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Number of bidders and the winner's curse

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Number of bidders and the winner's curse

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Abstract

Within an affiliated value auction setting, we study the relationship between the number of bidders and the winner's curse in terms of its occurrence and its expected harm. From a design perspective, we find that both the number of bidders and the level of affiliation are instrumental when choosing an auction format and whether to encourage or discourage bidder participation.

JEL classification codes: D44, D82, H57.

Keywords: Winner's curse; number of bidders; affiliated value auctions.

1 Introduction

Popular intuition suggests that more bidders participating in a common-value auction should increase the probability of the adverse selection problem referred to as "winner's curse", the possibility that the winner pays more than the value (cf. Thaler, 1988; Krishna, 2001), since this increases the winner's overestimating problem and induces more aggressive bidding. However, due to the imperfect information in common-value auctions, one also deliberately tries not to overbid, especially with fiercer competition. Therefore, the number of active bidders is an instrumental variable in the auction design, one that significantly affects the outcome. It is important that policy-makers understand this when they implicitly restrict the number of bidders in an auction, for instance by selling a product that only a couple of well-known incumbents can afford, or in the opposite case when auction entry is highly encouraged. The results in this paper suggest that under certain conditions both can be justified.

The relationship between the winner's curse and the number of bidders in common-value auctions is not explored in great detail in the literature. Investigating real life auction data,

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Capen et al. (1971) suggest that bidders should have shaded their bids much more in first-price common-value auctions with more participants. Experimental literature supports the claim that the winner's curse increases with the number of bidders and the profits for the winners decrease (cf. Kagel and Levin, 1986; Kagel and Dyer, 1988; Dyer et al., 1989). Kagel et al. (1989) suggest that even experienced bidders cannot significantly mitigate the probability of being struck by the winner's curse.

In this paper, the effects of changing the number of active bidders in sealed-bid affiliated value auctions are examined by means of numerical simulations in a simplified setting with symmetric linear bidding strategies. The number of bidders is shown to be a key factor affecting not only the probability of occurrence of the winner's curse but also the relative losses of the winner and hence it can determine the most desirable auction format.

2 Model and equilibrium

There are n risk neutral active bidders, each of whom independently receives a private signal x_i drawn from a uniform distribution on the unit interval, i.e. $x_i \sim U[0;1]$. We assume that the value of the item to each bidder i is a convex combination of his private signal and the average over all bidders' signals:

$$v_i = (1-\alpha)\,x_i + \alpha\,v^c$$

with $\alpha \in [0,1]$ and where

$$v^c = \frac{1}{n}\sum_{j=1}^{n} x_j$$

is the component of the value that is common to all bidders. Hence, one of the extremes (at $\alpha = 0$) implies the independent private valuations model (IPV), while the other extreme (at $\alpha = 1$) determines a pure common value model (CV).

The unique symmetric linear equilibrium bidding functions for the first- and the second-price sealed-bid auctions in this affiliated value setup are given in Klemperer (1999; p.258-259):

(i) The unique symmetric linear equilibrium bidding function for the first-price sealed-bid affiliated value auction is

$$b_i^1(x_i) = \tfrac{n-1}{n}\,(1 - \tfrac{n-2}{2\,n}\,\alpha)\,x_i. \tag{1}$$

(ii) The unique symmetric linear equilibrium bidding function for the second-price sealed-bid affiliated value auction is

$$b_i^2(x_i) = (1 - \tfrac{n-2}{2\,n}\,\alpha)\,x_i. \tag{2}$$

In both auction formats, relative to the pure IPV model, bidders shade their equilibrium bid by $\frac{n-2}{2n}\,\alpha\,x_i$, which increases in both the level of affiliation α and the number of bidders n, in order to accommodate the probability of the winner's curse.

Indeed, a posteriori, after having won the auction but not being informed about other bidders' bids (or signals), and only inferring from winning that others have received a signal lower than his signal x_w, the winning bidder still expects a positive payoff of

$$\tilde{\pi}_w = \frac{(1-\alpha)\,n+\alpha}{n^2}\,x_w$$

in both auction formats.[1] Nevertheless, the winning bidder may still be affected by the winner's curse in the sense of ending up with a negative ex-post payoff, after having learned the other bidders' bids and with that his true valuation. It is the likelihood of such events that is the focus of the next section.

3 Impact of the number of bidders on the winner's curse

The probability that the winner is cursed in the auction equals the probability that the highest (second-highest) bid, which is the price being paid for the object, is above the winner's valuation. Deriving this probability boils down to computing the volume of a polytope, that is described by a union of systems of inequalities, based on the bidding functions in Equation (1) or (2). It is possible to estimate this probability for the first- and second-price auction for low number of bidders, when the dimensions of the polytope are low. However, computing the volume of a polytope is "#P-hard ..., and a simple task only for low dimensions." (Peña et al., 2016). For this reason we resort to simulations to numerically estimate it.

Figure 1 shows the probability for the winner being cursed for various numbers of bidders $n \in \{2, \ldots, 12\}$ and weights $\alpha \in \{0.25, 0.50, 0.75, 1.00\}$ for the first- and second-price auction formats.[2] For both auction formats the incidence of the winner's curse is increasing in both the number of bidders n and the level of affiliation α.

Beyond the bid shading in order to account for the winner's curse, which was discussed in the previous section, the standard bid shading (by factor $\frac{n-1}{n}$) further suppresses this phenomenon in the first-price auction, while having to pay a price equal to the second-highest bid has an analogous impact in the second-price auction setting. Which effect dominates depends on both the level of affiliation α and the number of bidders n.

[1] In fact, in the second-price auction, even after being informed about the price p and inferring the second-highest bidder's signal $x_p \leq x_w$, the winner has a positive payoff expectation of $\tilde{\pi}^2_{w,p} = \frac{(1-\alpha)\,n+\alpha}{n}\,(x_w - x_p)$.

[2] The probabilities reported are, for all combinations of values for n and α considered, based on 10^7 random draws for the bidders' signals.

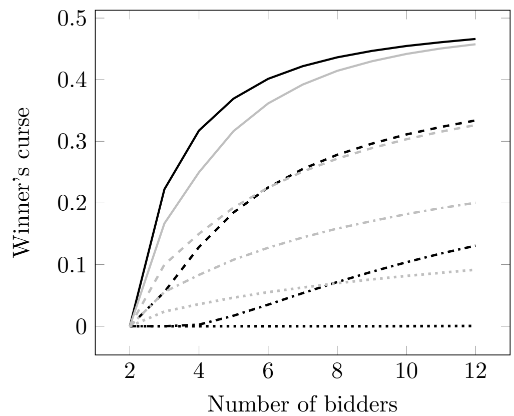

Figure 1: Incidence of the winner's curse as a function of the number of bidders for the first-price auction (black) and the second-price auction (gray) for various levels of α (dotted: $\alpha = 0.25$; dashdotted: $\alpha = 0.50$; dashed: $\alpha = 0.75$; solid $\alpha = 1.00$).

We find that for low levels of α the winner's curse is more severe in the second-price auction than in the first-price auction for any n. For values of α around 0.75, the winner's curse is more severe in the second-price auction for low values of n while it is more severe in the first-price auction for high values of n. For high values of α, the winner's curse is more severe in the first-price auction than in the second-price auction for any n.

While Figure 1 focused on the winner's curse occurring or not, Figure 2 highlights the size of the potential loss conditional on there being a loss. The expected conditional losses follow

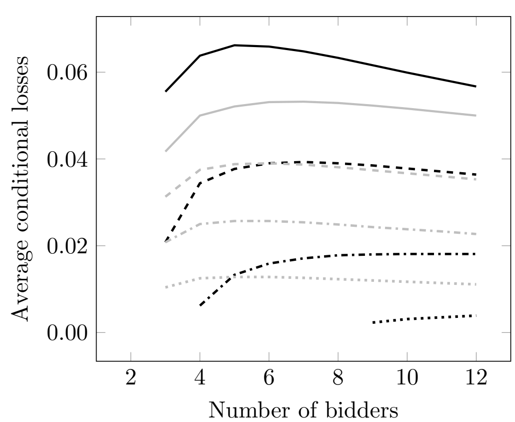

Figure 2: Average loss in case of occurrence of a winner's curse as a function of the number of bidders for the first-price auction (black) and the second-price auction (gray) for various levels of α (dotted: $\alpha = 0.25$; dashdotted: $\alpha = 0.50$; dashed: $\alpha = 0.75$; solid $\alpha = 1.00$).

the same ranking over the two auction formats as found for the incidence of the winner's curse, with the format being inferior in terms of the winner's curse incidence also being inferior in terms of expected losses conditional on there being a loss.

For both auction formats, given any number of bidders n, the expected conditional losses are increasing in the level of affiliation α. More surprisingly, for both auction formats, given a level of affiliation α, expected conditional losses increase up to a certain number of bidders n, while beyond this number they decrease.

4 Conclusion

From a design perspective, the first takeaway from this investigation is that in order to mitigate either the incidence of the winner's curse or the expected loss in cases with winner's curse, the second-price auction is preferred only in situations where the individual valuations for the object being auctioned are highly affiliated; for low levels of affiliation, the first-price auction is preferred. Interestingly, there is an intermediate range of levels of affiliation where the preferred auction format depends on the number of bidders, with the first-price auction performing better only when the number of bidders is not too high.[3]

The second takeaway concerns stimulating participation in the auction. For both auction formats and all levels of affiliation, the incidence of the winner's curse is increasing, while the expected loss in cases with winner's curse is inverse U-shaped in the number of bidders. This implies that beyond a certain threshold for the number of bidders, the decision to encourage further participation is a trade-off between increasing the likelihood of an ex post loss and decreasing the expected ex post losses. Up to this threshold, a further decrease in the number of bidders is in favour of the (remaining) bidders in the sense that their potential losses decrease in two ways: both the loss in case of a winner's curse and the likelihood of a winner's curse decrease. Here, implicitly or explicitly stimulating a higher number of active bidders is equivalent to exposing the winners to much higher risk of serious harm, which should be a warning sign for auction designers. For example, if a government does not want to scare off potential investors with a record of bankruptcies in a specific industry or increase its bailout budget, this effect must be taken into consideration.

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[3]Note that the model meets all conditions for revenue equivalence enumerated in Proposition 3.1 of Krishna (2002) and hence both auction formats generate equal expected revenues.

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