Leadership, Horizontal Integration, Bundling, and Market Performance in Price-Setting Markets: Integer Effects Matter*

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Abstract

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We examine market performance in a market with linear demand for two classes of differentiated products. One product is produced by a single firm, which may or may not act as a Stackelberg price leader. One class of products, differentiated from each other and from the singleton product, is produced by an endogenous number of firms. We contrast simultaneous price setting and Stackelberg price leadership, and consider three leadership scenarios. In the base case, the leader produces one variety, while followers each produce one variety of a class of products that are better substitutes for each other than for the leader's variety. In the second scenario, the leader is horizontally integrated into the class of products produced by followers. In the third scenario, the leader is horizontally integrated and bundles its two varieties.

JEL categories:D43, L13, L41.

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Contents

1 Introduction

We examine the determinants of market performance in a market with linear demand for two classes of differentiated products. One product (variety A) is produced by a single firm (firm A). Each variety in a second class of products (the B varieties), differentiated from each other and from variety A, is produced by one of an endogenous number of firms (the B firms). We compare outcomes if firm A acts as a Stackelberg price leader with outcomes if firm A sets price simultaneously with other firms, which for simplicity we refer to as a Bertrand market. We consider two alternative firm structures, and two types of conduct, of firm A. In the base case, we contrast simultaneous price setting with Stackelberg leadership by firm A if there are an endogenously-determined number $n + 1$ of B firms. In the second case, firm A is horizontally integrated into the B group, and produces one B variety along with variety A. In the third case, firm A is once again horizontally integrated into B group and in addition bundles its two products.

The endogenous number of follower firms is determined by market size, entry cost,[1] and the structure (horizontally integrated or not) and conduct (price leader or not) of the firm that produces variety A. A general result is that if fixed cost is small relative to market size, so that the number of follower firms is large, differences in market performance due to differences in leader structure and conduct are small. Product differentiation implies that markets with the demand and cost structure we examine are not, and not even approximately, perfectly competitive. But conditional on the existence of product differentiation, if entry is easy, consumer surplus and net social welfare are much the same whether firms set prices or quantities, whether firms set choice variables simultaneously or sequentially, and whether firm A is or is not integrated into group B.

Results are otherwise if fixed cost are large, so that the number of B firms is small.[2] There are some ranges of fixed cost for which entry deterrence is not a factor, in the sense that the equilibrium number of B firms is invariant to small changes in the Stackelberg leader's decision variable. In such cases, Stackelberg leadership is profitable for the leader, and reduces both consumer surplus and net social welfare. For other ranges of fixed cost, the Stackelberg leader will lower price to deter entry. There are ranges of fixed cost for which such entry deterrence maximizes consumer surplus and net social welfare. But in the bulk of the cases we examine, either consumer surplus, or net social welfare, or both, would be greater with simultaneous price setting than with Stackelberg leadership.

Qualitatively similar, but stronger, relationships appear if the Stackelberg leader is horizontally integrated into the B group and sells its products as a bundle.

[1] See Stigler (1968), Sutton (1991).

[2] We present numerical results for an endogenous number 0 to 3 or 4 B firms. Partly this is a practical matter: equilibrium conditions cannot be solved analytically. But it coincides with the intuition that whatever policy issues are raised by leadership, horizontal integration, and bundling are likely to manifest themselves when the equilibrium number of firms is small.

2 Related literature

Our analysis of market performance bears some relation to two streams of literature in industrial economics, that on bundling and foreclosure and that on Stackelberg leadership and market performance with free entry.

The literature on bundling is characterized by contrasting opinions on its possibilities to foreclose markets. According to the leverage theory (a.o., Kaysen and Turner, 1959), a monopolist can use bundling to leverage its monopoly power from the primary (monopoly) market in a secondary market, in which it faces *ex ante* competition, to foreclose and hence monopolize that secondary market, thereby reducing social welfare. The leverage theory has been criticized by the Chicago School.[3] According to its "single monopoly profit theorem," only one monopoly profit can be extracted from a market: if the secondary market is competitive,[4] the monopolist in the primary market cannot increase its profits in the secondary market by bundling its two products. Hence, if bundling occurs, it must have other, efficiency, motivations, such as economies of scale, price discrimination, or risk sharing. The Chicago view fails if the target market is imperfectly competitive.[5] Whinston (1990) argues that, with price competition, bundling is only profitable when it deters entry or induces a firm to exit the market and bundling is thus a mechanism to foreclose the secondary market.

Second, in his wide-ranging work on the implications of leadership for market performance, Etro (2007, 2008, and elsewhere) highlights that standard results of oligopoly models often hinge on the assumption that market structure is exogenous. Endogenizing market structure can reverse familiar findings.

> Elaborate on Etro's contribution. Discuss how our results relate to each of these literatures.

3 Model setup

3.1 Demand

We work with the linear demand product differentiation specification of Spence (1976), Dixit (1979), and Vives (1984). With one firm producing variety A and $n+1$ firms each producing one group B variety, inverse demand equations are

$$p_A = a - \left(q_A + s \sum_{i=0}^{n} q_{Bi} \right) \tag{1}$$

for variety A and

$$p_{Bi} = a - \left(sq_A + q_{Bi} + \sigma \sum_{j=i}^{n} q_{Bj} \right) \tag{2}$$

for variety Bi.

[3] See for example Director and Levi (1956), Bowman (1957), Posner (1976) and Bork (1978).

[4] And the "good approximation" assumption of the Chicago School is that most industries, most of the time, can be treated as if they are perfectly competitive (Reder, 1982).

[5] See Martin (1999), Choi and Stefanidis (2001), Carlton and Waldman (2002), Nalebuff (2004).

s measures differentiation between variety A and the B group of products. σ measures differentiation within the B group. We assume

$$0 < s < \sigma, \tag{3}$$

which implies that all varieties are substitutes and that the B varieties are closer substitutes one for another than for variety A.[6] For simplicity, we refer to the firm that produces variety A as firm A and to firms (other than firm A, if firm A is horizontally integrated into the B group) that produce one of the B varieties as B firms.

As is well known, inverse demand equations of the form (1), (2) can be derived from a quadratic aggregate welfare function,[7]

$$W = a\left(q_{A1} + \sum_{j=0}^{n} q_{Bj}\right)$$

$$-\frac{1}{2}\left[q_{A1}^2 + \sum_{j=0}^{n} q_{Bj}^2 + 2sq_{A1}\sum_{j=0}^{n} q_{Bj} + 2\sigma\sum_{j=0}^{n} q_{Bj}\sum_{k=j+1}^{n} q_{Bk}\right]. \tag{4}$$

We will use (4) for welfare calculations and to derive the inverse demand equations that apply if, when firm A produces variety A and variety B0, it bundles the two varieties.

We examine the case of price-setting firms. Inverse demand equations of the form (1), (2) imply demand equations[8]

$$\left[1-\sigma+(n+1)\left(\sigma-s^2\right)\right]q_A =$$

$$\left[1-\sigma+(n+1)(\sigma-s)\right]a-(1+n\sigma)p_A+s\sum_{0}^{n}p_{Bi} \tag{5}$$

$$(1-\sigma)\left[1-\sigma+(n+1)\left(\sigma-s^2\right)\right]q_{Bi} = (1-\sigma)(1-s)a$$

$$+(1-\sigma)sp_A-\left[1-\sigma+(n+1)\left(\sigma-s^2\right)-\left(\sigma-s^2\right)\right]p_{Bi}+\left(\sigma-s^2\right)\sum_{j\neq i}^{n}p_{Bj}. \tag{6}$$

If all B varieties set the same price, (6) simplifies to

$$q_B = \frac{(1-s)a+sp_A-p_B}{1-\sigma+(n+1)\left(\sigma-s^2\right)}. \tag{7}$$

[6] Much of the formal modelling goes through for $s < 0 < \sigma$, making variety A and the B varieties complementary, provided $0 < |s| < \sigma$. We limit attention to the $s > 0$ case for ease of interpretation.

[7] The linear demand product differentiation model can also be derived from a discrete choice model in which individuals' reservation prices for one variety depend on the prices of other varieties on the market; see Martin (2009).

[8] An outline of derivations is given in the Appendix. Full details of derivations are available on request from the authors.

3.1.1 One B firm

A special case, to which we will refer from time to time below, deserves particular mention: If there is one B firm ($n = 0$), σ drops out of the demand equations, which become

$$\left(1 - s^2\right) q_A = (1 - s)\, a - p_A + s p_{B0} \tag{8}$$

$$\left(1 - s^2\right) q_{B0} = (1 - s)\, a + s p_A - p_{B0} \tag{9}$$

These are demand equations for a symmetric duopoly with linear aggregate demands and product differentiation parameter s.

3.2 Supply

We assume that all varieties (A and B) are produced with identical and constant marginal cost, which (without loss of generality) we normalize to be zero. We assume each B variety is produced by one firm. Each active B firm incurs a sunk entry cost F. The number of B firms/varieties satisfies a free-entry condition: B firms enter until entry of one additional B firms would imply losses for all B firms. If firm A is horizontally integrated into the B group, it too incurs sunk cost F.[9]

4 Base case: n continuous

We begin by examining Bertrand (simultaneous price setting) and Stackelberg price leadership cases, treating n as a continuous variable.

4.1 Simultaneous price setting

4.1.1 Best responses

Firm A's objective function is

$$\pi_A = p_A q_A, \tag{10}$$

where q_A is given by (5).

The first-order condition to maximize π_A with respect to p_A can be written as

$$2\,(1 + n\sigma)\, p_A - (n + 1)\, s p_B = [1 - \sigma + (n + 1)\,(\sigma - s)]\, a \tag{11}$$

Firm B0's objective function is

$$\pi_{B0} = p_{B0} q_{B0} - F, \tag{12}$$

where q_{B0} is given by (6) for $i = 0$.

The first-order condition to maximize π_{B0} can be written

$$(1 - \sigma)\,(1 - s)\, a + (1 - \sigma)\, s p_A - 2\left[1 - \sigma + n\left(\sigma - s^2\right)\right] p_{B0} + \left(\sigma - s^2\right) \sum_{j=1}^{n} p_{Bj} \equiv 0, \tag{13}$$

[9] Since we simply assume that firm A is the unique producer of variety A, we can neglect fixed cost of producing variety A without affecting our results.

or (making all B variety prices the same and collecting terms)[10]

$$-(1-\sigma)\,sp_A + \left[2(1-\sigma) + n\left(\sigma - s^2\right)\right] p_B = (1-\sigma)(1-s)\,a \tag{14}$$

Solving (14) for p_B gives the B-firm best response price,

$$p_B = (1-\sigma)\frac{(1-s)\,a + sp_A}{2(1-\sigma) + n(\sigma - s^2)} \tag{15}$$

4.1.2 Equilibrium prices, payoffs, and number of B firms

Solving the first-order equations (11) and (14) gives Bertrand equilibrium prices with n+1 B firms,

$$p_A^{Bert} = \frac{\left[2(1-\sigma) + n\left(\sigma - s^2\right)\right]\left[1-\sigma+(n+1)(\sigma-s)\right] + (n+1)(1-\sigma)\,s(1-s)}{2(1+n\sigma)\left[2(1-\sigma)+n(\sigma-s^2)\right] - (n+1)(1-\sigma)\,s^2}a. \tag{16}$$

$$p_B^{Bert} = (1-\sigma)\frac{s\left[1-\sigma+(n+1)(\sigma-s)\right] + 2(1+n\sigma)(1-s)}{2(1+n\sigma)\left[2(1-\sigma)+n(\sigma-s^2)\right] - (n+1)(1-\sigma)\,s^2}a. \tag{17}$$

Equilibrium payoffs are

$$\pi_A^{Bert} = \frac{1+n\sigma}{1-\sigma+(n+1)(\sigma-s^2)}\left(p_A^{Bert}\right)^2 \tag{18}$$

and

$$\pi_B^{Bert} = \frac{1}{1-\sigma}\frac{1-\sigma+n\left(\sigma-s^2\right)}{1-\sigma+(n+1)(\sigma-s^2)}\left(p_B^{Bert}\right)^2 - F. \tag{19}$$

Substituting for p_B^{Bert}, if n is treated as a continuous variable and entry occurs until profit per B firm equals zero, the condition that determines the equilibrium number of B varieties is

$$\pi_B^{Bert} = \frac{(1-\sigma)\left[1-\sigma+n\left(\sigma-s^2\right)\right]}{1-\sigma+(n+1)(\sigma-s^2)}\times$$

$$\left[\frac{s\left[1-\sigma+(n+1)(\sigma-s)\right]+2(1+n\sigma)(1-s)}{2(1+n\sigma)\left[2(1-\sigma)+n(\sigma-s^2)\right]-(n+1)(1-\sigma)\,s^2}a\right]^2 - F \equiv 0. \tag{20}$$

(20) is a fifth-degree equation in n.

4.1.3 F_{01}^{B}: first entry of a B firm if prices are set simultaneously

Let $n = 0$, $n+1 = 1$, so there is one B firm. As noted above (Section 3.1.1), this makes the market a symmetric price-setting duopoly. Equilibrium prices are

$$p_A^{Bert}(1) = p_B^{Bert}(1) = \frac{1-s}{2-s}a. \tag{21}$$

From (19), the equilibrium profit of one B firm is

$$\pi_B^{Bert}(1) = \frac{1-s}{(1+s)(2-s)^2}a^2 - F. \tag{22}$$

[10] This is without loss of generality, since (in view of the underlying symmetry of the model) all B firms set the same price in equilibrium.

F	$n+1$	p_A^{Bert}	p_B^{Bert}	π_A^{Bert}	CS^{Bert}	NSW^{Bert}
91429	1	130.83	64.767	10284.	3.6571	3.7599
80000	1.0867	128.58	61.125	9189.2	3.8730	3.9649
70000	1.1703	126.61	57.978	8290.9	4.0407	4.1236
60000	1.2686	124.5	54.664	7393.2	4.2010	4.2749
50000	1.3905	122.16	51.042	6468.6	4.3597	4.4244
40000	1.5519	119.45	46.920	5488.9	4.5225	4.5774
30000	1.7843	116.16	42.024	4426.4	4.695	4.7393
20000	2.1696	111.86	35.817	3238.1	4.8873	4.9197
10000	3.0323	105.41	26.897	1846.7	5.1231	5.1416
5000	4.2478	100.09	19.899	1020.6	5.2822	5.2924
1000	9.3689	91.659	9.4808	235.53	5.4871	5.4895
500	13.205	89.382	6.8082	122.03	5.5347	5.5359
100	29.391	86.142	3.1089	25.619	5.5976	5.5979

Table 1: Equilibrium characteristics, different values of fixed cost, simultaneous price setting, n treated as a continuous variable. Values for CS^{Bert} and NSW^{Bert} are 10^5 times the reported values. ($a = 1000$, $\sigma = 9/10$, $s = 3/4$).

Thus one B firm would just break even, if prices are set simultaneously, for fixed cost

$$F_{01}^{B} = \frac{1-s}{(1+s)(2-s)^2} a^2. \tag{23}$$

4.1.4 Numerical results

In the absence of an analytical solution for the equilibrium number of firms, we solve (20)) numerically for specific parameter values[11] and different values of F. With numerical values for the number of B firms $(n+1)$, it is straightforward to obtain results for equilibrium characteristics of interest. Representative results are shown in Table 1.

As F falls from F_{01}^{B}, the equilibrium number of B firms rises. The higher the number of B firms, the lower is the equilibrium price of the B varieties. With the upward-sloping best-response equations that are characteristic of price-setting oligopoly with linear demand and constant marginal cost (see (11), (15)), the lower is p_B^{Bert}, the lower is p_A^{Bert}.

As F falls, q_A^{Bert} also falls.[12] Lower p_A^{Bert} tends to increase the quantity demanded of firm A, but this effect is dominated by the inward movement of A's residual demand as the number of B varieties increases. Since p_A^{Bert} and q_A^{Bert} both fall as F falls, their product, π_A^{Bert}, necessarily falls as F falls as well.

As F falls, prices and outputs (per variety, for the B firms) fall, and the number of B varieties increases. The net impact on consumer surplus as F falls

[11] The parameter values used throughout the paper are $a = 1000$, $\sigma = 9/10$, $s = 3/4$. Maple programs that produce numerical results for general parameter values are available on request from the authors.

[12] This is not shown in Table 1, but firm A's first-order condition implies that in equilibrium q_A^{Bert} is positively proportional to p_A^{Bert}.

is, in principle, ambiguous. Direct evaluation from (4)[13] verifies the intuitive result, that consumer surplus rises as F falls.

Since firm A's profit falls as F falls, while n is determined so that B-firm profit is zero, and consumer surplus rises as F falls, the overall impact on net social welfare, the sum of profits and consumer surplus, is once again in principle ambiguous. Direct evaluation again verifies the intuitive result, that net social welfare increases as F falls.

4.2 Stackelberg price leadership

If firm A acts as a Stackelberg price leader, and n is a continuous variable, it can be modelled as maximizing profit subject to two constraints, B-firm best-response pricing (15) and the free-entry constraint (set (19) equal to zero). A Lagrangian for this constrained optimization problem is

$$\max_{p_A, p_B, n} \mathcal{L} = p_A \frac{\left[1-\sigma+(n+1)(\sigma-s)\right]a-(1+n\sigma)p_A+(n+1)sp_B}{1-\sigma+(n+1)\left(\sigma-s^2\right)}$$

$$+\lambda\left[p_B-(1-\sigma)\frac{(1-s)a+sp_A}{2(1-\sigma)+n\left(\sigma-s^2\right)}\right]+\mu\left[F-\frac{1}{1-\sigma}\frac{1-\sigma+n\left(\sigma-s^2\right)}{1-\sigma+(n+1)\left(\sigma-s^2\right)}p_B^2\right]. \tag{24}$$

where λ is the Lagrangian multiplier associated with the B-firm best response equation and μ is the Lagrangian multiplier associated with the free-entry constraint.

The Kuhn-Tucker first-order conditions for the solution to (24) form a system of five equations in five unknowns (p_A, p_B, n, λ, and μ). Some details are given in the Appendix. Not surprisingly, there are no analytical results. We illustrate the properties of the solution, and compare with the Bertrand case, using numerical evaluation.

Table 2 compares the equilibrium number of B firms and prices for the Bertrand and Stackelberg regimes. For large F, the equilibrium number of B firms is small, and (as one expects) firm A's leadership price is substantially higher than its Bertrand equilibrium price. The B-firm Stackelberg follower price is also higher than the B-firm Bertrand price. With a higher p_B, the Stackelberg leadership number of B varieties must be greater, all else equal, to satisfy the B-firm zero-profit constraint. As for the Bertrand case, Stackelberg prices fall, and the Stackelberg number of B firms rises, as F falls. For low F, there is little difference between the Bertrand and Stackelberg outcomes.

Table 3 compares welfare results for the Bertrand and Stackelberg regimes. As expected, firm A's Stackelberg payoff is substantially greater than its Bertrand payoff. Bertrand consumer surplus is greater than Stackelberg leadership consumer surplus for $F = 91428.57$, otherwise less, but the amount of consumer surplus is essentially the same under either regime for all values of fixed cost. With consumer surplus essentially the same under both regimes, firm A's Stackelberg profit greater than its Bertrand profit, and B-firm profit zero in both cases, Stackelberg net social welfare is greater than Bertrand net social welfare.

[13] And subtracting the amount consumers pay for the quantities demanded.

F	$(n+1)^{Bert}$	$(n+1)^{SL}$	p_A^{Bert}	p_A^{SL}	p_B^{Bert}	p_B^{SF}
91429	1	1.0399	130.83	209.23	64.767	190.62
80000	1.0867	1.1178	128.58	195.05	61.125	165.28
70000	1.1703	1.1970	126.61	183.91	57.978	145.56
60000	1.2686	1.2923	124.5	173.39	54.664	127.26
50000	1.3905	1.4121	122.16	163.14	51.042	109.81
40000	1.5519	1.5719	119.45	152.82	46.920	92.776
30000	1.7843	1.8031	116.16	142.05	42.024	75.688
20000	2.1696	2.1877	111.86	130.19	35.817	57.860
10000	3.0323	3.0500	105.41	115.76	26.897	37.765
5000	4.2478	4.2656	100.09	106.02	19.899	25.306
1000	9.3689	9.3869	91.659	93.379	9.4808	10.560
500	13.205	13.223	89.382	90.425	6.8082	7.3481
100	29.391	29.409	86.142	86.499	3.1089	3.2169

Table 2: Equilibrium number of firms, different values of fixed cost, simultaneous price setting and Stackelberg leader continuous n approach ($a = 1000$, $\sigma = 9/10$, $s = 3/4$).

F	π_A^{Bert}	π_A^{SL}	CS^{Bert}	CS^{SL}	NSW^{Bert}	NSW^{SL}
91429	10284	87182	3.6571	3.666	3.7599	4.5378
80000	9189.2	77856	3.8730	3.8647	3.9649	4.6433
70000	8290.9	70686	4.0407	4.0276	4.1236	4.7345
60000	7393.2	64092	4.2010	4.1865	4.2749	4.8274
50000	6468.6	57851	4.3597	4.3457	4.4244	4.9242
40000	5488.9	51791	4.5225	4.5097	4.5774	5.0276
30000	4426.4	45729	4.695	4.684	4.7393	5.1413
20000	3238.1	39403	4.8873	4.8785	4.9197	5.2725
10000	1846.7	32241	5.1231	5.117	5.1416	5.4394
5000	1020.6	27763	5.2822	5.2780	5.2924	5.5556
1000	235.53	22405	5.4871	5.4853	5.4895	5.7094
500	122.03	21228	5.5347	5.5334	5.5359	5.7457
100	25.619	19708	5.5976	5.597	5.5979	5.7941

Table 3: Welfare comparisons, different values of fixed cost, simultaneous price setting and Stackelberg leadership, n treated as a continuous variable. Values for CS^{Bert} and NSW^{Bert} are 10^5 times the reported values. ($a = 1000$, $\sigma = 9/10$, $s = 3/4$).

Threshold	$F_{i,i+1}^{Bert}$	$F_{i,i+1}^{LP}$
0 to 1	91429	105337
1 to 2	23680	24513
2 to 3	10224	10417
3 to 4	5656	5728
4 to 5	3582	3616

Table 4: Threshold values for B-firm entry, Bertrand (simultaneous price setting) and Stackelberg regimes, $n+1 = 1, 2, 3, 4, 5$ ($a = 1000$, $\sigma = 9/10$, $s = 3/4$).

Once again, the two regimes differ little for low F (large n, although n is of course endogenous).

Thus, treating the number of B firms as a continuous variable, Stackelberg leadership improves market performance.

5 Base case: integer-valued n

5.1 Simultaneous price-setting

Treating n as an integer-valued variable requires one change in the simultaneous-price-setting model of Section 4.1: suppressing all parameters except n, for notational compactness, the free-entry value of $n+1$ satisfies the pair of inequalities

$$\pi_B^{Bert}(n+1) \geq 0 > \pi_B^{Bert}(n+2), \tag{25}$$

rather than the zero-profit condition (20). B-firm profit is zero only for specific values of fixed cost. Otherwise, active B firms earn some economic profit, but not so much that entry of another B firm would be profitable.

Setting n successively equal to 0, 1, 2, 3, and 4 in $\pi_B^{Bert}(n+1) = 0$ gives the threshold values at which the first, second, ..., fifth B firm could enter and just break even. We denote these entry-threshold levels of fixed cost as F_{01}^{Bert}, F_{12}^{Bert}, ..., F_{45}^{Bert}. The second column of Table 4 reports these threshold levels for our test parameter values.

5.2 Stackelberg leadership

To present the intuition behind the Stackelberg outcome, consider first a fixed cost so large that firm A can set the monopoly price without entry being profitable for a B firm.

As F falls, a level (see below) is reached at which a single B firm could enter the market and just break even, if A sets the Stackelberg leader price. We make the tie-breaking assumption that if a B firm would just break even, it does not enter.

The Stackelberg leader price maximizes firm A's profit on the residual demand curve with one B firm in the market. From (15), if there is one B firm ($n = 0$), the B price is

$$p_B = \frac{(1-\sigma)(1-s)a + (1-\sigma)sp_A}{2(1-\sigma) + (0)(\sigma - s^2)} = \frac{1}{2}\left[(1-s)a + sp_A\right]. \tag{26}$$

Substituting $n = 0$ and (26) in the demand equation (5) and rearranging terms, A's residual demand equation with one active B firm is

$$q_A = \frac{1}{2}\frac{(1-s)(2+s)a - \left(2-s^2\right)p_A}{1-s^2}. \tag{27}$$

Firm A's objective function, Stackelberg leader price, and payoff with one B-firm follower are

$$\pi_A^{SL} = \frac{1}{2}p_A\frac{(1-s)(2+s)a - \left(2-s^2\right)p_A}{1-s^2}, \tag{28}$$

$$p_A^{SL} = \frac{1}{2}\frac{(1-s)(2+s)}{2-s^2}a, \tag{29}$$

and

$$\pi_A^{SL} = \frac{1}{8}\frac{(1-s)(2+s)^2}{2-s^2}a^2, \tag{30}$$

respectively.

For p_A^{SL} given by (29), the B firm's payoff is

$$\pi_B^{SF} = \frac{1}{16}\frac{(1-s)\left(4+2s-s^2\right)^2}{(1+s)\left(2-s^2\right)^2}a^2 - F, \tag{31}$$

from which it follows that the value of F at which one B firm would just break even if firm A sets the Stackelberg leader price ignoring the possibility of entry deterrence is[14]

$$F_{01}^{LP} = \frac{1}{16}\frac{(1-s)\left(4+2s-s^2\right)^2}{(1+s)\left(2-s^2\right)^2}a^2. \tag{32}$$

But firm A need not ignore the possibility of entry deterrence. As a Stackelberg leader that takes account of the impact the price it sets has on the equilibrium number of firms, firm A can set the price that makes the B-firm's follower profit equal to zero:

$$\pi_B = \frac{1}{4}\frac{1}{1-s^2}\left[(1-s)a + sp_A\right]^2 - F = 0. \tag{33}$$

This entry-limiting price is

$$p_A^{LP1} = \frac{1}{s}\left[2\sqrt{\left(1-s^2\right)F} - (1-s)a\right]. \tag{34}$$

Under our tie-breaking assumption, if firm A sets price p_A^{LP1}, the B firm stays out of the market. Firm A, by setting a price below what one might call its "naive" (ignoring the endogeneity of entry) Stackelberg price, deters entry.[15] The cost of deterring entry is the need to reduce price below the Stackelberg

[14] For our test parameter values, $F_{01}^{LP} = 105337$ (see Table 4). Entry would occur at a higher level of fixed cost with Stackelberg than Bertrand pricing because (in the absence of entry-deterrence), the Stackelberg price is greater than the Bertrand price, all else equal.

[15] One can think of this pricing strategy as a version of limit pricing (Marshall [1890] 1920; Sylos-Labini 1957; Modigliani, 1958), where the force of potential competition induces an incumbent to depart from what would otherwise be its profit-maximizing strategy.

level.[16] The benefit of deterring entry is that demand is that the quantity demanded at this lower price is determined by the monopoly demand,

$$q_A = a - p_A, \tag{35}$$

not the residual demand if there is one active B firm, (27).

For a range of fixed cost immediately below F_{01}^{LP}, A's most profitable option is to set the entry-deterring price p_A^{LP1}. As F falls, p_A^{LP1} falls as well: firm A must set a lower price to make entry unprofitable. By setting a lower price, the incumbent reduces its entry-deterring payoff. For a sufficiently low level of fixed cost, the incumbent would make the same profit setting the price that makes the equilibrium profit of one B-firm Stackelberg follower equal to zero that it would make as a Stackelberg leader with one B-firm follower in the market.

For a range of fixed cost beginning with this threshold value, firm A and its B-firm follower interact as in the standard model of price leadership with differentiated products. Modest reductions in fixed cost increase the B firm's profit, but not so much as to induce entry by a second B firm.

If F falls sufficiently, a level is reached below which a second B firm could profitably enter, if firm B sets the Stackelberg leadership price for two followers. Once again, firm A can deter entry of the second B firm by setting a price that would make B-firm post-entry profit zero. Firm A's profit is less than it would be with one follower, but greater than it would be as a Stackelberg leader with two followers.[17]

As F falls further, A's entry-deterrence profit falls. Eventually a level of F is reached below which firm A earns a greater profit with two followers in the market than it would earn keeping the second B firm out. Below this second entry threshold, firm A maximizes profit with two B firms in the market.

As F falls further, the cycle repeats itself: Stackelberg leadership with a given number of followers, entry deterrence, entry of an additional follower.

Values of fixed cost $F_{i,i+1}^{LP}$ at which firm A finds it profitable to price to deter entry or additional entry are given in the third column of Table 4. Interleaved between these values are the values of fixed cost at which firm A finds it profitable to give entry deterrence and act as the Stackelberg leader of a market with one additional B-firm follower (Table 5).

These thresholds describe firm A's profit-maximizing conduct, given that it acts as a Stackelberg price leader. If firm A would earn a greater profit in a simultaneous-pricing regime, it can accomplish this when it sets price by setting the Bertrand price for equilibrium number of follower firms. Follower firms, setting prices on their best-response functions, will of necessity set their Bertrand prices as well. In this sense, firm A can choose between Bertrand and Stackelberg pricing regimes.

	$(n+1)^{SL}$	*Deter?*
$105337 \leq F$	0	NO
$61747 \leq F < 105337$	0	YES
$24513 \leq F < 61747$	1	NO
$17134 \leq F < 24513$	1	YES
$10417 \leq F < 17134$	2	NO
$8436 \leq F < 10417$	2	YES
$5728 \leq F < 8436$	3	NO
$4924 \leq F < 5728$	3	YES
$3616 \leq F < 4924$	4	NO
$3214 \leq F < 3616$	4	YES
$2489 \leq F < 32148$	5	NO

Table 5: Fixed cost thresholds, Stackelberg price leader movement into and out of entry deterrence pricing.

F	$n+1$	π_A		CS		NSW	
	$Bert, SL$	$Bert$	SL	$Bert$	SL	$Bert$	SL
$105337 < F$	0	250000	250000	125000	125000	375000	375000
$24513 < F \leq 61747$	1	91429	93944	365710	342130	$548570 - F$	$541161 - F$
$10417 < F \leq 17134$	2	41962	42105	481330	477970	$570650 - 2F$	$569100 - 2F$
$5728 < F \leq 8436$	3	32484	32525	511700	510230	$574860 - 3F$	$574010 - 3F$

Table 6: Bertrand and Stackelberg comparison, small n ($a = 1000$, $\sigma = 9/10$, $s = 3/4$).

5.2.1 Welfare: entry deterrence not a factor

Table 6 shows welfare results, for our test parameter values, for ranges of fixed cost where entry deterrence is not an issue: firm A does not find it profitable to deter entry, and the endogenously-determined number of B firms is the same in either pricing regime.

For these values of F, Stackelberg leadership is profitable for firm A. Having firm A act as a Stackelberg leader is profitable for such B firms as are in the market, as one would expect with price-setting firms and profit differentiation. The Stackelberg leader sets a price above the Bertrand level, and the firm B Stackelberg follower price is greater than its Bertrand price.[18] Consumers, who pay higher prices for the same number of varieties are worse off with Stackelberg pricing. Stackelberg leadership is advantageous firm firms, disadvantageous for consumers, and on balance reduces net social welfare is less than Bertrand net social welfare.

[16] The difference in prices is

$$p_A^{SL} - p_A^{LP1} = \frac{1}{2}(1-s)\frac{4+2s-s^2}{s(2-s^2)}a - \frac{2}{s}\sqrt{(1-s^2)F},$$

which is zero for $F = F_{01}^{LP}$ and positive for $F < F_{01}^{LP}$.

[17] The one B firm that is in the market also earns greater profit if firm A deters entry of a second B firm.

[18] We omit π_B from the table for expositional compactness. Complete sets of results are available from the authors on request.

5.2.2 Pricing to deter entry

Interleaved between the ranges of fixed cost covered in Table 6 are successive pairs of ranges of fixed cost. The number of B firms may be one less in the Stackelberg market than in the Bertrand market: for a first range of F, firm A sets price to deter entry. For lower ranges of F, A gives up entry deterrence and the number of B firms is the same in both types of markets.

For our test parameter values, for $F \geq 105337$ there are no B firms in either a Bertrand or a Stackelberg market; firm A maximizes its payoff by setting the unconstrained monopoly price.

For $91429 < F \leq 105337$ there are no B firms in either a Bertrand or a Stackelberg market, and in a Stackelberg market, firm A sets a price to deter entry. The corresponding lower price makes consumer surplus greater than it would be with one Stackelberg follower on the market. Since entry deterrence leaves both firm A and consumers better off, it necessarily improves net social welfare (row 1, Table 7).

For $61747 < F \leq 91429$, there is one B firm in a Bertrand market, there are no B firms in a Stackelberg market, and in a Stackelberg market, firm A sets price to deter entry. For $24513 < F \leq 61747$ there is one B firm under either pricing regime, and in the Stackelberg regime, firm A does not set price to deter entry. We illustrate the profit-maximizing and welfare maximizing choices for regimes for $61747 < F \leq 91429$ in Figures 1, 2, and 3.

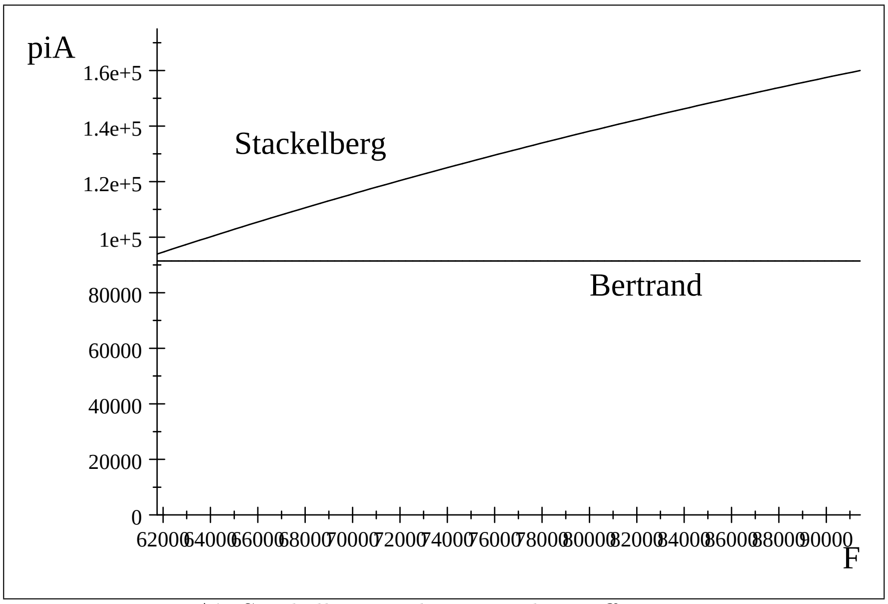

Figure 1: Firm A's Stackelberg and Bertrand payoffs, $61747 < F \leq 91429$.

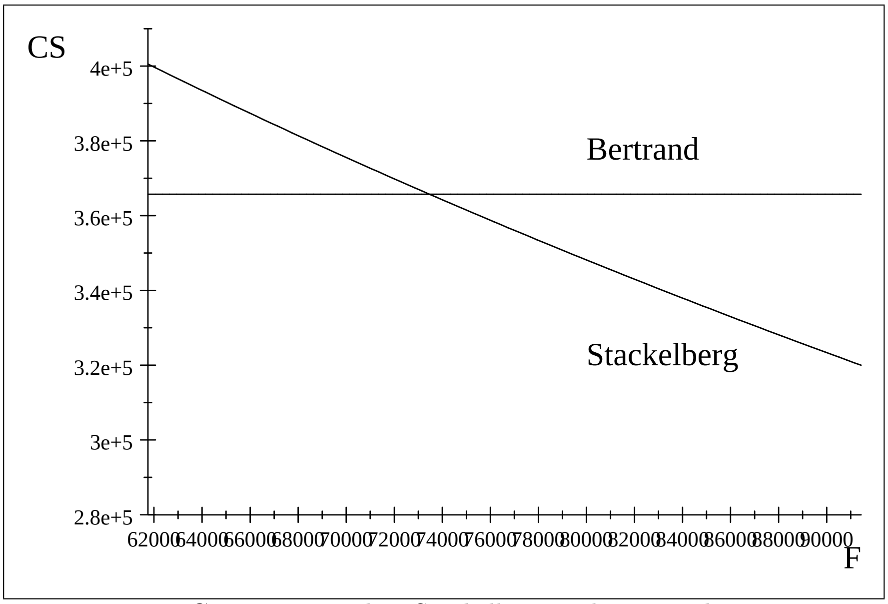

Figure 2: Consumer surplus, Stackelberg and Bertrand regimes, $61747 < F \leq 91429$.

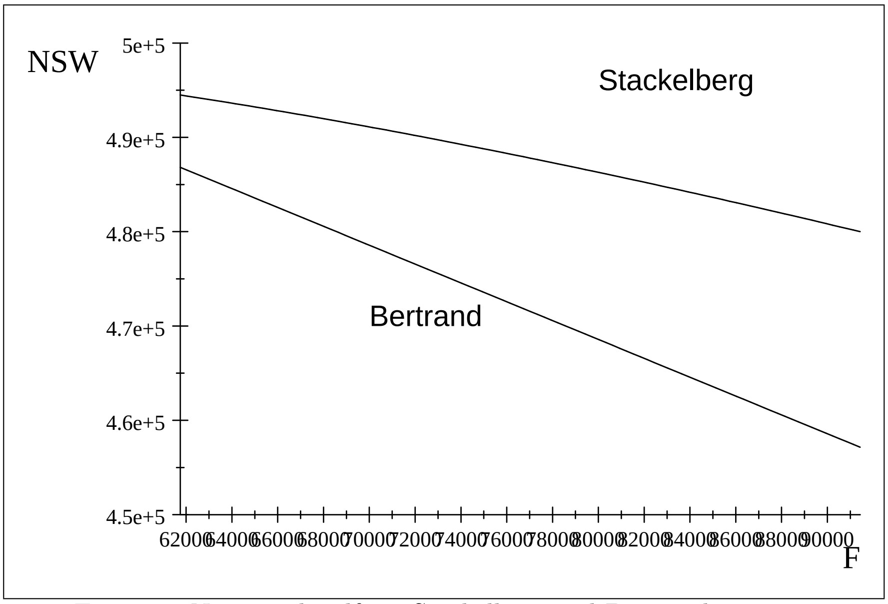

Figure 3: Net social welfare, Stackelberg and Bertrand regimes, $61747 < F \leq 91429$.

For this range of F, firm A always earns greater profit by acting as a Stackelberg leader, and this is the regime that maximizes net social welfare. Consumer surplus is higher in a Bertrand market with one B firm than in a Stackelberg entry-deterrence market with no B firms for $73473 < F \leq 91429$, and highest in the Stackelberg market for $61747 < F \leq 73473$.

As shown in Table 7, for the first two ranges of fixed cost where the Stackelberg leader prices to deter entry, its profit-maximizing price regime maximizes

	π_A	CS	NSW
$91429 < F \leq 105337$	SL	SL	SL
$73473 < F \leq 91429$	SL	B	SL
$61747 < F \leq 73473$	SL	SL	SL

Table 7: Maximizing regime, p_A, CS, and NSW, $61747 < F \leq 105337$ For these values of fixed cost, the Stackelberg leader prices to deter entry.

	π_A	CS	NSW
$23680 < F \leq 24513$	B	SL	SL
$17134 < F \leq 23680$	SL	B	B

Table 8: Maximizing regime, p_A, CS, and NSW, $17134 < F \leq 24513$. For these values of fixed cost, the Stackelberg leader prices to deter entry.

consumer welfare for one of the three ranges of fixed cost, and maximizes net social welfare for two of the three ranges of fixed cost.

For the immediately lower range of fixed cost, $24513 < F \leq 61747$, as shown in Table 6, the leader's most profitable pricing regime is never the regime that maximizes consumer surplus or net social welfare.

There follow two ranges of fixed cost where there is successively 1 or 2 B firms in the Bertrand market, 1 B firm in the Stackelberg market, and in the Stackelberg market firm A sets price to deter entry. As reported in Table 8, the Stackelberg leader's most profitable pricing regime is never the regime that would be preferred by consumers or maximize net social welfare.

5.2.3 Discussion

Our results are for markets where fixed cost is large, relative to market size, and the equilibrium number of firms small. In such markets, if price leadership does not affect the equilibrium number of follower firms, it is profitable for firms but means higher prices for a given number of varieties, lower consumer surplus, and lower net social welfare.

Otherwise, entry deterrence by a price leader may increase net social welfare, compared with simultaneous price setting. If so, it may be at the expense of consumer welfare (row 2, 7). If Stackelberg entry deterrence would improve consumer surplus and net social welfare, the Stackelberg leader may find simultaneous pricing most profitable (row 1, Table 8). If Stackelberg entry deterrence is most profitable for the leader, it may reduce consumer surplus and net social welfare (row 2, Table 8).

6 Horizontal integration

6.1 Analytics

We now contrast simultaneous price-setting (Bertrand) behavior with Stackelberg leadership by firm A, if firm A produces variety B0 as well as variety A. There are thus an endogenous number n of independent firms producing varieties of product B.

	n		CS		Π_A	
	$Bert$	SL	$Bert$	SL	$Bert$	SL
$3396.8 < F \leq 3803.1$	4		532440		$30104 - F$	
$3803.1 < F \leq 5203.4$	3		523170		$34682 - F$	
$5203.4 < F \leq 6140.2$	3		523170		$34682 - F$	
$6140.2 < F \leq 8763.4$	2		506270		$44133 - F$	
$8763.4 < F \leq 11617$	2		506270		$44133 - F$	
$11617 < F \leq 15684$	1		464780		$71614 - F$	
$15684 < F \leq 30697$	1		464780		$71614 - F$	
$30697 < F$	0		62500		$500000 - F$	

Table 9: Simultaneous price-setting and Stackelberg leadereship comparisons, horizontal integration by firm A ($a = 1000$, $\sigma = 9/10$, $s = 3/4$).

Firm A's objective function is now

$$\pi_A = p_A q_A + p_{B0} q_{B0} - F. \tag{36}$$

In equilibrium ($i.e.$, $p_{Bi} = p_B$ for $i = 1, 2, ..., n$) the first-order conditions to maximize (36) with respect to p_A and p_{B0} are

$$2(1-\sigma)(1+n\sigma)p_A - 2(1-\sigma)sp_{B0} - (1-\sigma)nsp_{Bi} = (1-\sigma)\left[1 - s + n(\sigma - s)\right]a \tag{37}$$

and

$$-2(1-\sigma)sp_A + 2\left[1 - s^2 + (n-1)\left(\sigma - s^2\right)\right]p_{B0} - \left(\sigma - s^2\right)np_B = (1-\sigma)(1-s)a, \tag{38}$$

respectively. As expected, with horizontal integration firm A internalizes cross-price effects on the quantities demanded of its varieties.

The equilibrium B-firm best-response equation is

$$p_B = \frac{(1-\sigma)(1-s)a + (1-\sigma)sp_A\left(\sigma - s^2\right)p_{B0}}{2(1-s^2) + (n-1)(\sigma - s^2)}. \tag{39}$$

(37), (38), and (39) are linear in prices, and can be solved for equilibrium prices, given n. The free-entry remains (25).

6.2 Numerical results

Tables 9 and 10 make some basic comparisons of the simultaneous price-setting and leadership regimes when firm A is integrated into production of one of the B varieties.

This section is in preparation.

	n		NSW		
	$Bert$	SL	$Bert$	SL	$SL - Bert$
$3396.8 < F \leq 3803.1$	4		$577750 - 5F$		
$3803.1 < F \leq 5203.4$	3		$576280 - 4F$		
$5203.4 < F \leq 6140.2$	3		$576280 - 4F$		
$6140.2 < F \leq 8763.4$	2		$573640 - 3F$		
$8763.4 < F \leq 11617$	2		$573640 - 3F$		
$11617 < F \leq 15684$	1		$567100 - 2F$		
$15684 < F \leq 30697$	1		$567100 - 2F$		
$30697 < F$	0		$562\,500 - F$		

Table 10: Simultaneous price-setting and Stackelberg leadereship comparisons, horizontal integration by firm A ($a = 1000$, $\sigma = 9/10$, $s = 3/4$).

7 Bundling

7.1 Analytics

When firm A is horizontally integrated in the B market, it can offer its two products as a bundle if it is profitable to do so. Here we assume firm A bundles one unit of variety A with one unit of variety B, and contrast outcomes if it acts or does not act as a Stackelberg price leader in the market for differentiated bundles.

We let x_0 denote the number of bundles sold by firm A, and x_j the quantity of the pseudo-bundle of firm Bj, $j = 1, 2, ..., n$. Firm Bj's pseudo-bundle contains zero units of variety and one unit of variety Bj. Thus

$$q_A = q_{B_0} = x_0 \tag{40}$$

and, for $j = 1, 2, ..., n$,

$$q_{B_j} = x_j. \tag{41}$$

Inserting these relations in (4) yields the aggregate welfare function if firm A bundles:

$$W^B = a\left(2x_0 + \sum_{j=1}^{n} x_j\right)$$

$$-\frac{1}{2}\left[2\left(1+s\right)x_0^2 + \sum_{j=1}^{n} x_j^2 + 2\left(s+\sigma\right)x_0\sum_{j=1}^{n} x_j + 2\sigma\sum_{j=1}^{n} x_j \sum_{k=j+1}^{n} x_k\right]. \tag{42}$$

Maximizing this utility function under appropriate budget constraints and inverting the inverse demand functions gives direct demand equations for bundles:

$$\left\{2\left[1+\left(n-1\right)\sigma\right]\left(1+s\right) - n\left(s+\sigma\right)^2\right\}x_0 =$$

$$\left[2\left(1-\sigma\right) + n\left(\sigma - s\right)\right]a - \left[1+\left(n-1\right)\sigma\right]p_0 + \left(s+\sigma\right)\sum_{j=1}^{n} p_j, \tag{43}$$

and

$$\left\{2\left[1+(n-1)\sigma\right](1+s)-n(s+\sigma)^2\right\}x_j =$$

$$2(1-\sigma)a+(s+\sigma)p_0-\frac{1}{1-\sigma}Xp_j+\frac{1}{1-\sigma}\left[\sigma-s^2+\sigma(1-\sigma)\right]\sum_{\substack{i=1\\i\neq j}}^{n}p_i, \quad (44)$$

where

$$X =$$

$$2\left[1+(n-1)\sigma\right](1+s)-n(s+\sigma)^2-\left[\left(\sigma-s^2\right)+\sigma(1-\sigma)\right]. \quad (45)$$

The payoff functions of the two types of firms are

$$\pi_A = p_0x_0 - F \quad (46)$$

and

$$\pi_{B_J} = p_jx_j - F, \quad (47)$$

respectively.

7.2 Outcomes

Given the altered demand structure of equations (43) and (44), the equilibrium outcome can be characterized in the same way as for the base case of Section 5. The Bertrand outcome is a price-setting equilibrium of a market with differentiated products. In the Stackelberg market, there are ranges of fixed cost where firm A can set the Stackelberg leader price without inducing entry. There are ranges of fixed cost where the number of B firms is the same in a Bertrand or Stackelberg market, and in the Stackelberg market firm A sets price to deter entry. There are ranges of fixed cost where there is one more B firm in the Bertrand than the Stackelberg market, and firm A sets price to deter entry. For some values of fixed cost, firm A earns a greater profit setting the Bertrand equilibrium price in a market with a larger number of B firms than it would earn setting the entry-deterring price in a market with one fewer B firms.

7.3 Numerical results

7.4 Exclusion

Table 11 simply compares the number of B firms/varieties for a given level of fixed cost under the two alternative regimes. For the indicated ranges of fixed cost, Stackelberg leadership is exclusionary in the sense that for some ranges of fixed cost, the equilibrium number of firms is less under Stackelberg leadership than under Bertrand competition (simultaneous price setting).

7.5 Entry deterrence not a factor

Exclusion, or the lack of it, is a topic that appears in the policy literature, and this may be justified to the extent that there is a one-to-one relation between number of active firms and market performance. It is market performance that is of primary interest.[19]

[19] For much of its long existence, U.S. antitrust policy objected to actions that were profitable only on the condition that they excluded equally-efficient competitors.

	n		
	Bertrand	Stackelberg	Deter?
$49785 < F$	0	0	No
$33820 < F \leq 49785$	0	0	Yes
$19452 < F \leq 33820$	1	0	Yes
$13999 < F \leq 19452$	1	1	No
$11589 < F \leq 13999$	1	1	Yes
$7199.6 < F \leq 11589$	2	1	Yes
$6226.5 < F \leq 7199.6$	2	2	No
$5698 < F \leq 6226.5$	2	2	Yes
$4018.4 < F \leq 5698$	3	2	Yes
$3674.8 < F \leq 4018.4$	3	3	No
$3368 < F \leq 3674.8$	3	3	Yes
$2562.2 < F \leq 3368$	4	3	Yes

Table 11: Number of B firms, Bertrand and Stackelberg regimes, by range of fixed cost, firm A bundles. $a = 1000$, $\sigma = 9/10$, $s = 3/4$, $n + 1 = 1, 2, 3, 4, 5$. D indicates firm A prices to deter further entry.

	n	π_A		CS		NSW	
		Bert	*SL*	*Bert*	*SL*	*Bert*	*SL*
$49785 < F$	0	$285174 - F$	$285714 - F$	142857	142857	$428571 - F$	$428\,571 - F$
$13999 < F \leq 19452$	1	$44336 - F$	$82993 - F$	456169	425719	$568130 - 2F$	$558\,501 - 2F$
$6226.5 < F \leq 7199.6$	2	$33117 - F$	$45673 - F$	505000	495470	$572890 - 3F$	$569140 - 3F$
$3674.8 < F \leq 4018.4$	3	$28223 - F$	$34288 - F$	523320	518490	$574330 - 4F$	$571460 - 4F$

Table 12: Payoffs, Bertrand and Stackelberg regimes, by range of fixed cost, firm A bundles, entry deterrence not a factor. $a = 1000$, $\sigma = 9/10$, $s = 3/4$, $n = 0, 1, 2, 3$.

	n^{Bert}	n^{SL}	*SL Deter?*	π_A	CS	NSW
$11589 < F \leq 13999$	1	1	Yes	B	SL	SL
$7199.6 < F \leq 11589$	2	1	Yes	B	SL	SL
$5698 < F \leq 6226.5$	2	2	Yes	B	SL	SL
$4018.4 < F \leq 5698$	3	2	Yes	SL	B^*	SL

Table 13: Maximizing regime, p_A, CS, and NSW, $4018.4 < F \leq 5698$. *For $4018.4 \leq F \leq 4118.4$, consumer surplus is greater under Stackelberg price leadership.

Table 12 shows results that correspond to those of Table 6 for the base case. For these ranges of fixed cost, Stackelberg leadership is more profitable for firm A than simultaneous price setting, but leaves consumers worse off and reduces net social welfare.

7.6 Pricing to deter entry

The picture for firm A, if it bundles, is somewhat different if entry deterrence is a factor. When firm A bundles and the endogenous number of B firms is small, it is generally not profitable for firm A to act as a Stackelberg leader (Table 13). Consumers would be better off, and net social welfare greatest, if firm A were a Stackelberg leader. Except for the lowest ranges of fixed cost we examine,[20] however, firm A's interests and consumer/social interests are not aligned.

8 Conclusion

Economists and policymakers should both keep in mind, not only that market structure is endogenous, but that incumbent firms are aware that market structure is endogenous, and set their own strategies accordingly. There are circumstances in which exclusionary, above marginal-cost leadership prices, because they are lower prices, improve market performance. We show that this is not a general result, and in so doing, our work joins a small set[21] of papers highlighting circumstances in which full understanding of the determinants of market performance requires examination of discrete changes in numbers of firms.

Price leadership generally worsens market performance if it does *not* involve exclusion. Where the relation between fixed cost and market means a price leader could exclude rivals, it might prefer not to act as a price leader, although consumer welfare and net social welfare would be greater if it were to do so. Where the relation between fixed cost and market means a price leader could exclude rivals, it might prefer to exclude, although this reduces consumer welfare and net social welfare.

[20] That is, for $4018.4 < F \leq 4018.4$ if one takes consumer surplus as a measure of market performance, $4018.4 < F \leq 5698$ if one looks at net social welfare. In this range of F, there are 3 B firms in the Bertrand regimes, 2 in the Stackelberg regime, and firm A sets price to deter entry.

[21] Among which one may include Selten (1973), Lambson (1987), and Martin and Valbonesi (2008).

For the specification investigated here, if entry deterrence is not a factor, a firm that bundles will find it profitable to act as a price leader, although this leaves consumers and society as a whole worse off. If a bundling firm can exclude rivals by acting as a Stackelberg leader, it may not find it profitable to do so, although leadership would benefit consumers and society as a whole. It may find it profitable to act as a price leader, although this leaves consumers worse off.

These results show that there is no general conduct-structure-market performance relationship. Leadership may improve market performance, or it may not. Bundling may improve market performance, or it may not. Leadership by a firm that bundles may improve market performance, or it may not. If fixed cost is small relative to market size, meaning that the equilibrium number of firms is large, differences in market performance under different firm structure and pricing regimes are likely to be small. If fixed cost is large relative to market size, which is when competition policy issues are most likely to arise, there is no substitute for an explicit evaluation of the impact of business practice on market performance.

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10 Appendix

10.1 Direct demand, no bundling

Write the system of inverse demand equations as

$$\begin{pmatrix} p_A \\ p_B \end{pmatrix} = a \begin{pmatrix} 1 \\ J_{n+1} \end{pmatrix} - \begin{pmatrix} 1 & sJ'_{n+1} \\ sJ_{n+1} & Z_{n+1} \end{pmatrix} \begin{pmatrix} q_A \\ q_B \end{pmatrix}, \tag{48}$$

where p_A and q_A are scalars, p_B and q_B are $n+1$-row column vectors of B-variety prices and quantities,[22] respectively, J_{n+1} is an $n+1$-row column vector of 1s and

$$Z_{n+1} = (1-\sigma)\, I_{n+1} + \sigma J_{n+1} J'_{n+1}. \tag{49}$$

The inverse of Z_{n+1} is known,

$$Z_{n+1}^{-1} = \frac{1}{1-\sigma}\left[I_{n+1} - \frac{\sigma}{1+n\sigma} J_{n+1} J'_{n+1} \right], \tag{50}$$

[22] There is some abuse of notation here, as we have used p_B in the text to denote the scalar value of the common equilibrium price of the B varieties.

and by using one of the standard formulae for the inverse of a partitioned matrix,

$$\begin{pmatrix} A & B \\ B' & C \end{pmatrix}^{-1} = \begin{pmatrix} D & -DBC^{-1} \\ -C^{-1}B'D & C^{-1} + C^{-1}B'DBC^{-1} \end{pmatrix}, \tag{51}$$

where

$$D = \left(A - BC^{-1}B'\right)^{-1}, \tag{52}$$

one obtains the demand equations (5), (6).

10.2 Monopoly

If there are no B firms and entry would be unprofitable, firm A's inverse demand equation is (35). Monopoly equilibrium characteristics are

Price:

$$p_A^m = \frac{1}{2}a. \tag{53}$$

Output:

$$q_A^m = \frac{1}{2}a. \tag{54}$$

Profit:

$$\pi_A^m = \frac{1}{4}a^2. \tag{55}$$

Net social welfare:

$$W = aq_A - \frac{1}{2}q_A^2 = a\left(\frac{1}{2}a\right) - \frac{1}{2}\left(\frac{1}{2}a\right)^2 = \frac{3}{8}a^2. \tag{56}$$

Consumer surplus:

$$CS^m = W^m - p_A q_A = \frac{3}{8}a^2 - \frac{1}{4}a^2 = \frac{1}{8}a^2. \tag{57}$$

10.3 Base case

10.3.1 Simultaneous price-setting

Firm A's price best-response equation is (11). Solving (11) and the B-firm first-order condition (**??**) gives firm A's equilibrium price,

$$p_A = \frac{\left[2(1-\sigma) + n\left(\sigma - s^2\right)\right]\left[1 - \sigma + (n+1)(\sigma - s)\right] + (n+1)(1-\sigma)s(1-s)}{2(1+n\sigma)\left[2(1-\sigma) + n\left(\sigma - s^2\right)\right] - (n+1)(1-\sigma)s^2}a, \tag{58}$$

and the B-firm equilibrium price, (**??**).

For numerical evaluation, we solve the first-order conditions and the free-entry condition $\pi_B(n+1) = 0$ for fixed values of $n+1$, obtaining the implied equilibrium prices and requisite value of F. Equilibrium quantities and welfare expressions follow from equilibrium prices and the value of F.[23]

[23] The Maple programs used to obtain numerical results are available on request from the authors.

10.4 Stackelberg leadership

If n is treated as a continuous variable, a Lagrangian for firm A's constrained optimization problem is (24). The first-order conditions to maximize (24) are

p_A:

$$\frac{[1-\sigma+(n+1)(\sigma-s)]\,a-2(1+n\sigma)\,p_A+(n+1)\,sp_B}{1-\sigma+(n+1)(\sigma-s^2)}+\lambda\frac{(1-\sigma)\,s}{2(1-\sigma)+n(\sigma-s^2)}\equiv 0. \tag{59}$$

p_B:

$$\frac{(n+1)\,sp_A}{1-\sigma+(n+1)(\sigma-s^2)}-\lambda-2\mu\frac{1}{1-\sigma}\frac{1-\sigma+n\left(\sigma-s^2\right)}{1-\sigma+(n+1)(\sigma-s^2)}p_B\equiv 0 \tag{60}$$

n:

$$-p_A\frac{s(1-\sigma)\left[(1-s)\,a-p_B+sp_A\right]}{\left[1-\sigma+(n+1)(\sigma-s^2)\right]^2}$$

$$+\lambda(1-\sigma)\frac{\left(\sigma-s^2\right)\left[(1-s)\,a+sp_A\right]}{\left[2(1-\sigma)+n(\sigma-s^2)\right]^2}-\mu\frac{1}{1-\sigma}\frac{\left(\sigma-s^2\right)^2}{\left[1-\sigma+(n+1)(\sigma-s^2)\right]^2}p_B^2=0, \tag{61}$$

along with the B-firm best-response equation and the free-entry condition. These are the coefficients of λ and μ, respectively, in (24), set equal to zero. Clearing fractions, we write these equations as

λ:

$$\left[2(1-\sigma)+n\left(\sigma-s^2\right)\right]p_B-(1-\sigma)\left[(1-s)\,a+sp_A\right]=0 \tag{62}$$

μ:

$$(1-\sigma)\,F\left[1-\sigma+(n+1)\left(\sigma-s^2\right)\right]-\left[1-\sigma+n\left(\sigma-s^2\right)\right]p_B^2=0 \tag{63}$$

Numerically solving the first-order conditions for parameter values and n gives threshold values of F at which an additional B firm could enter and just break even.