3.2	−4.15	−4.02	−3.93	−3.90	−3.79	−3.72	−3.69
3.3	−4.13	−4.00	−3.91	−3.86	−3.75	−3.70	−3.67
3.4	−4.09	−3.95	−3.86	−3.81	−3.73	−3.67	−3.64
3.5	−4.09	−3.93	−3.83	−3.78	−3.69	−2.27	−3.62
3.6	−4.04	−3.90	−3.80	−3.75	−3.67	−3.63	−3.61
3.7	−4.04	−3.86	−3.77	−3.74	−3.65	−3.61	−3.61
3.8	−4.02	−3.86	−3.77	−3.71	−3.65	−3.61	−3.60
3.9	−4.00	−3.85	−3.75	−3.70	−3.63	−3.59	−3.59
4.0	**−4.00**	**−3.86**	**−3.75**	**−3.70**	**−3.61**	**−3.57**	**−3.57**
4.1	−4.00	−3.86	−3.72	−3.69	−3.61	−3.56	−3.55
4.2	−4.01	−3.84	−3.74	−3.69	−3.59	−3.54	−3.55
4.3	−4.02	−3.86	−3.75	−3.68	−3.58	−3.53	−3.53
4.4	−4.02	−3.86	−3.73	−3.67	−3.55	−3.51	−3.52
4.5	−4.01	−3.86	−3.73	−3.67	−3.54	−3.49	−3.50
4.6	−4.02	−3.86	−3.74	−3.65	−3.54	−3.49	−3.50
4.7	−4.02	−3.85	−3.73	−3.65	−3.52	−3.49	−3.49
4.8	−4.02	−3.86	−3.73	−3.65	−3.51	−3.49	−3.50
4.9	−4.01	−3.85	−3.71	−3.65	−3.52	−3.46	−3.47
5.0	**−4.01**	**−3.85**	**−3.73**	**−3.65**	**−3.51**	**−3.47**	**−3.48**

Table 12: 1% critical values of τ_{DF} (*j* selected by Hall's, 1994, general-to-specific method)

k	*T* = 40	*T* = 60	*T* = 80	*T* = 100	*T* = 200	*T* = 500	*T* = 2500
0.1	−5.77	−5.50	−5.39	−5.29	−5.08	−4.93	−4.76
0.2	−5.79	−5.51	−5.39	−5.28	−5.10	−4.94	−4.76
0.3	−5.79	−5.51	−5.37	−5.28	−5.10	−4.93	−4.78
0.4	−5.81	−5.51	−5.39	−5.29	−5.12	−4.94	−4.76
0.5	−5.82	−5.51	−5.38	−5.31	−5.13	−4.95	−4.77
0.6	−5.81	−5.53	−5.40	−5.31	−5.13	−4.95	−4.79
0.7	−5.84	−5.55	−5.43	−5.34	−5.16	−4.98	−4.81
0.8	−5.85	−5.55	−5.43	−5.35	−5.15	−4.97	−4.80
0.9	−5.91	−5.57	−5.45	−5.38	−5.17	−5.00	−4.80
1.0	**−5.91**	**−5.61**	**−5.46**	**−5.36**	**−5.20**	**−5.01**	**−4.83**
1.1	−5.88	−5.63	−5.49	−5.41	−5.21	−5.00	−4.84
1.2	−5.91	−5.62	−5.50	−5.40	−5.20	−5.02	−4.82
1.3	−5.96	−5.64	−5.49	−5.39	−5.21	−5.03	−4.85
1.4	−5.96	−5.66	−5.51	−5.40	−5.22	−5.00	−4.85
1.5	−5.94	−5.62	−5.47	−5.38	−5.17	−4.98	−4.79
1.6	−5.92	−5.61	−5.44	−5.35	−5.12	−4.95	−4.76
1.7	−5.89	−5.58	−5.42	−5.31	−5.09	−4.89	−4.71
1.8	−5.83	−5.51	−5.36	−5.27	−5.04	−4.86	−4.68
1.9	−5.79	−5.48	−5.31	−5.21	−4.97	−4.80	−4.65
2.0	**−5.71**	**−5.38**	**−5.25**	**−5.16**	**−4.94**	**−4.74**	**−4.61**
2.1	−5.65	−5.33	−5.17	−5.09	−4.89	−4.71	−4.56
2.2	−5.56	−5.29	−5.13	−5.05	−4.84	−4.68	−4.52
2.3	−5.48	−5.21	−5.07	−4.99	−4.82	−4.63	−4.51
2.4	−5.42	−5.16	−5.04	−4.95	−4.82	−4.63	−4.48
2.5	−5.32	−5.10	−4.99	−4.92	−4.77	−4.61	−4.48
2.6	−5.25	−5.06	−4.93	−4.89	−4.76	−4.58	−4.46
2.7	−5.23	−5.01	−4.91	−4.86	−4.69	−4.57	−4.43
2.8	−5.16	−4.98	−4.90	−4.81	−4.66	−4.53	−4.40
2.9	−5.11	−4.93	−4.86	−4.80	−4.62	−4.49	−4.39
3.0	**−5.11**	**−4.91**	**−4.83**	**−4.75**	**−4.60**	**−4.46**	**−4.36**
3.1	−5.08	−4.88	−4.77	−4.72	−4.56	−4.47	−4.35
3.2	−5.03	−4.84	−4.72	−4.70	−4.54	−4.41	−4.32
3.3	−5.01	−4.82	−4.71	−4.64	−4.49	−4.36	−4.29
3.4	−5.00	−4.76	−4.66	−4.61	−4.46	−4.36	−4.28
3.5	−4.99	−4.75	−4.61	−4.57	−4.45	−2.39	−4.24
3.6	−4.91	−4.72	−4.58	−4.51	−4.42	−4.33	−4.25
3.7	−4.92	−4.66	−4.56	−4.51	−4.40	−4.31	−4.25
3.8	−4.88	−4.67	−4.53	−4.47	−4.39	−4.30	−4.23
3.9	−4.85	−4.65	−4.51	−4.44	−4.38	−4.30	−4.23
4.0	**−4.85**	**−4.65**	**−4.54**	**−4.46**	**−4.35**	**−4.25**	**−4.21**
4.1	−4.84	−4.61	−4.51	−4.45	−4.32	−4.25	−4.17
4.2	−4.86	−4.63	−4.50	−4.43	−4.30	−4.23	−4.16
4.3	−4.86	−4.64	−4.50	−4.42	−4.27	−4.20	−4.14
4.4	−4.86	−4.61	−4.47	−4.38	−4.26	−4.18	−4.14
4.5	−4.83	−4.60	−4.44	−4.38	−4.24	−4.17	−4.13
4.6	−4.86	−4.62	−4.44	−4.35	−4.22	−4.15	−4.12
4.7	−4.83	−4.58	−4.43	−4.35	−4.21	−4.14	−4.11
4.8	−4.84	−4.61	−4.45	−4.34	−4.19	−4.15	−4.15
4.9	−4.81	−4.60	−4.42	−4.33	−4.19	−4.10	−4.09
5.0	**−4.82**	**−4.59**	**−4.43**	**−4.35**	**−4.17**	**−4.11**	**−4.08**

Table 13: Critical values of $F(\hat{k})$ – FLM test

	$T = 40$			$T = 60$			$T = 80$			$T = 100$			$T = 200$			$T = 500$			$T = 2500$		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
(a) Integer values of k and $j = 0$ by assumption																					
	4.92	5.95	8.46	4.49	5.41	7.59	4.31	5.19	7.21	4.20	5.05	6.92	3.99	4.79	6.60	3.87	4.63	6.26	3.86	4.61	6.29
										(7.50)	(8.80)	(11.79)	(7.34)	(8.60)	(11.32)	(7.24)	(8.45)	(11.03)	(7.18)	(8.37)	(10.90)
(b) Fractional values of k and $j = 0$ by assumption																					
	6.58	8.00	11.31	5.93	7.12	9.93	5.65	6.76	9.29	5.50	6.57	8.96	5.15	6.16	8.30	4.99	5.92	7.99	4.97	5.90	7.96
(c) Fractional values of k and j selected by Hall's (1994) general-to-specific method																					
	8.04	10.17	15.51	6.79	8.54	12.39	6.15	7.65	11.05	5.77	7.13	10.28	5.22	6.37	9.05	4.94	5.98	8.39	4.96	5.93	8.08

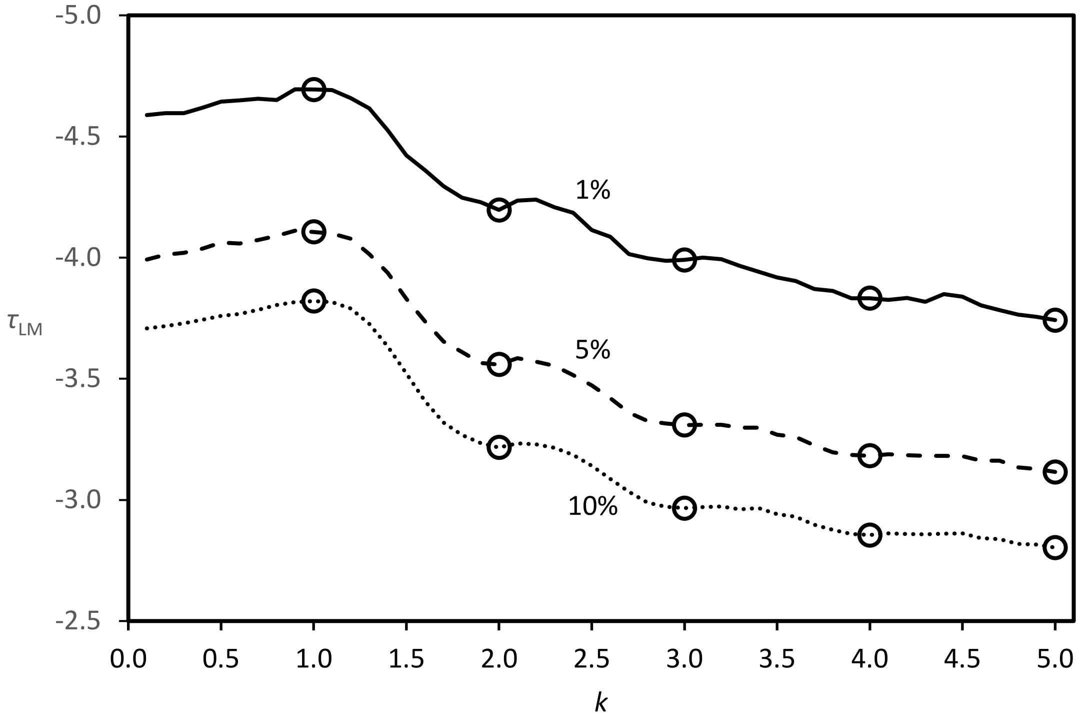
Notes: Enders and Lee's (2012a, Table 1) values are shown in parentheses.

Table 14: Critical values of F($\hat{k}$) – FDF test

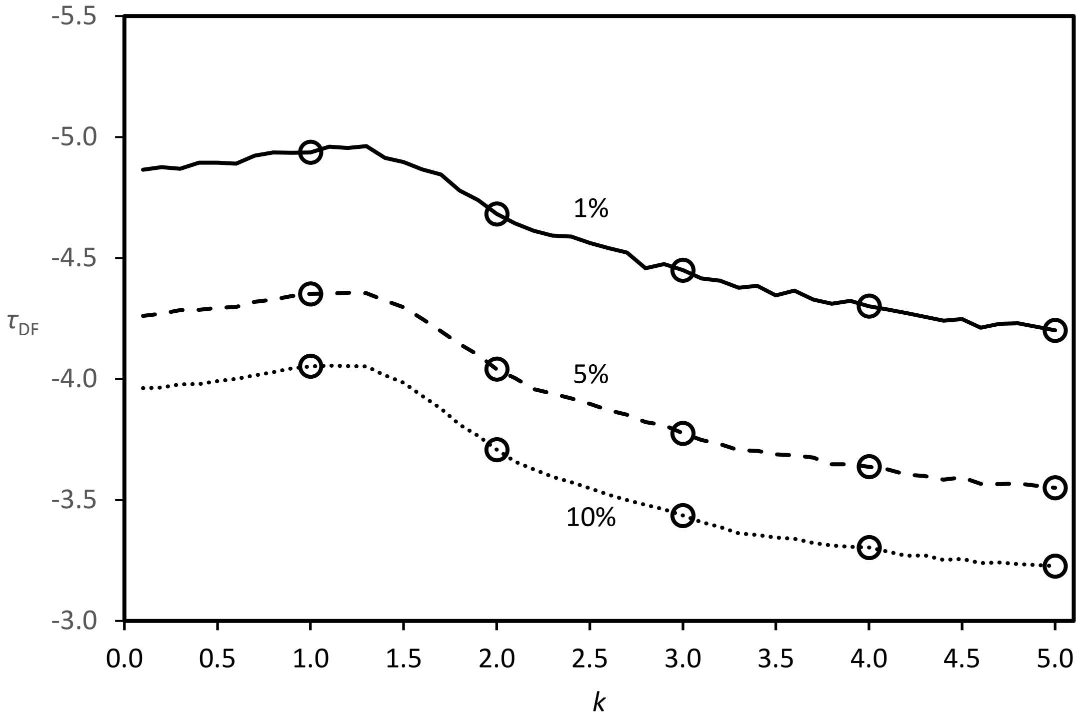
	T = 40			*T* = 60			*T* = 80			*T* = 100			*T* = 200			*T* = 500			*T* = 2500		
	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
(a) Integer values of *k* and *j* = 0 by assumption																					
	8.30	9.97	13.86	8.02	9.49	12.92	7.80	9.22	12.32	7.81	9.16	12.17	7.63	8.92	11.70	7.56	8.80	11.48	7.46	8.67	11.34
										(7.78)	(9.14)	(12.21)	(7.62)	(8.88)	(11.70)	(7.53)	(8.76)	(11.52)	(7.50)	(8.71)	(11.35)
(b) Fractional values of *k* and *j* = 0 by assumption																					
	10.75	12.82	17.66	10.18	11.90	15.95	9.85	11.52	15.14	9.72	11.27	14.72	9.47	10.92	14.08	9.29	10.68	13.66	9.18	10.51	13.31
(c) Fractional values of *k* and *j* selected by Hall's (1994) general-to-specific method																					
	14.93	17.72	24.60	13.17	15.37	20.25	12.22	14.21	18.42	11.69	13.55	17.43	10.69	12.28	15.71	10.03	11.53	14.62	9.43	10.82	13.72

Notes: Enders and Lee's (2012b, Table 1a) values are shown in parentheses.

Figure 1: The effect of k on the critical values of τ_{LM} and τ_{DF}



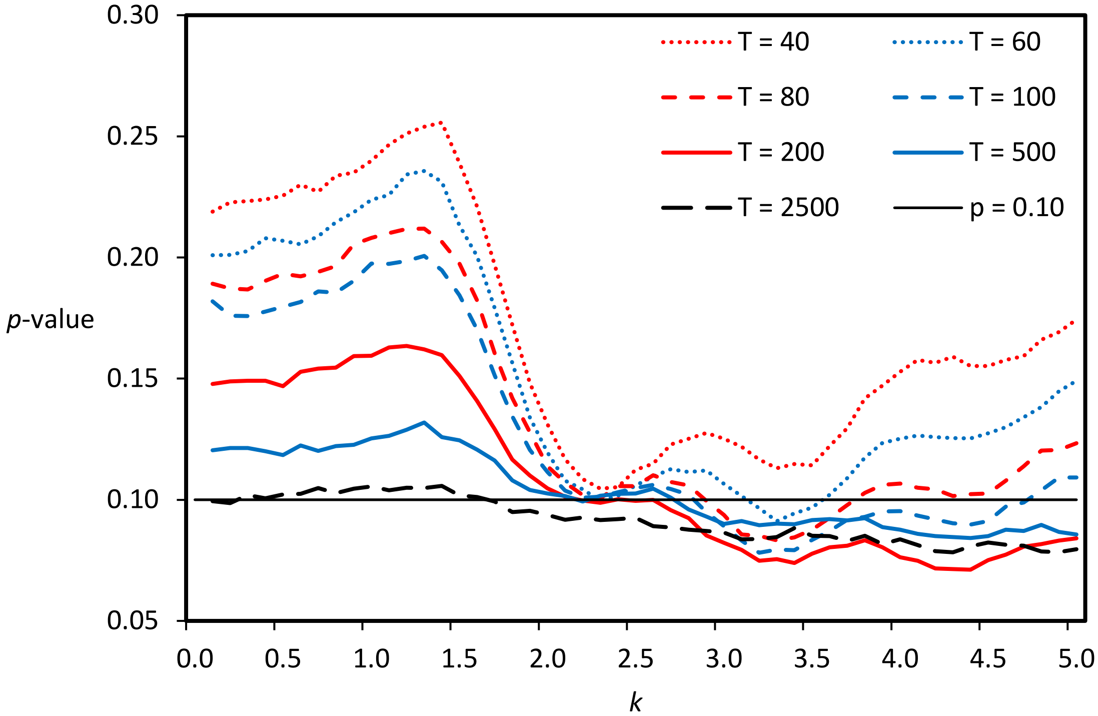
(a) τ_{LM} ($T = 100$)



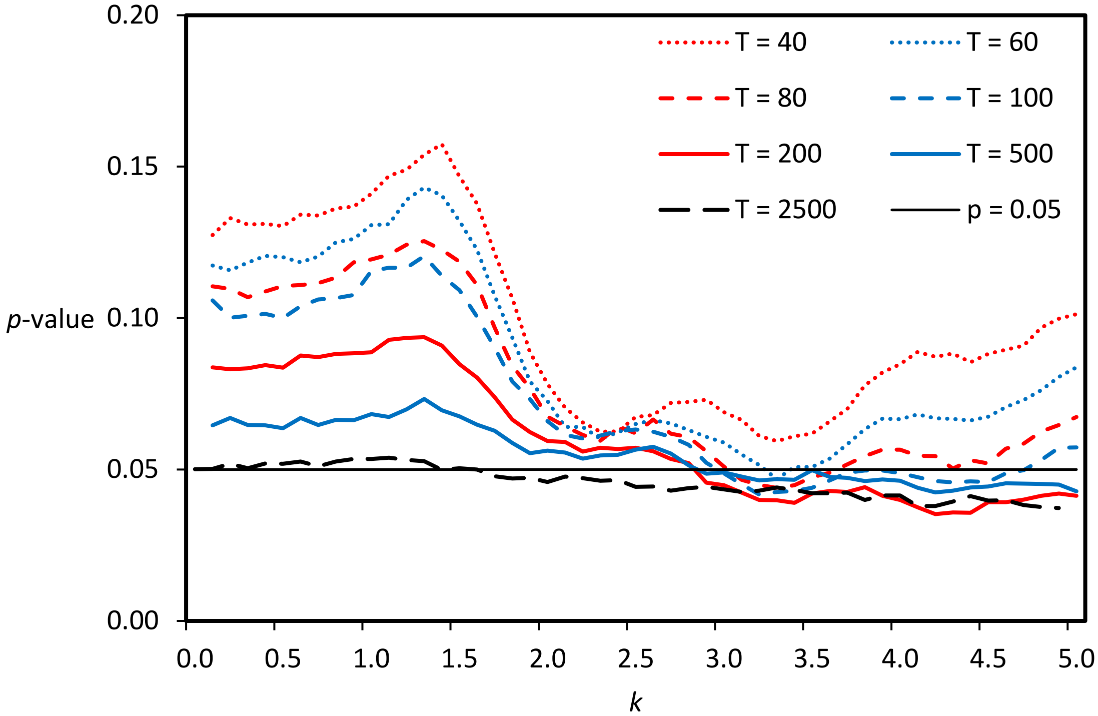
(b) τ_{DF} ($T = 100$)

Note: Circles denote the critical values for integer values of k.

Figure 2: Critical values of τ_{LM} (for $j = 0$) – p-values implied by the distribution estimated using Hall's (1994) general-to-specific lag selection method

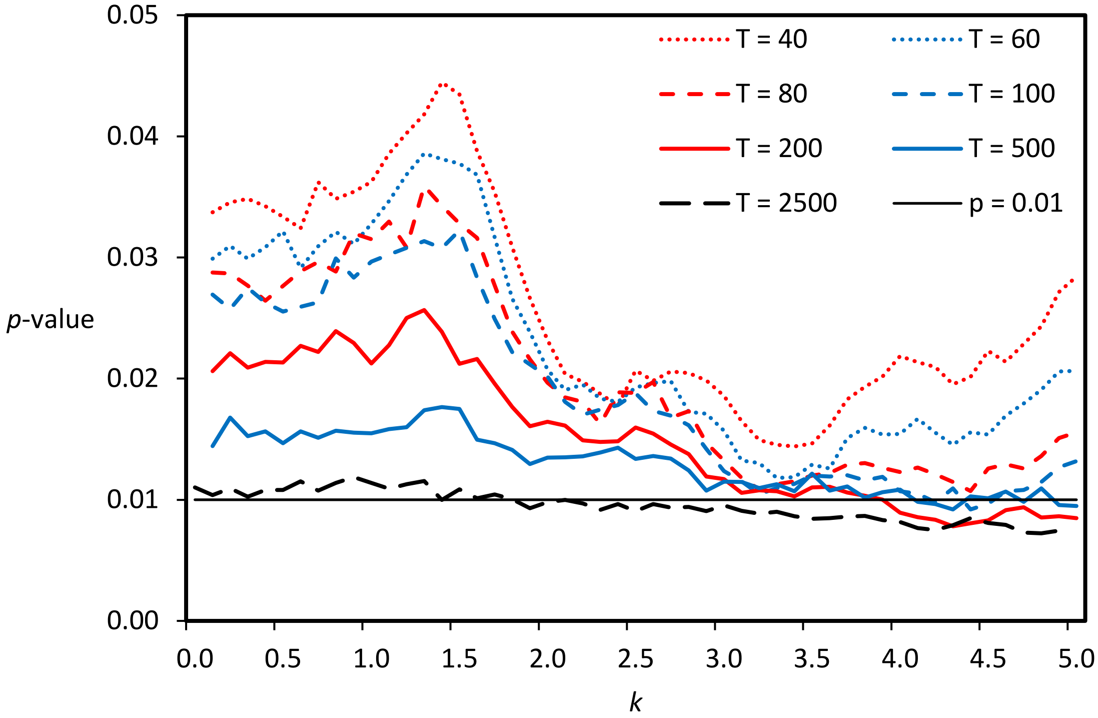


(a) 10% critical values from Table 1



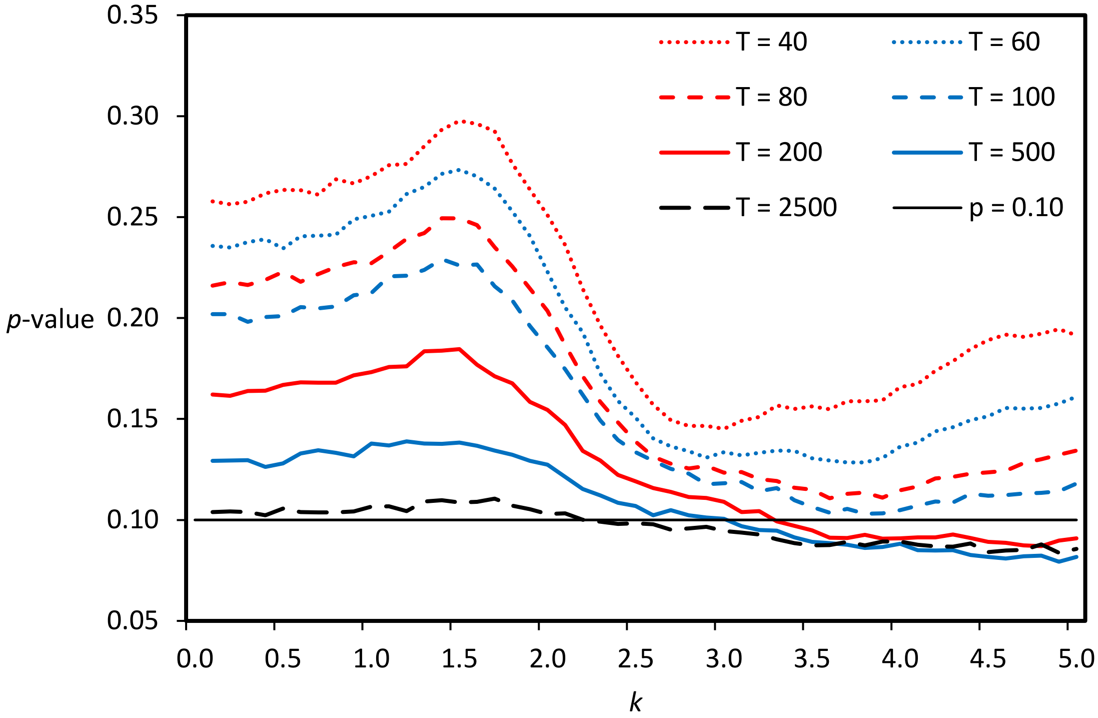
(b) 5% critical values from Table 2

Figure 2 (continued): Critical values of τ_{LM} (for $j = 0$) – p-values implied by the distribution estimated using Hall's (1994) general-to-specific lag selection method

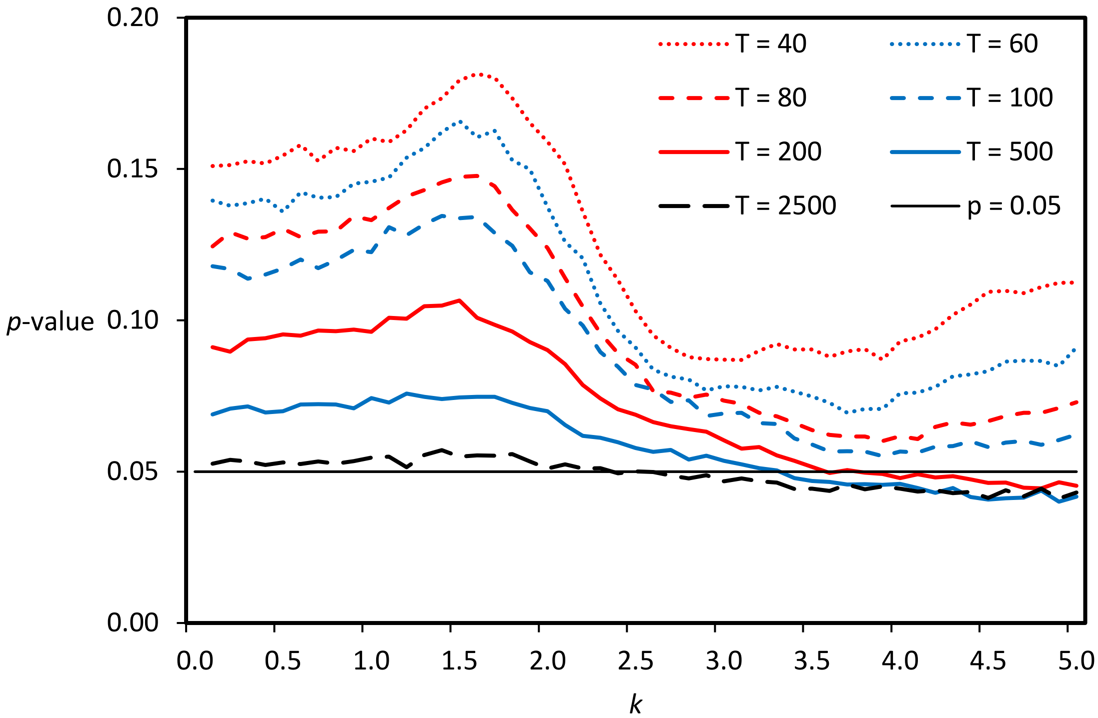


(c) 1% critical values from Table 3

Figure 3: Critical values of τ_{DF} (for $j = 0$) – p-values implied by the distribution estimated using Hall's (1994) general-to-specific lag selection method

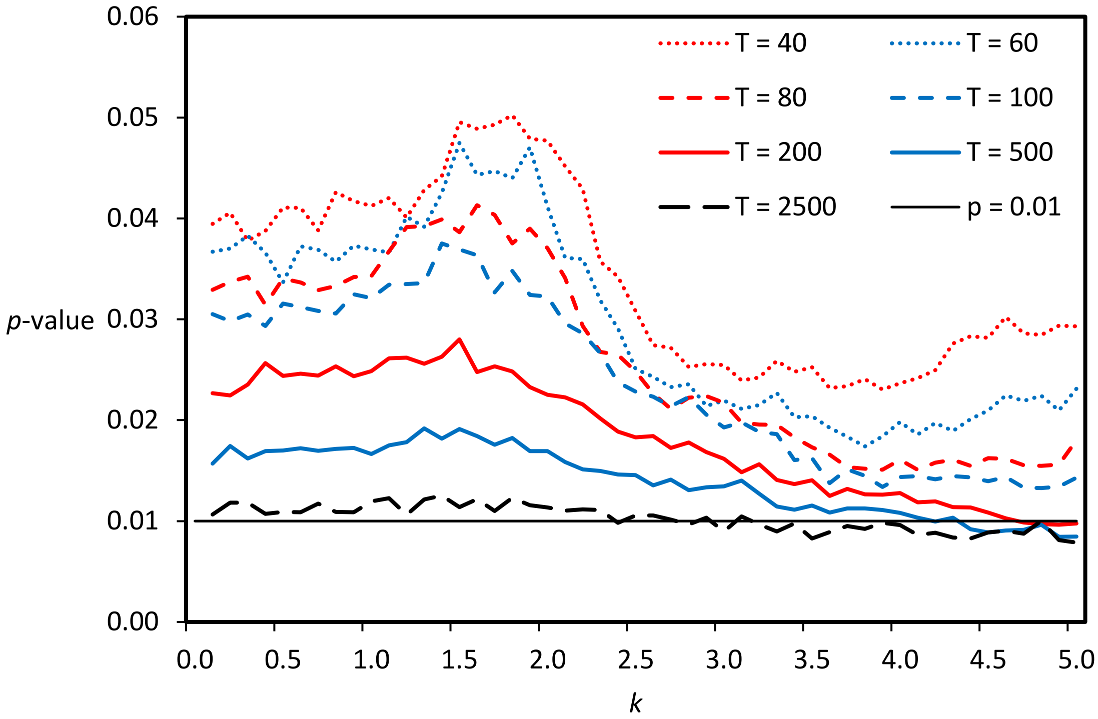


(a) 10% critical values from Table 4



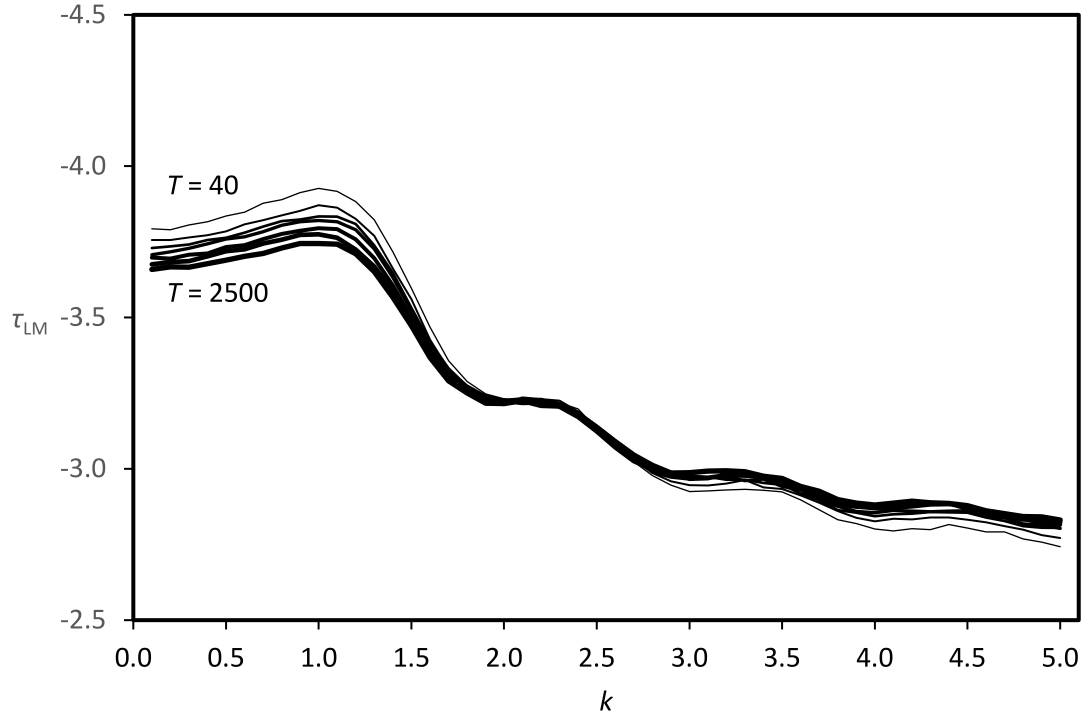
(b) 5% critical values from Table 5

Figure 3 (continued): Critical values of τ_{DF} (for j = 0) – p-values implied by the distribution estimated using Hall's (1994) general-to-specific lag selection method

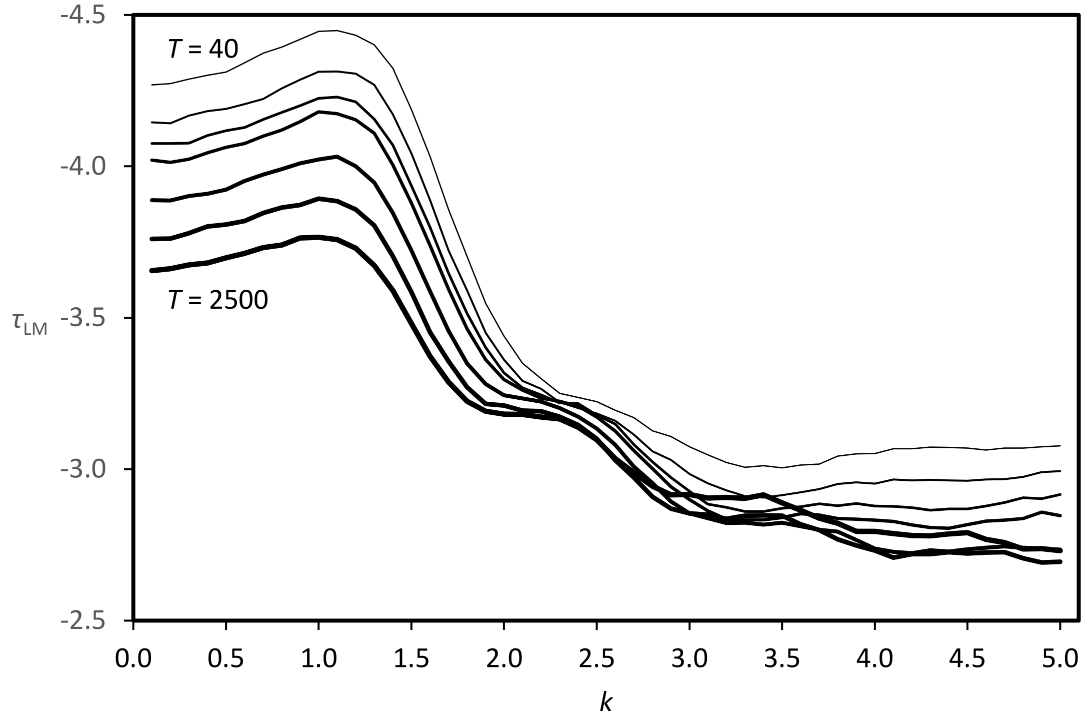


(c) 1% critical values from Table 6

Figure 4: 10% critical values of τ_{LM} (T = 40, 60, 80, 100, 200, 500, 2500)

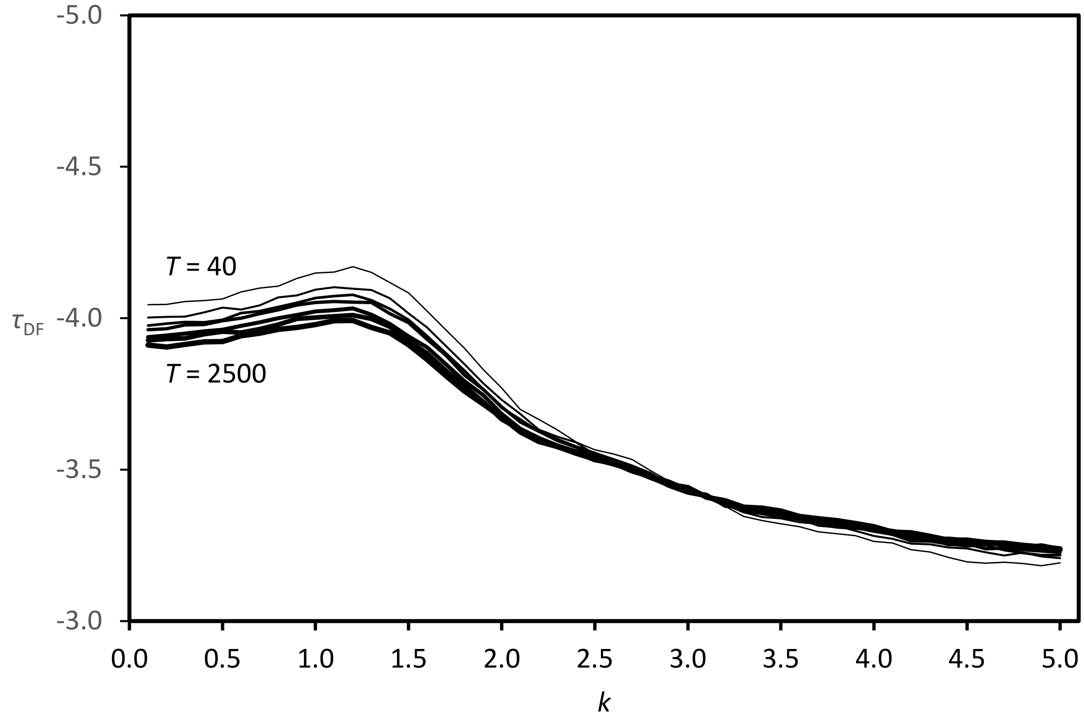


(a) No lagged terms in the test equation

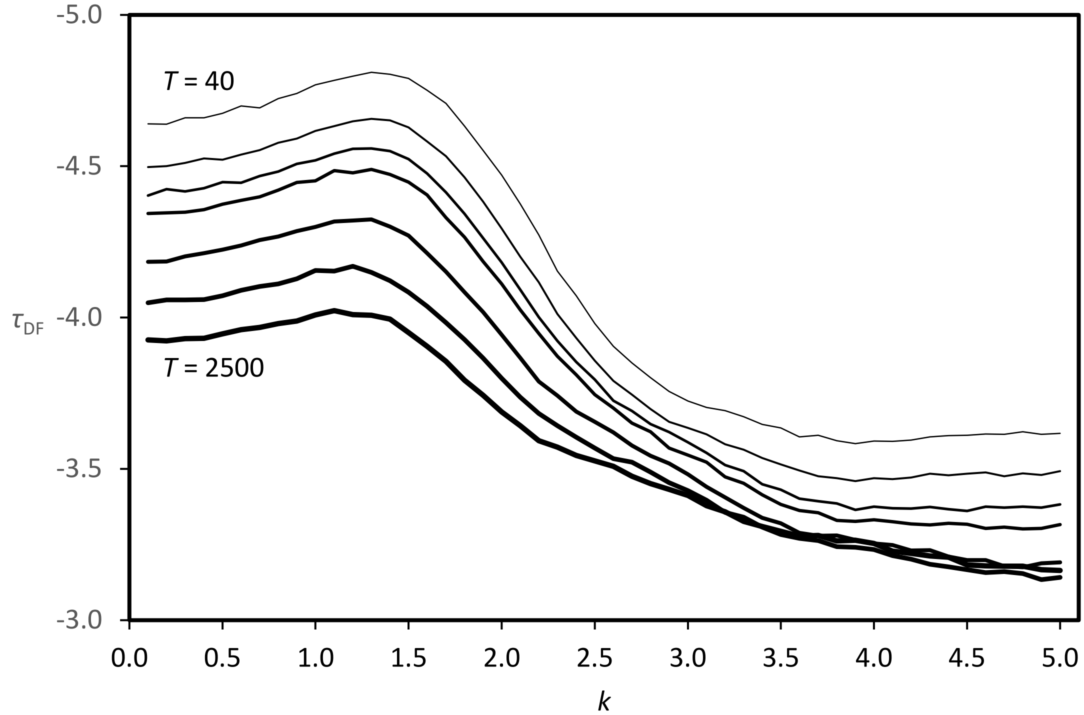


(b) Hall's (1994) general-to-specific method used to select the number of lagged terms

Figure 5: 10% critical values of τ_{DF} (T = 40, 60, 80, 100, 200, 500, 2500)



(a) No lagged terms in the test equation



(b) Hall's (1994) general-to-specific method used to select the number of lagged terms

Appendix

A1. E&L's (2012a) RATS code

The RATS code used to compute the critical values of F($\hat{k}$) presented in E&L (2012a, Table 1, Panel c) is given below.[5]

```
com length = 2500
all length
seed 12345
com iters = 100000
dec vector[series] ftests(4)
dec vector[integer] reglist
do j = 1,4
set tstats(j) 1 iters = 0.
   set ftests(j) 1 iters = 0.
end do
do i = 1,iters
   set(first=%ran(1)) v = v{1} + %ran(1)
   set y = v
   dif y / dy
com j = 0
dofor cap_T = 100 200 500 2500
   set sin1 1 cap_T = sin(2*%pi*1*t/cap_T)
      set cos1 1 cap_T = cos(2*%pi*1*t/cap_T)
      set sin2 1 cap_T = sin(2*%pi*2*t/cap_T)
      set cos2 1 cap_T = cos(2*%pi*2*t/cap_T)
      set sin3 1 cap_T = sin(2*%pi*3*t/cap_T)
      set cos3 1 cap_T = cos(2*%pi*3*t/cap_T)
      set sin4 1 cap_T = sin(2*%pi*4*t/cap_T)
      set cos4 1 cap_T = cos(2*%pi*4*t/cap_T)
      set sin5 1 cap_T = sin(2*%pi*5*t/cap_T)
      set cos5 1 cap_T = cos(2*%pi*5*t/cap_T)
   com j = j+1
   do k = 1,5
      if k == 1  { ; set x1 1 cap_T = sin1 ; set x2 1 cap_T = cos1 ; }
      if k == 2  { ; set x1 1 cap_T = sin2 ; set x2 1 cap_T = cos2 ; }
      if k == 3  { ; set x1 1 cap_T = sin3 ; set x2 1 cap_T = cos3 ; }
      if k == 4  { ; set x1 1 cap_T = sin4 ; set x2 1 cap_T = cos4 ; }
      if k == 5  { ; set x1 1 cap_T = sin5 ; set x2 1 cap_T = cos5 ; }
      dif x1 * cap_T dx1
      dif x2 * cap_T dx2
      linreg(noprint) dy * cap_T ; # constant
      com phi1 = y(1) - %beta(1)
      set s_tilde * cap_T = y - phi1 - %beta(1)*t
      dif s_tilde * cap_T ds_tilde
      lin(noprint) dy * cap_T; # s_tilde{1} dx1 dx2 constant
      if k==1 ; com kmin = 1 , rss_min = %rss
      if %rss<rss_min ; com kmin = k , rss_min = %rss
    end do k
```

[5] I am grateful to Professor Enders for providing this code.

```
    set x1 1 cap_T = sin(2*%pi*kmin*t/cap_T)
    set x2 1 cap_T = cos(2*%pi*kmin*t/cap_T)
    dif x1 * cap_T dx1
    dif x2 * cap_T dx2
    linreg(noprint) dy * cap_T ; # constant
    com phi1 = y(1) - %beta(1)
    set s_tilde * cap_T = y - phi1 - %beta(1)*t
    dif s_tilde * cap_T ds_tilde
    lin(noprint) dy * cap_T; # s_tilde{1} dx1 dx2 constant
    exc(noprint); # dx1 dx2
    com ftests(j)(i) = %cdstat
end dofor cap_T
end do i
do j = 1,4
sta(noprint,fractiles) ftests(j)
   dis 'Ftests' ###.### %fract90 %fract95 %fract99
end do
```

The lines highlighted above remove a linear trend from the original data series to create $\tilde{S}$. However, equation (9) in E&L (2012a) shows that $\tilde{S}$ is constructed by removing a single frequency Fourier-form (i.e., nonlinear) trend. The following code does this:

```
linreg(noprint) dy * cap_T ; # constant dx1 dx2
com phi1 = y(1) - %beta(1) - %beta(2)*x1(1) - %beta(3)*x2(1)
set s_tilde * cap_T = y - phi1 - %beta(1)*t - %beta(2)*x1 - %beta(3)*x2
```

When this code is used, the critical values of F($\hat{k}$) generated are consistent with those reported in Table 13.

A2. SHAZAM code used in this study

The following examples of code relate to the $T = 100$ and (when relevant) $k = 1.2$ case, but are easily adapted to any value of T and/or k. Each programme is designed to replicate a test only 10,000 times. However, as the SET RANFIX command is not used, each run of a programme produces a unique set of output values. Therefore, the desired number of replications can be achieved by rerunning the programme the required number of times.

A2.1 Code for estimating the distribution of τ_{LM} (j = 0)

(Used for Tables 1–3.)

```
SIZE 99000
PAR 90000
GEN1 T=100
GEN1 K=1.2
SAMPLE 1 T
GENR TIME=TIME(0)
GENR SIN=SIN(2*3.141592654*K*TIME/T)
GENR COS=SIN((3.141592654/2)-(2*3.141592654*K*TIME/T))
GENR DSIN=SIN-LAG(SIN,1)
GENR DCOS=COS-LAG(COS,1)
SE:1
SET NODOECHO NOWARN
DIM TR0 4
   ?DO # = 1,10000
   SAMPLE 1 T
   GENR U=NOR([SE])
   GEN1 E0=SAMP(U)
   GENR Y=SUM(U)+E0
   GENR DY=Y-LAG(Y,1)
   SAMPLE 2 T
   ?OLS DY DSIN DCOS/ COEF=DELTILDE
   MATRIX DEL0=DELTILDE(3,1)
   MATRIX DEL1=DELTILDE(1,1)
   MATRIX DEL2=DELTILDE(2,1)
   SAMPLE 1 1
   GENR PSI=Y-DEL0-(DEL1*SIN)-(DEL2*COS)
   SAMPLE 1 T
   GENR S=Y-PSI-(DEL0*TIME)-(DEL1*SIN)-(DEL2*COS)
   GENR S1=LAG(S,1)
   SAMPLE 2 T
   ?OLS DY S1 DSIN DCOS/TRATIO=TR0
   GEN1 TAU#=TR0:1
   ?ENDO
SAMPLE 1 10000
DIM TAULM 10000
   ?DO # = 1,10000
   GENR TAULM:#=TAU#
   ?ENDO
WRITE(C:\SHAZAM\output1.txt) TAULM TAULM
DELETE/ALL
COMPRESS
STOP
```

A2.2 Code for estimating the distribution of τ_{DF} (j = 0)

(Used for Tables 4–6.)

```
SIZE 99000
PAR 90000
GEN1 T=100
SAMPLE 1 T
GENR TIME=TIME(0)
GEN1 K=1.2
GENR SIN=SIN(2*3.141592654*K*TIME/T)
GENR COS=SIN((3.141592654/2)-(2*3.141592654*K*TIME/T))
SE:1
SET NODOECHO NOWARN
DIM TR0 5
   ?DO # = 1,10000
   SAMPLE 1 T
   GENR U=NOR([SE])
   GEN1 E0=SAMP(U)
   GENR Y=SUM(U)+E0
   GENR DY=Y-LAG(Y,1)
   GENR Y1=LAG(Y,1)
   SAMPLE 2 T
   ?OLS DY Y1 SIN COS TIME/TRATIO=TR0
   GEN1 TAU#=TR0:1
   ?ENDO
SAMPLE 1 10000
DIM TAUDF 10000
   ?DO # = 1,10000
   GENR TAUDF:#=TAU#
   ?ENDO
WRITE(C:\SHAZAM\output1.txt) TAUDF TAUDF
DELETE/ALL
COMPRESS
STOP
```

A2.3 Code for estimating the distribution of τ_{LM} (j selected by Hall's, 1994, method)

(Used for Tables 7–9.)

```
SIZE 99000
PAR 90000
GEN1 T=100
GEN1 K=1.2
SAMPLE 1 T
GENR TIME=TIME(0)
GENR SIN=SIN(2*3.141592654*K*TIME/T)
GENR COS=SIN((3.141592654/2)-(2*3.141592654*K*TIME/T))
GENR DSIN=SIN-LAG(SIN,1)
GENR DCOS=COS-LAG(COS,1)
SE:1
SET NODOECHO NOWARN
DIM TR10 14 TR9 13 TR8 12 TR7 11 TR6 10 TR5 9 TR4 8 TR3 7 TR2 6 TR1 5 TR0 4
   ?DO # = 1,10000
   SAMPLE 1 T
   GENR U=NOR([SE])
   GEN1 E0=SAMP(U)
   GENR Y=SUM(U)+E0
   GENR DY=Y-LAG(Y,1)
   GEN1 CHECK=99
   SAMPLE 2 T
   ?OLS DY DSIN DCOS/ COEF=DELTILDE
   MATRIX DEL0=DELTILDE(3,1)
   MATRIX DEL1=DELTILDE(1,1)
   MATRIX DEL2=DELTILDE(2,1)
   SAMPLE 1 1
   GENR PSI=Y-DEL0-(DEL1*SIN)-(DEL2*COS)
   SAMPLE 1 T
   GENR S=Y-PSI-(DEL0*TIME)-(DEL1*SIN)-(DEL2*COS)
   GENR S1=LAG(S,1)
   GENR DS=S-S1
      ?DO ! = 1,10
      GENR DS!=LAG(DS,!)
      ?ENDO
   SAMPLE 12 T
   ?OLS DY S1 DS10 DS9 DS8 DS7 DS6 DS5 DS4 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR10
   ?OLS DY S1 DS9 DS8 DS7 DS6 DS5 DS4 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR9
   ?OLS DY S1 DS8 DS7 DS6 DS5 DS4 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR8
   ?OLS DY S1 DS7 DS6 DS5 DS4 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR7
   ?OLS DY S1 DS6 DS5 DS4 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR6
   ?OLS DY S1 DS5 DS4 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR5
   ?OLS DY S1 DS4 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR4
   ?OLS DY S1 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR3
   ?OLS DY S1 DS2 DS1 DSIN DCOS/TRATIO=TR2
   ?OLS DY S1 DS1 DSIN DCOS/TRATIO=TR1
   ?OLS DY S1 DSIN DCOS/TRATIO=TR0
   IF((CHECK .EQ. 99) .AND. (ABS(TR10:2) .GT. 1.65))
   GEN1 CHECK=10
   IF(CHECK .EQ. 10)
```

```
   GEN1 TAU#=TR10:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR9:2) .GT. 1.65))
   GEN1 CHECK=9
   IF(CHECK .EQ. 9)
   GEN1 TAU#=TR9:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR8:2) .GT. 1.65))
   GEN1 CHECK=8
   IF(CHECK .EQ. 8)
   GEN1 TAU#=TR8:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR7:2) .GT. 1.65))
   GEN1 CHECK=7
   IF(CHECK .EQ. 7)
   GEN1 TAU#=TR7:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR6:2) .GT. 1.65))
   GEN1 CHECK=6
   IF(CHECK .EQ. 6)
   GEN1 TAU#=TR6:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR5:2) .GT. 1.65))
   GEN1 CHECK=5
   IF(CHECK .EQ. 5)
   GEN1 TAU#=TR5:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR4:2) .GT. 1.65))
   GEN1 CHECK=4
   IF(CHECK .EQ. 4)
   GEN1 TAU#=TR4:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR3:2) .GT. 1.65))
   GEN1 CHECK=3
   IF(CHECK .EQ. 3)
   GEN1 TAU#=TR3:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR2:2) .GT. 1.65))
   GEN1 CHECK=2
   IF(CHECK .EQ. 2)
   GEN1 TAU#=TR2:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR1:2) .GT. 1.65))
   GEN1 CHECK=1
   IF(CHECK .EQ. 1)
   GEN1 TAU#=TR1:1
   IF(CHECK .EQ. 99)
   GEN1 TAU#=TR0:1
   ?ENDO
SAMPLE 1 10000
DIM TAULM 10000
   ?DO # = 1,10000
   GENR TAULM:#=TAU#
   ?ENDO
WRITE(C:\SHAZAM\output1.txt) TAULM TAULM
DELETE/ALL
COMPRESS
STOP
```

A2.4 Code for estimating the distribution of τ_{DF} (j selected by Hall's, 1994, method)

(Used for Tables 10–12.)

```
SIZE 99000
PAR 90000
GEN1 T=100
SAMPLE 1 T
GENR TIME=TIME(0)
GEN1 K=1.2
GENR SIN=SIN(2*3.141592654*K*TIME/T)
GENR COS=SIN((3.141592654/2)-(2*3.141592654*K*TIME/T))
SE:1
SET NODOECHO NOWARN
DIM TR10 15 TR9 14 TR8 13 TR7 12 TR6 11 TR5 10 TR4 9 TR3 8 TR2 7 TR1 6 TR0
5
   ?DO # = 1,10000
   SAMPLE 1 T
   GENR U=NOR([SE])
   GEN1 E0=SAMP(U)
   GENR Y=SUM(U)+E0
   GENR Y1=LAG(Y,1)
   GENR DY=Y-Y1
      ?DO ! = 1,10
      GENR DY!=LAG(DY,!)
      ?ENDO
   GEN1 CHECK=99
   SAMPLE 12 T
   ?OLS  DY  Y1  DY10  DY9  DY8  DY7  DY6  DY5  DY4  DY3  DY2  DY1  SIN  COS
TIME/TRATIO=TR10
   ?OLS DY Y1 DY9 DY8 DY7 DY6 DY5 DY4 DY3 DY2 DY1 SIN COS TIME/TRATIO=TR9
   ?OLS DY Y1 DY8 DY7 DY6 DY5 DY4 DY3 DY2 DY1 SIN COS TIME/TRATIO=TR8
   ?OLS DY Y1 DY7 DY6 DY5 DY4 DY3 DY2 DY1 SIN COS TIME/TRATIO=TR7
   ?OLS DY Y1 DY6 DY5 DY4 DY3 DY2 DY1 SIN COS TIME/TRATIO=TR6
   ?OLS DY Y1 DY5 DY4 DY3 DY2 DY1 SIN COS TIME/TRATIO=TR5
   ?OLS DY Y1 DY4 DY3 DY2 DY1 SIN COS TIME/TRATIO=TR4
   ?OLS DY Y1 DY3 DY2 DY1 SIN COS TIME/TRATIO=TR3
   ?OLS DY Y1 DY2 DY1 SIN COS TIME/TRATIO=TR2
   ?OLS DY Y1 DY1 SIN COS TIME/TRATIO=TR1
   ?OLS DY Y1 SIN COS TIME/TRATIO=TR0
   IF((CHECK .EQ. 99) .AND. (ABS(TR10:2) .GT. 1.65))
   GEN1 CHECK=10
   IF(CHECK .EQ. 10)
   GEN1 TAU#=TR10:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR9:2) .GT. 1.65))
   GEN1 CHECK=9
   IF(CHECK .EQ. 9)
   GEN1 TAU#=TR9:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR8:2) .GT. 1.65))
   GEN1 CHECK=8
   IF(CHECK .EQ. 8)
   GEN1 TAU#=TR8:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR7:2) .GT. 1.65))
```

```
   GEN1 CHECK=7
   IF(CHECK .EQ. 7)
   GEN1 TAU#=TR7:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR6:2) .GT. 1.65))
   GEN1 CHECK=6
   IF(CHECK .EQ. 6)
   GEN1 TAU#=TR6:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR5:2) .GT. 1.65))
   GEN1 CHECK=5
   IF(CHECK .EQ. 5)
   GEN1 TAU#=TR5:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR4:2) .GT. 1.65))
   GEN1 CHECK=4
   IF(CHECK .EQ. 4)
   GEN1 TAU#=TR4:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR3:2) .GT. 1.65))
   GEN1 CHECK=3
   IF(CHECK .EQ. 3)
   GEN1 TAU#=TR3:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR2:2) .GT. 1.65))
   GEN1 CHECK=2
   IF(CHECK .EQ. 2)
   GEN1 TAU#=TR2:1
   IF((CHECK .EQ. 99) .AND. (ABS(TR1:2) .GT. 1.65))
   GEN1 CHECK=1
   IF(CHECK .EQ. 1)
   GEN1 TAU#=TR1:1
   IF(CHECK .EQ. 99)
   GEN1 TAU#=TR0:1
   ?ENDO
SAMPLE 1 10000
DIM TAUDF 10000
   ?DO # = 1,10000
   GENR TAUDF:#=TAU#
   ?ENDO
WRITE(C:\SHAZAM\output1.txt) TAUDF TAUDF
DELETE/ALL
COMPRESS
STOP
```

A2.5 Code for estimating the distribution of $F(\hat{k})$ *(FLM test, integer values of k, j = 0)*

(Used for Table 13(a).)

```
SIZE 99000
PAR 90000
GEN1 T=100
SAMPLE 1 T
GENR TIME=TIME(0)
SET NODOECHO NOWARN
SE:1
   ?DO # = 1,10000
   SAMPLE 1 T
   GENR U=NOR([SE])
   GEN1 E0=SAMP(U)
   GENR Y=SUM(U)+E0
   GENR DY=Y-LAG(Y,1)
   GEN1 SSRMIN=1000000000
   GEN1 KHAT=0
      ?DO % = 1,5
      GEN1 K=%
      SAMPLE 1 T
      GENR SIN=SIN(2*3.141592654*K*TIME/T)
      GENR COS=SIN((3.141592654/2)-(2*3.141592654*K*TIME/T))
      GENR DSIN=SIN-LAG(SIN,1)
      GENR DCOS=COS-LAG(COS,1)
      SAMPLE 2 T
      ?OLS DY DSIN DCOS/ COEF=DELTILDE
      MATRIX DEL0=DELTILDE(3,1)
      MATRIX DEL1=DELTILDE(1,1)
      MATRIX DEL2=DELTILDE(2,1)
      SAMPLE 1 1
      GENR PSI=Y-DEL0-(DEL1*SIN)-(DEL2*COS)
      SAMPLE 1 T
      GENR S=Y-PSI-(DEL0*TIME)-(DEL1*SIN)-(DEL2*COS)
      GENR S1=LAG(S,1)
      SAMPLE 2 T
      ?OLS DY S1 DSIN DCOS/
      GEN1 SSR=$SSE
      IF(SSR .LT. SSRMIN)
      GEN1 KHAT=K
      IF(SSR .LT. SSRMIN)
      GEN1 SSRMIN=SSR
      ?ENDO
   SAMPLE 1 T
   GENR SIN=SIN(2*3.141592654*KHAT*TIME/T)
   GENR COS=SIN((3.141592654/2)-(2*3.141592654*KHAT*TIME/T))
   GENR DSIN=SIN-LAG(SIN,1)
   GENR DCOS=COS-LAG(COS,1)
   SAMPLE 2 T
   ?OLS DY DSIN DCOS/ COEF=DELTILDE
   MATRIX DEL0=DELTILDE(3,1)
   MATRIX DEL1=DELTILDE(1,1)
```

```
   MATRIX DEL2=DELTILDE(2,1)
   SAMPLE 1 1
   GENR PSI=Y-DEL0-(DEL1*SIN)-(DEL2*COS)
   SAMPLE 1 T
   GENR S=Y-PSI-(DEL0*TIME)-(DEL1*SIN)-(DEL2*COS)
   GENR S1=LAG(S,1)
   SAMPLE 2 T
   ?OLS DY S1 DSIN DCOS /
   ?TEST
   ?TEST DSIN=0
   ?TEST DCOS=0
   ?END
   GEN1 FKH#=$F
   ?ENDO
SAMPLE 1 10000
DIM FKHAT 10000
   ?DO # = 1,10000
   GENR FKHAT:#=FKH#
   ?ENDO
WRITE(C:\SHAZAM\output1.txt) FKHAT FKHAT
DELETE/ALL
COMPRESS
STOP
```

A2.6 Code for estimating the distribution of $\mathrm{F}(\hat{k})$ *(FLM test, fractional values of k, j = 0)*

(Used for Table 13(b).)

Replace the highlighted lines in A1.5 with the following:

```
?DO % = 1,50
GEN1 K=%/10
```

A2.7 Code for estimating the distribution for $\mathrm{F}(\hat{k})$ *(FLM test, fractional values of k, j selected by Hall's, 1994, method)*

(Used for Table 13(c).)

```
SIZE 99000
PAR 90000
GEN1 T=100
SAMPLE 1 T
GENR TIME=TIME(0)
SET NODOECHO NOWARN
DIM TR10 14 TR9 13 TR8 12 TR7 11 TR6 10 TR5 9 TR4 8 TR3 7 TR2 6 TR1 5
SE:1
   ?DO # = 1,10000
   SAMPLE 1 T
   GENR U=NOR([SE])
   GEN1 E0=SAMP(U)
   GENR Y=SUM(U)+E0
   GENR DY=Y-LAG(Y,1)
   GEN1 SSRMIN=1000000000
   GEN1 KHAT=0
      ?DO % = 1,50
      GEN1 K=%/10
      SAMPLE 1 T
      GENR SIN=SIN(2*3.141592654*K*TIME/T)
      GENR COS=SIN((3.141592654/2)-(2*3.141592654*K*TIME/T))
      GENR DSIN=SIN-LAG(SIN,1)
      GENR DCOS=COS-LAG(COS,1)
      SAMPLE 2 T
      ?OLS DY DSIN DCOS/ COEF=DELTILDE
      MATRIX DEL0=DELTILDE(3,1)
      MATRIX DEL1=DELTILDE(1,1)
      MATRIX DEL2=DELTILDE(2,1)
      SAMPLE 1 1
      GENR PSI=Y-DEL0-(DEL1*SIN)-(DEL2*COS)
      SAMPLE 1 T
      GENR S=Y-PSI-(DEL0*TIME)-(DEL1*SIN)-(DEL2*COS)
      GENR S1=LAG(S,1)
      GENR DS=S-S1
         ?DO ! = 1,10
         GENR DS!=LAG(DS,!)
         ?ENDO
      SAMPLE 12 T
      ?OLS DY S1 DSIN DCOS DS1 DS2 DS3 DS4 DS5 DS6 DS7 DS8 DS9 DS10/
      GEN1 SSR=$SSE
      IF(SSR .LT. SSRMIN)
      GEN1 KHAT=K
      IF(SSR .LT. SSRMIN)
      GEN1 SSRMIN=SSR
      ?ENDO
   SAMPLE 1 T
   GENR SIN=SIN(2*3.141592654*KHAT*TIME/T)
   GENR COS=SIN((3.141592654/2)-(2*3.141592654*KHAT*TIME/T))
```

```
GENR DSIN=SIN-LAG(SIN,1)
GENR DCOS=COS-LAG(COS,1)
SAMPLE 2 T
?OLS DY DSIN DCOS/ COEF=DELTILDE
MATRIX DEL0=DELTILDE(3,1)
MATRIX DEL1=DELTILDE(1,1)
MATRIX DEL2=DELTILDE(2,1)
SAMPLE 1 1
GENR PSI=Y-DEL0-(DEL1*SIN)-(DEL2*COS)
SAMPLE 1 T
GENR S=Y-PSI-(DEL0*TIME)-(DEL1*SIN)-(DEL2*COS)
GENR S1=LAG(S,1)
GENR DS=S-S1
      ?DO ! = 1,10
      GENR DS!=LAG(DS,!)
      ?ENDO
GEN1 CHECK=99
SAMPLE 12 T
?OLS DY S1 DS10 DS9 DS8 DS7 DS6 DS5 DS4 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR10
?TEST
?TEST DSIN=0
?TEST DCOS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR10:2) .GT. 1.65))
GEN1 CHECK=10
IF(CHECK .EQ. 10)
GEN1 FKH#=$F
?OLS DY S1 DS9 DS8 DS7 DS6 DS5 DS4 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR9
?TEST
?TEST DSIN=0
?TEST DCOS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR9:2) .GT. 1.65))
GEN1 CHECK=9
IF(CHECK .EQ. 9)
GEN1 FKH#=$F
?OLS DY S1 DS8 DS7 DS6 DS5 DS4 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR8
?TEST
?TEST DSIN=0
?TEST DCOS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR8:2) .GT. 1.65))
GEN1 CHECK=8
IF(CHECK .EQ. 8)
GEN1 FKH#=$F
?OLS DY S1 DS7 DS6 DS5 DS4 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR7
?TEST
?TEST DSIN=0
?TEST DCOS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR7:2) .GT. 1.65))
GEN1 CHECK=7
IF(CHECK .EQ. 7)
GEN1 FKH#=$F
```

```
?OLS DY S1 DS6 DS5 DS4 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR6
?TEST
?TEST DSIN=0
?TEST DCOS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR6:2) .GT. 1.65))
GEN1 CHECK=6
IF(CHECK .EQ. 6)
GEN1 FKH#=$F
?OLS DY S1 DS5 DS4 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR5
?TEST
?TEST DSIN=0
?TEST DCOS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR5:2) .GT. 1.65))
GEN1 CHECK=5
IF(CHECK .EQ. 5)
GEN1 FKH#=$F
?OLS DY S1 DS4 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR4
?TEST
?TEST DSIN=0
?TEST DCOS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR4:2) .GT. 1.65))
GEN1 CHECK=4
IF(CHECK .EQ. 4)
GEN1 FKH#=$F
?OLS DY S1 DS3 DS2 DS1 DSIN DCOS/TRATIO=TR3
?TEST
?TEST DSIN=0
?TEST DCOS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR3:2) .GT. 1.65))
GEN1 CHECK=3
IF(CHECK .EQ. 3)
GEN1 FKH#=$F
?OLS DY S1 DS2 DS1 DSIN DCOS/TRATIO=TR2
?TEST
?TEST DSIN=0
?TEST DCOS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR2:2) .GT. 1.65))
GEN1 CHECK=2
IF(CHECK .EQ. 2)
GEN1 FKH#=$F
?OLS DY S1 DS1 DSIN DCOS/TRATIO=TR1
?TEST
?TEST DSIN=0
?TEST DCOS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR1:2) .GT. 1.65))
GEN1 CHECK=1
IF(CHECK .EQ. 1)
GEN1 FKH#=$F
```

```
   ?OLS DY S1 DSIN DCOS /
   ?TEST
   ?TEST DSIN=0
   ?TEST DCOS=0
   ?END
   IF(CHECK .EQ. 99)
   GEN1 FKH#=$F
   ?ENDO
SAMPLE 1 10000
DIM FKHAT 10000
   ?DO # = 1,10000
   GENR FKHAT:#=FKH#
   ?ENDO
WRITE(C:\SHAZAM\output1.txt) FKHAT FKHAT
DELETE/ALL
COMPRESS
STOP
```

A2.8 Code for estimating the distribution of $\mathrm{F}(\hat{k})$ *(FDF test, integer values of k, j = 0)*

(Used for Table 14(a).)

```
SIZE 99000
PAR 90000
GEN1 T=100
SAMPLE 1 T
GENR TIME=TIME(0)
SET NODOECHO NOWARN
   ?DO # = 1,10000
   SE:1
   SAMPLE 1 T
   GENR U=NOR([SE])
   GEN1 E0=SAMP(U)
   GENR Y=SUM(U)+E0
   GENR DY=Y-LAG(Y,1)
   GENR Y1=LAG(Y,1)
   GEN1 SSRMIN=1000000000
   GEN1 KHAT=0
      ?DO % = 1,5
      GEN1 K=%
      SAMPLE 1 T
      GENR SIN=SIN(2*3.141592654*K*TIME/T)
      GENR COS=SIN((3.141592654/2)-(2*3.141592654*K*TIME/T))
      SAMPLE 2 T
      ?OLS DY Y1 SIN COS TIME/
      GEN1 SSR=$SSE
      IF(SSR .LT. SSRMIN)
      GEN1 KHAT=K
      IF(SSR .LT. SSRMIN)
      GEN1 SSRMIN=SSR
      ?ENDO
   SAMPLE 1 T
   GENR SIN=SIN(2*3.141592654*KHAT*TIME/T)
   GENR COS=SIN((3.141592654/2)-(2*3.141592654*KHAT*TIME/T))
   SAMPLE 2 T
   ?OLS DY Y1 SIN COS TIME/
   ?TEST
   ?TEST SIN=0
   ?TEST COS=0
   ?END
   GEN1 FKH#=$F
   ?ENDO
SAMPLE 1 10000
DIM FKHAT 10000
   ?DO # = 1,10000
   GENR FKHAT:#=FKH#
   ?ENDO
WRITE(C:\SHAZAM\output1.txt) FKHAT FKHAT
DELETE/ALL
COMPRESS
STOP
```

A2.9 Code for estimating the distribution of $\mathrm{F}(\hat{k})$ *(FDF test, fractional values of k, j = 0)*

(Used for Table 14(b).)

Replace the highlighted lines in A1.8 with the following:

```
?DO % = 1,50
GEN1 K=%/10
```

A2.10 Code for estimating the distribution for $\mathrm{F}(\hat{k})$ *(FDF test, fractional values of k, j selected by Hall's, 1994, method)*

(Used for Table 14(c).)

```
SIZE 99000
PAR 90000
GEN1 T=100
SAMPLE 1 T
GENR TIME=TIME(0)
SET NODOECHO NOWARN
DIM TR10 15 TR9 14 TR8 13 TR7 12 TR6 11 TR5 10 TR4 9 TR3 8 TR2 7 TR1 6
   ?DO # = 1,10000
   SE:1
   SAMPLE 1 T
   GENR U=NOR([SE])
   GEN1 E0=SAMP(U)
   GENR Y=SUM(U)+E0
   GENR Y1=LAG(Y,1)
   GENR DY=Y-Y1
      ?DO ! = 1,10
      GENR DY!=LAG(DY,!)
      ?ENDO
   GEN1 SSRMIN=1000000000
   GEN1 KHAT=0
      ?DO % = 1,50
      GEN1 K=%/10
      SAMPLE 1 T
      GENR SIN=SIN(2*3.141592654*K*TIME/T)
      GENR COS=SIN((3.141592654/2)-(2*3.141592654*K*TIME/T))
      SAMPLE 12 T
      ?OLS DY Y1 SIN COS TIME DY1 DY2 DY3 DY4 DY5 DY6 DY7 DY8 DY9 DY10/
      GEN1 SSR=$SSE
      IF(SSR .LT. SSRMIN)
      GEN1 KHAT=K
      IF(SSR .LT. SSRMIN)
      GEN1 SSRMIN=SSR
      ?ENDO
   SAMPLE 1 T
   GENR SIN=SIN(2*3.141592654*KHAT*TIME/T)
   GENR COS=SIN((3.141592654/2)-(2*3.141592654*KHAT*TIME/T))
   GEN1 CHECK=99
   SAMPLE 12 T
   ?OLS  DY  Y1  DY10  DY9  DY8  DY7  DY6  DY5  DY4  DY3  DY2  DY1  SIN  COS
TIME/TRATIO=TR10
   ?TEST
   ?TEST SIN=0
   ?TEST COS=0
   ?END
   IF((CHECK .EQ. 99) .AND. (ABS(TR10:2) .GT. 1.65))
   GEN1 CHECK=10
   IF(CHECK .EQ. 10)
   GEN1 FKH#=$F
```

```
?OLS DY Y1 DY9 DY8 DY7 DY6 DY5 DY4 DY3 DY2 DY1 SIN COS TIME/TRATIO=TR9
?TEST
?TEST SIN=0
?TEST COS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR9:2) .GT. 1.65))
GEN1 CHECK=9
IF(CHECK .EQ. 9)
GEN1 FKH#=$F
?OLS DY Y1 DY8 DY7 DY6 DY5 DY4 DY3 DY2 DY1 SIN COS TIME/TRATIO=TR8
?TEST
?TEST SIN=0
?TEST COS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR8:2) .GT. 1.65))
GEN1 CHECK=8
IF(CHECK .EQ. 8)
GEN1 FKH#=$F
?OLS DY Y1 DY7 DY6 DY5 DY4 DY3 DY2 DY1 SIN COS TIME/TRATIO=TR7
?TEST
?TEST SIN=0
?TEST COS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR7:2) .GT. 1.65))
GEN1 CHECK=7
IF(CHECK .EQ. 7)
GEN1 FKH#=$F
?OLS DY Y1 DY6 DY5 DY4 DY3 DY2 DY1 SIN COS TIME/TRATIO=TR6
?TEST
?TEST SIN=0
?TEST COS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR6:2) .GT. 1.65))
GEN1 CHECK=6
IF(CHECK .EQ. 6)
GEN1 FKH#=$F
?OLS DY Y1 DY5 DY4 DY3 DY2 DY1 SIN COS TIME/TRATIO=TR5
?TEST
?TEST SIN=0
?TEST COS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR5:2) .GT. 1.65))
GEN1 CHECK=5
IF(CHECK .EQ. 5)
GEN1 FKH#=$F
?OLS DY Y1 DY4 DY3 DY2 DY1 SIN COS TIME/TRATIO=TR4
?TEST
?TEST SIN=0
?TEST COS=0
?END
IF((CHECK .EQ. 99) .AND. (ABS(TR4:2) .GT. 1.65))
GEN1 CHECK=4
IF(CHECK .EQ. 4)
GEN1 FKH#=$F
```

```
    ?OLS DY Y1 DY3 DY2 DY1 SIN COS TIME/TRATIO=TR3
    ?TEST
    ?TEST SIN=0
    ?TEST COS=0
    ?END
    IF((CHECK .EQ. 99) .AND. (ABS(TR3:2) .GT. 1.65))
    GEN1 CHECK=3
    IF(CHECK .EQ. 3)
    GEN1 FKH#=$F
    ?OLS DY Y1 DY2 DY1 SIN COS TIME/TRATIO=TR2
    ?TEST
    ?TEST SIN=0
    ?TEST COS=0
    ?END
    IF((CHECK .EQ. 99) .AND. (ABS(TR2:2) .GT. 1.65))
    GEN1 CHECK=2
    IF(CHECK .EQ. 2)
    GEN1 FKH#=$F
    ?OLS DY Y1 DY1 SIN COS TIME/TRATIO=TR1
    ?TEST
    ?TEST SIN=0
    ?TEST COS=0
    ?END
    IF((CHECK .EQ. 99) .AND. (ABS(TR1:2) .GT. 1.65))
    GEN1 CHECK=1
    IF(CHECK .EQ. 1)
    GEN1 FKH#=$F
    ?OLS DY Y1 SIN COS TIME/
    ?TEST
    ?TEST SIN=0
    ?TEST COS=0
    ?END
    IF(CHECK .EQ. 99)
    GEN1 FKH#=$F
    ?ENDO
SAMPLE 1 10000
DIM FKHAT 10000
    ?DO # = 1,10000
    GENR FKHAT:#=FKH#
    ?ENDO
WRITE(C:\SHAZAM\output1.txt) FKHAT FKHAT
DELETE/ALL
COMPRESS
STOP
```