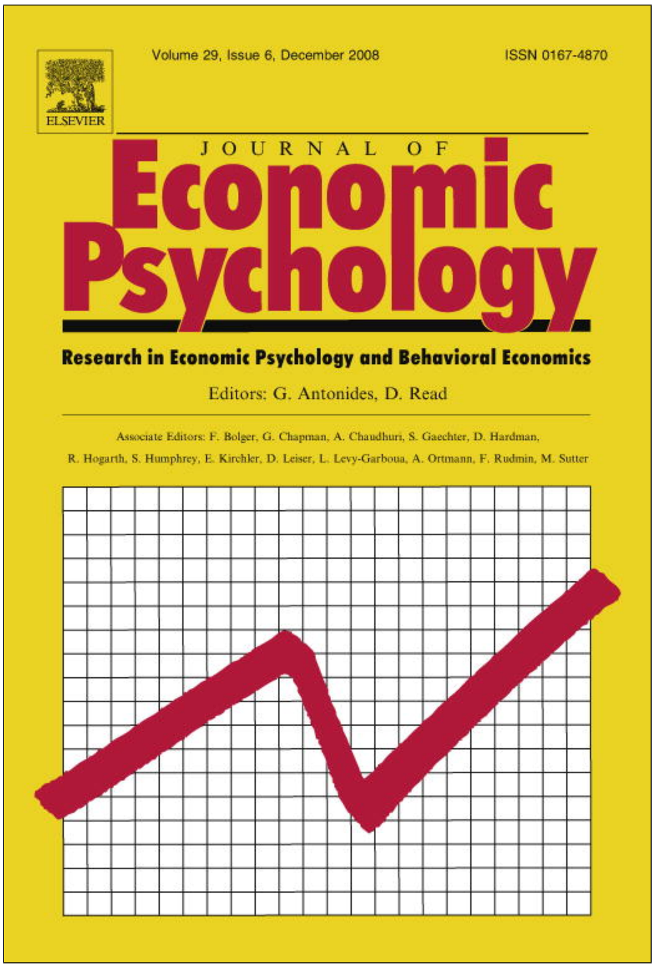
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Rational ignoring with unbounded cognitive capacity

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In canonical decision problems with standard assumptions, we demonstrate that inversely related payoffs and probabilities can produce expected-payoff-maximizing decisions that are independent of payoff-relevant information. This phenomenon of rational ignoring, where expected-payoff maximizers ignore costless and genuinely predictive information, arises because the conditioning effects of such signals disappear on average (i.e., under the expectations operator) even though they exert non-trivial effects on payoffs and probabilities considered in isolation (i.e., before integrating). Thus, rational ignoring requires no decision costs, cognitive constraints, or other forms of bounded rationality. This implies that simple decision rules relying on small subsets of the available information can, depending on the environment in which they are used, achieve high payoffs. Ignoring information is therefore rationalizable solely as a consequence of the shape of the stochastic payoff distribution.

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1. Introduction

Do environments exist in which unboundedly rational optimizers ignore payoff-relevant information? If so, can these environments be characterized in terms of visible markers found in real-word decision-making environments? This paper provides an affirmative answer to the first question and makes a new prediction that may help address the second.

The bounded rationality literature provides abundant theoretical and empirical support for the claim that information-frugal decision rules, which rely on small subsets of information available in the decision maker's environment, can come close to matching the performance of information-hungry decision rules based on constrained optimization (Baucells, Carrasco, & Hogarth, in preparation; Conlisk, 1996; Gigerenzer and Goldstein, 1996; Gigerenzer and Selten, 2001; Martignon and Hoffrage, 2002; Rodepeter and Winter, 1999; Simon, 1982). Such findings support explanations for the success of information-frugal decision rules that emphasize savings of decision time, effort in computation, and other cognitive resources in scarce supply.

There is another explanation, however – one that has been largely overlooked – that may account for why successful decision rules can rely on only small subsets of the non-redundant predictors available in the environment. This new explanation focuses on the joint structure of information and payoffs (as described by the joint distribution of stochastic payoffs and observable conditioning information) which – apart from any limitations or bounds on human cognition – is capable of rewarding decisions that ignore objectively predictive information.[1] The focus of this paper is on signals that have zero, or near-zero, value even though they are non-trivially correlated with future events affecting payoffs. In such cases, the signal is genuinely predictive of payoff-relevant outcomes but, because of inversely related effects on probabilities and payoffs that cancel under the expectations operator, do not affect maximized expected utility, and therefore rationalize ignoring.

In pursuing mechanisms that rationalize ignoring without cognitive constraints, we do not mean to suggest that cognitive constraints are uninteresting or play an unimportant role. Rather, our motivation stems from the observation that many, if not all, aspects of human cognition can be alternatively viewed as enabling or limiting, depending on the specifics of the particular task environment. For example, limitations on human memory play an important role in enabling individuals to detect broad patterns, discover generalizations, and engage in abstract thinking (Cosmides & Tooby, 1996; Gigerenzer, 2005; Hertwig & Gigerenzer, 1999; Kareev, 2000; Schooler & Hertwig, 2005). Conversely, very large endowments of working memory may be disabling in some settings, leading to pathologically poor performance in task domains requiring summarization, encapsulation and compression of informational stimuli (Luria, 1968).

Together, internal cognitive constraints and the external structure of the environment are likely to play complementary roles in explaining conditions under which informational frugality is adaptive or beneficial. To isolate the role of the environment, however, in selecting for informationally frugal strategies (referred to in the biology literature as *coarse behavior* (Bookstaber & Langsam, 1985)), this paper begins by assuming that decision makers are unboundedly rational. In most of the models developed below, cognitive constraints and decision-making costs play no role, and subjective beliefs are assumed to coincide perfectly with objective frequencies. These unrealistic assumptions serve as controls in what is intended as a thought experiment whose goal is to cleanly identify environmental structure that can account for behavioral insensitivity to objectively predictive information. One version of the model reintroduces cognitive limitations in the form of just-noticeable differences to demonstrate their positive interaction with environmental structure that favors ignoring, which results in significant enlargement of the rational ignoring set.

This approach is consistent with Herbert Simon's research program on bounded rationality, which emphasized the interplay between external environment and internal cognitive processing, rather than the exclusive focus on errors and biases commonly identified with bounded rationality today. Simon states (1990, p. 7): "Human rational behavior is shaped by a scissors whose blades are the struc-

[1] There is a closely related literature on the value of information (Blackwell, 1953; Delquié, 2006; Hilton, 1981; Lawrence, 1999), where the value of a signal is defined as expected utility (conditioning on the signal) minus unconditional expected utility, with expectations taken a second time with respect to the signal's marginal pdf.

ture of task environments and the computational capabilities of the actor." Studying the roles of both blades, Simon argues, is needed if economics and psychology are to explain how decision rules come into existence, proliferate, and recede.

The plan of this paper is as follows. Section 2 provides background from respective economics and psychology literatures relating to the phenomenon of ignoring information. Section 3 describes the general decision model. Section 4 analyzes the case in which actions are chosen from a discrete set, and Section 5 analyzes the case in which the choice set is a continuum. Section 6 concludes with a discussion of how these results can be interpreted and their implications for bounded rationality.

2. Background

There is abundant evidence that human subjects in psychological experiments systematically ignore information. Explanations for why patterned ignoring is an empirical regularity in human subjects go back at least to Stroop (1935), with antecedents in William James' (1890) implicitly probabilistic interpretation of memory. The work of Gerard (1960), Walker and Bourne (1961) and Posner (1964) established that information reduction is a critical feature of cognitive function in signal detection tasks. Later studies of information reduction and signal extraction led to theories of automaticity (Hasher & Zacks, 1979; Kahneman & Treisman, 1984; Logan, 1980; Logan & Zbrodoff, 1998), which implied an underlying purpose for sub-maximum utilization of available information, namely, re-allocation of scarce cognitive resources to higher priority components of the task. Recent evidence (Dishon-Berkovits & Algom, 2000; Godijn & Theeuwes, 2002; Hutchison, 2002) confirms the existence and manipulability of automaticity, perceptive imperviousness, and Stroop effects.

Within economics, systematic information-processing irregularities have become a priority because of links to important policy issues such as pension finance, healthcare decisions, and immigration (Dow, 1991; Cutler, Porterba, & Summers, 1989; Mullainathan, 2002). Conlisk (1996) surveys the bounded rationality literature within economics, describing how problem-solving costs have been incorporated into neoclassical theory. One observes in this literature that frugality is not a necessary consequence of bounded rationality. On the contrary, constrained optimization problems with explicit decision costs frequently lead to more complicated analytical problems and decision rules with greater informational requirements (e.g., Sargent, 1993). Lipman (1991) identifies a different problem apart from increasing complexity in modeling bounded rationality as cognitively constrained optimization.

Given this paper's interest in environments that reward individuals who ignore, it is relevant to note that normative claims in economics about ignoring information have been mostly negative. Aside from exceptions such as Carrillo and Mariotti (2000) and Aghion et al.'s (1991) work on optimal experimentation, the standard view interprets information suppression as deriving from limitations standing in the way of full optimization rather than as an adaptive response facilitating enhanced performance. Mullainathan (2002), for example, acknowledges that his economic approach to memory does not deal at all with the possibility of beneficial or adaptive information suppression. In contrast, computer scientists (Emran & Ye, 2001; Schein, Popescul, Ungar, & Pennock, 2002) and decision theorists (Jones & Brown, 2002) report that ignoring can be beneficial in some models of computer memory. In other specialized applications, such as music listening (Bigand, McAdams, & Foret, 2000) and chess playing (Reingold, Charness, Schultetus, & Stampe, 2001), there is a clear association between ignoring information and high levels of performance. Psychological work on robustness and flexibility when moving from one environment to another also suggests the possibility of beneficial ignoring (Ginzburg, Janson, & Ferson, 1996; Czerlinski, Gigerenzer, & Goldstein, 1999; Hertwig & Todd, 2003). Hogarth and Karelaia (2005) demonstrate less-is-more effects in simulated binary choice with continuous cues. Extending these interesting simulation studies with formal analysis, Baucells et al. (in preparation), and Hogarth and Karelaia (2006, 2007) characterize large classes of decision problems according to the structure of the environment in which simplifying heuristics perform well, emphasizing the practical implication for managers – that it is often more beneficial to identify the most important decision factors (and rank them) than to compute optimal decision weights for each.

This paper's approach to ignoring differs from its predecessors in that virtually all the idealized assumptions of standard neoclassical theory are present. The setup allows us to show that bounded rationality, distorted beliefs, and anomalous preferences are not required to rationalize ignoring. Also not required are non-linear probability weights or other departures from expected utility theory (c.f., Kahneman & Tversky, 1973). In the analysis that follows, special emphasis is given to the domain-specific nature of rationality by evaluating decision procedures in terms of whether they are well matched to the environments in which they are used, referred to elsewhere as ecological rationality (Gigerenzer & Todd, 1999; Smith, 2003).

3. The decision problem

Let ω represent a continuous or discrete random variable on the set Ω indexing states of nature, which are assumed to be *ex ante* unobservable. The vector of cues X represents observable environmental factors freely available for formulating expectations about ω. Assumption 1 rules out redundancy, or perfect collinearity, among cues:

A1 (Non-redundancy). *$E[XX']$ exists and is full rank.*

The trivial case of cues with no predictive power is similarly ruled out by requiring that each component of X is relevant with respect to state probabilities (but not necessarily with respect to expected payoffs). For a cue to be state-relevant, there must be a value in its support at which the conditional expectation of ω is non-constant with respect to X:

A2 (State-relevancy). *$E[\omega|X]$ is non-constant in each component of X for some value on the support of X.*

The decision variable labeled a (for action) takes on values in the action space, denoted $\mathscr{A}$. It turns out that the cardinality of $\mathscr{A}$ is itself an important parameter in the general decision problem, and the distinction between discrete-action versus continuous-action cases is highlighted in the analysis below.

The function $f_{\omega|X}(\omega,X,a)$ denotes the conditional density of ω given X, which may or may not depend on a. If, for example, the decision maker's choice of how much to eat this year, a, directly affects the probability of a heart attack (denoted by ω = heart attack) conditional on the high blood pressure reading (X = high), then the conditional density $f_{\omega|X}(\omega,X,a)$ would naturally depend on a.

The payoff function $\pi(\omega,X,a)$ serves to rank conditional distributions of ω by the expected-payoff criterion. The function $\pi(\omega,X,a)$ may or may not depend on X. For example if the event of no heart attack yields higher payoffs conditional on a low blood pressure reading than on a high reading, then the payoff function $\pi(\omega,X,a)$ depends non-trivially on X. Thus, endogeneity of frequencies (i.e., $f_{\omega|X}$ dependent on a) and signal-dependent utility (i.e., π dependent on X) are possible, but not required, elements of the set-up.

The decision-making *environment* is defined as any pair of conditional density and payoff functions, denoted $\{f_{\omega|X}(\omega,X,a),\pi(\omega,X,a)\}$. Assuming that the necessary technical requirements for existence of a unique maximum hold, the decision problem is specified in the usual way, with constraints on action embedded in the definition of $\mathscr{A}$, and payoff-maximizing decision rules defined as solutions to the problem:

$$\max_{a\in\mathscr{A}} \int_{\Omega} \pi(\omega,X,a) f_{\omega|X}(\omega,X,a)\mathrm{d}\Omega, \tag{1}$$

where the integral above is a Lebesgue integral accommodating both discrete and continuous random variables as special cases. The solution to (1) is denoted a^*, which is in general a function of the full vector of decision cues X (i.e., $a^* = a^*(X)$).

Rational ignoring arises when $a^*(X)$ is independent of at least one element in X. The goal then is to identify conditions on $\{f_{\omega|X}(\omega,X,a),\pi(\omega,X,a)\}$ under which the link between X and $a^*(X)$ is broken, despite the dependence of $E[\omega|X]$ on X. Note that the variables ω,X, and a can be discrete or continuous. Subsequent sections show that the cardinality of the supports of ω and X matters very little compared to whether a is discrete or continuous, which profoundly affects the size of the set of rational ignoring environments. In the next two sections, we first investigate the case in which a is discrete, followed by the case in which a is continuous.

4. Environments where action is a discrete variable

The simplest version of the model is when the state $\omega \in \{L, R\}$, the signal $X \in \{0,1\}$, and the action $a \in \{U, D\}$ are all binary variables. Consider the payoff matrix below in which rows represent actions and columns represent states of nature:

$$\begin{array}{c|c|c|} & \omega = L & \omega = R \\ \hline a = U & \delta + g_L & 0 \\ \hline a = D & \delta & g_R \\ \hline \end{array}. \tag{2}$$

Zero in the northeast cell is an innocent normalization. The symbols g_L and g_R represent the marginal payoffs of moving from suboptimal to optimal action in each state, and δ represents an unavoidable state-dependent component of payoffs. Obviously, if one action is payoff dominant (which, in this payoff matrix, requires that g_L and g_R have opposite signs), then states of nature are irrelevant to the decision maker, and information that helps predict those states is worthless. To guarantee that states are relevant, we assume (without loss of generality) that $g_L > 0$ and $g_R > 0$. This assumption implies that $a = U$ is the optimal action in state $\omega = L$, and $a = D$ is the optimal action in state $\omega = R$:

A3 (No uniformly best action across states). *Conditional on ω, the optimal action is non-constant with respect to ω.*

A3 implies that states of nature are non-trivially relevant when choosing actions, because the best action will be different depending on which state of nature is realized. In the two-state, two-action model above, assumption A3 reduces to the two inequalities $g_L > 0$ and $g_R > 0$.

4.1. Unconditional action

As a benchmark against which to compare the optimal action rule when conditioning on X, this section develops an expression for the unconditionally optimal action rule when X is not used. Denote the unconditional probability of state L as:

$$p \equiv Pr(\omega = L). \tag{3}$$

The expected payoff as a function of actions is given by:

$$E[\pi(\omega, a)] = \begin{cases} p(g_L + \delta) & \text{if } a = U, \\ p\delta + (1-p)g_R & \text{if } a = D. \end{cases} \tag{4}$$

Assuming expected-payoff maximization, the unconditional action rule is

$$a_0^* = U \text{ if } \frac{p}{1-p} > \frac{g_R}{g_L}, \quad a_0^* = L \text{ if } \frac{p}{1-p} < \frac{g_R}{g_L}, \tag{5}$$

and indeterminate in case $\frac{p}{1-p} = \frac{g_R}{g_L}$. Unconditional action depends on the difference between the odds of state L and the ratio of marginal payoffs with respect to action.

4.2. Conditional action

Now suppose the decision maker facing payoff matrix (2) observes X, which is known to be correlated with ω and is therefore useful for helping predict states. For now, assume that X has a two-element support $\{0,1\}$. (The case of continuous signals is described in a later section.) Denote the conditional probabilities of state L given X as

$$f_i \equiv Pr(\omega = L | X = i), \quad i \in \{0,1\}, \quad \text{such that } f_1 > p > f_0. \tag{6}$$

The inequalities $f_1 > p > f_0$ imply that the signal X has strictly positive correlation with states of nature, indicating a greater chance of state L (relative to the unconditional frequency p) when $X = 1$, and a lower chance of L when $X = 0$.

We investigate conditions under which a^* is independent of X even though X objectively helps predict action-relevant states of nature in the sense of (A3). Conditional on X, the expected-payoff matrix is as follows, where rows correspond to actions, and columns correspond to realized values of X that determine the conditional probabilities used in computing expectations

	$X = 1$	$X = 0$
$a = U$	$f_1(g_L + \delta)$	$f_0(g_L + \delta)$
$a = D$	$f_1\delta + (1 - f_1)g_R$	$f_0\delta + (1 - f_0)g_R$

(7)

In the conditional model, the marginal expected payoff of moving from $a = D$ to $a = U$ is denoted Δ_1 or Δ_0, depending on the observed value of X through the index i:

$$\Delta_i \equiv f_i g_L - (1 - f_i)g_R = \left(\frac{f_i}{1 - f_i} - \frac{g_R}{g_L}\right)(1 - f_i)g_L, \quad i = 0, 1. \tag{8}$$

Thus, Δ_1 and Δ_0 represent marginal gains by moving from $a = D$ to $a = U$, conditioning on $X = 1$ and $X = 0$ respectively. The first term on the right-hand side, $f_i g_L$, is the expected gain from correctly choosing $a = U$ in state L. The negative term that follows, $-(1 - f_i)g_R$, is the expected loss incurred by incorrectly choosing $a = U$ in state R.

Using the notation above, one can write an expression for the optimal action conditional on X. The inequalities $f_1 > p > f_0$, together with the assumption that X and ω have non-zero correlation, imply that $\Delta_1 > \Delta_0$. Thus, there are three distinct action functions to consider, corresponding to the three subsets of the parameter space given by the cases $\Delta_1 > \Delta_0 > 0$, $0 > \Delta_1 > \Delta_0$, and $\Delta_1 > 0 > \Delta_0$. The boundary cases $\Delta_1 = 0$ or $\Delta_0 = 0$ imply obvious indeterminacies that follow from trivial cases where it does not matter (in expectation conditional on X) which action is chosen. Aside from these trivial boundary cases, the three action rules (Eqs. (9)–(11) below) cover the entire universe of environments parameterized by $\theta \equiv [g_L, g_R, f_1, f_0]$

$$\Delta_1 > \Delta_0 > 0 \;\Rightarrow\; a^*(X) = \begin{cases} U & \text{for } X = 1, \\ U & \text{for } X = 0. \end{cases} \tag{9}$$

$$0 > \Delta_1 > \Delta_0 \;\Rightarrow\; a^*(X) = \begin{cases} D & \text{for } X = 1, \\ D & \text{for } X = 0. \end{cases} \tag{10}$$

$$\Delta_1 > 0 > \Delta_0 \;\Rightarrow\; a^*(X) = \begin{cases} U & \text{for } X = 1, \\ D & \text{for } X = 0. \end{cases} \tag{11}$$

The main observation about the conditional action functions above is that, in two out of three cases, a^* is independent of X even though X helps predict payoff-relevant ω. Parameter values θ corresponding to the first two cases ($\Delta_1 > \Delta_0 > 0$ and $0 > \Delta_1 > \Delta_0$) are referred to as the *rational ignoring set*. It is important to recall that no best strategy across both values of ω exists, because of the assumption (A3) that g_R and g_L are strictly positive. Rather, state-invariant dominance of one action over the other seen in (9) and (10) emerges only because of uncertainty and, in particular, the interaction of conditional probabilities and the marginal gains from correct actions. In the payoff matrix (2), U is the correct action in state L, and D is the correct action in state R. This much is unambiguous. Moving to the conditional expected-payoff matrix (7), however, it becomes possible that Δ_1 and Δ_0 have the same sign. This implies (in case Δ_1 and Δ_0 are both positive) that U is the expected-payoff maximizer regardless of X, or (in case Δ_1 and Δ_0 are both negative) that D is dominant regardless of X.

4.2.1. Result

Given a state-dependent matrix of the form (2) that contains no payoff-dominant rows, and a binary decision cue X satisfying $Pr[\omega = L|X = 1] > Pr\,[\omega = L] > Pr[\omega = L|X = 0]$, the expected-payoff-maximizing action a^* is independent of X whenever $\Delta_1\Delta_0 > 0$, where $\Delta_i = f_i g_L - (1 - f_i)g_R$, $i = 0, 1$, which represent marginal expected-payoffs conditional on X when moving from action D to U.

4.3. Determinants of rational ignoring

Rational ignoring occurs in Eqs. (9) and (10), but not (11). In the two-state, two-action environment, the determinants of rational ignoring can be examined analytically by considering an ignoring index, denoted as F, which depends on the parameter $\theta = [g_L, g_R, f_1, f_0]$ given in the environment:

$$F(\theta) \equiv \Delta_1 \Delta_0 = \left(\frac{f_1}{1-f_1} - \frac{g_R}{g_L}\right)\left(\frac{f_0}{1-f_0} - \frac{g_R}{g_L}\right)(1-f_1)(1-f_0)g_L^2. \tag{12}$$

Rational ignoring if and only if $F(\theta) > 0$. To decide whether to pay attention to X, only the sign of $F(\theta)$ is required.

As one would expect, when $f_1 = f_0$ (i.e., the signal X offers no predictive benefit), then X should indeed be ignored:

$$F(\theta)|_{f_1=f_0=f} = [fg_L + (1-f)g_R]^2 > 0. \tag{13}$$

Apart from this trivial case, there are a large number of more interesting cases where (12) is positive. One can additionally show that the rational ignoring set covers a large subset in the universe of admissible values of θ.

For illustration, we describe one parameterization in the rational ignoring set represented by rational numbers: $g_R/g_L = 2/5$, $f_1/(1-f_1) = 2$, and $f_0/(1-f_0) = 1/2$ (or, equivalently, $f_1 = 2/3$, and $f_0 = 1/3$). Because the ratio of marginal benefits with respect to optimal action (g_R/g_L) is less than both odds ratios, the formula in (12) is clearly positive, and it is therefore optimal *ex ante* to ignore the signal. This is interesting because, as the large difference in conditional probabilities shows, the signal is quite informative about ω, and ω matters significantly in terms of payoffs. Nevertheless, because the marginal benefit of correct versus mistaken action is two and one half times as great in state $\omega = R$ as in $\omega = L$, expected payoffs are maximized by choosing the action that is correct in state $\omega = R$, regardless of which signal is observed (i.e., regardless of which state is anticipated). Thus, it pays to ignore the signal and play it safe by being correct in state $\omega = R$ and accepting being wrong when $\omega = L$, which costs only 2/5 what it costs to be wrong when $\omega = R$.

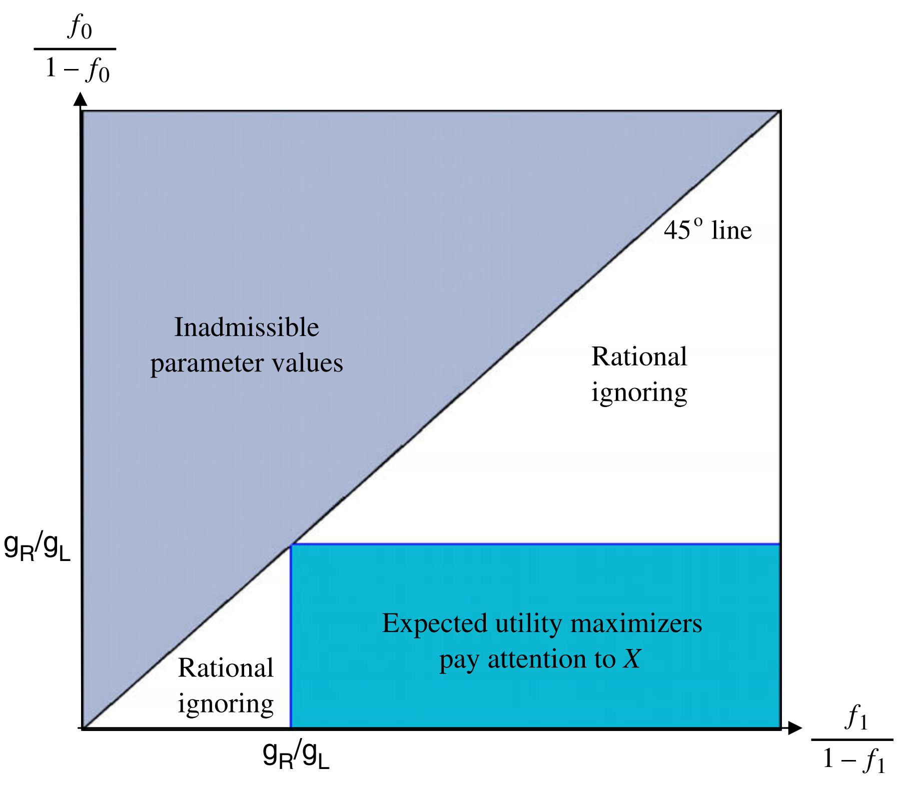

Fig. 1. Rational ignoring environments in the two-state, two-action mode.

Fig. 1 sketches the admissible range of parameter values, which is represented by the infinite triangle in the northwest quadrant between the x-axis and the 45-degree line, and the large rational ignoring set within it. Coordinates on the x-axis represents the odds of $\omega = L$ conditional on $X = 1$, $\left(\frac{f_1}{1-f_1}\right)$, and coordinates on the y-axis represent the odds of $\omega = L$ conditional on $X = 0$, $\left(\frac{f_0}{1-f_0}\right)$. By definition, the inequality $f_1 > f_0$ holds, and therefore $\frac{f_1}{1-f_1} > \frac{f_0}{1-f_0}$ holds, too, indicated in Fig. 1 by the fact that points above the 45-degree line are inadmissible. The ignoring set as depicted in Fig. 1 consists of pairs of odds ratios where both quantities are on the same side of the marginal benefit ratio g_R/g_L

$$f_1/(1-f_1) > f_0/(1-f_0) > g_R/g_L, \quad \text{or } g_R/g_L > f_1/(1-f_1) > f_0/(1-f_0). \tag{14}$$

Expected-payoff maximizers need to pay attention to the ω-correlated signal X only for odds ratios in the rectangle along the x-axis formed by the vertical line $f_1/(1-f_1) = g_R/g_L$ and horizontal line $f_0/(1-f_0) = g_R/g_L$. Everywhere else in the admissible parameter range is part of the rational ignoring set.

4.4. Seatbelting example

A real-world decision task that can be modeled as an instance of the simple two-state, two-action model above is whether or not to wear a seatbelt. Let L denote the state the world when a serious auto accident capable of producing an injury occurs, and R denote the complementary state of no such accident. Then f_1 denotes the probability of a serious auto accident conditional on a signal that indicates higher than average risk, and f_0 denotes the same accident probability conditional on a signal indicating lower than average risk.

Some drivers always wear seatbelts, and some never wear them. Others wear seatbelts conditionally, perhaps weighing risk factors such as time of day, day of the week, time of year, road conditions, the driver's level of sleepiness, and speed. The model provides an unambiguous explanation attributing this heterogeneity to variation in g_R/g_L, which reflects different subjective assessments of convenience, comfort and style premiums associated with not seatbelting in no-accident states of the world and the value of avoiding injury in states of the world in which accidents do occur. Those who experience a large convenience, comfort or style premium by not seatbelting in no-accident states of the world have relatively large values of g_R/g_L. On the other hand, those whose primary goal is to avoid injury in the event of an accident have very small values of g_R/g_L.

The behavioral phenomenon of habitual seatbelt usage, without weighing easily observable signals that shift conditional risks up and down, may appear trivial upon first consideration. Further reflection, however, reveals the high stakes involved in seatbelting decisions, with life and death contingencies hanging in the balance and the possibility of weighing an abundance of easy-to-observe risk factors.[2] Although the base rate of auto accident injuries is low, factors such as time of day, month of the year, state within the US and, most importantly, whether one is driving on a two-lane highway, condition fatality and injury probabilities by a factor of five or more.

Seatbelting decisions have been the target of intense policy interventions aimed at persuading more drivers to buckle up. According to the US Department of Transportation (National Highway Traffic Safety Administration, 2005), seatbelts reduce the risk of serious injury in auto accidents by 50% or more, and yet 5–35% of the population (depending on the state within the US) choose not to wear a seatbelt.[3] A simple economic model that accounts for behavioral variation would therefore seem desirable.

According to the model, the expected-payoff-maximizing action rule stops conditioning on X (i.e., rational ignoring prevails) in two cases. First, if the odds of an accident is uniformly greater than $\frac{g_R}{g_L}$ (regardless of whether X suggests high or low risk), then the driver automatically seatbelts for all real-

[2] The World Health Organization reports that auto accidents cause more than 100,000 injuries and 3000 fatalities worldwide each day (Peden et al., 2004). For evidence on signals that condition the probability of accidents, see Elvick and Vaa (2004) or the US Department of Transportation's National Highway Traffic Safety Administration (National Highway Traffic Safety Administration, 2005). NHTSA research papers, data server and additional links appear at http://www.nhtsa.dot.gov.

[3] Some economists dispute the claim that seatbelts save lives, based on methodological critiques and alternative data from the Center for Disease Control (Garbaez, 1990). Notwithstanding, there appears to be broad consensus among many researchers that seatbelts are of great benefit in the event of an accident.

izations of X and therefore stops paying attention to it. Similarly, if the odds of an accident is uniformly less than $\frac{g_R}{g_L}$, then the driver never wears a seatbelt regardless of X. Even though the odds of an accident are very small, it is worth investigating the magnitudes required for rational ignoring to be plausible, as indicated by the inequalities $\frac{f_0}{1-f_0} > \frac{g_R}{g_L}$ or $\frac{g_R}{g_L} > \frac{f_1}{1-f_1}$, versus conditional seatbelting, which occurs when $\frac{f_1}{1-f_1} > \frac{g_R}{g_L} > \frac{f_0}{1-f_0}$.

The NHTSA reports what are probably the most widely cited fatality and injury statistics in the US. Based on aggregation of all police-reported motor vehicle crashes in 2004, more than 33,000 were killed, and 2.5 million were injured. Injury rates appear quite different depending on whether one normalizes by population, registered cars, total miles driven, or the number of licensed drivers. We focus on one widely used statistic: The unconditional annual rate of auto injuries, which was 1402 injuries per 100,000 licensed drivers in 2004. Of course, this statistic averages over seatbelt users and non-users.

If the annual auto injury rate is 1402/100,000, then the unconditional daily risk of injury could be computed by dividing through by 365: 1402/36,500,000.[4] Since most accidents occur under high-risk conditions, we assume that the unconditional rate computed above is roughly equal to f_1, and then compute f_0 according to the assumption that the high-risk signal raises the conditional risk of injury by a factor of 5:

$$f_1 = 1402/36,500,000, \quad \text{and} \quad f_0 = f_1/5. \tag{15}$$

Given these conditional frequencies, one may then investigate the plausibility of the two rational ignoring conditions:

$$\frac{g_R}{g_L} > \frac{f_1}{1-f_1} = 0.0000384 \quad \text{or } 0.0000077 = \frac{f_0}{1-f_0} > \frac{g_R}{g_L}. \tag{16}$$

These inequalities imply that, if the marginal benefit of not seatbelting in no-accident states of the world (in terms of convenience, comfort and style) is 1/25,000 (i.e., 0.0000384 rounded to the nearest one hundred thousandth) times as beneficial as seatbelting in an accident state of the world – or more, then it is reasonable to ignore risk signals and never wear seatbelts. On the other hand, if the benefit of not wearing a seatbelt is less than 1/125,000 (i.e., an approximation of 0.0000077) times the seatbelting benefit, then expected-payoff maximization leads to unconditional seatbelting.

Unconditional seatbelting is a case of rationally ignoring signals that indicate lower than average risk. Even though these signals reduce conditional risk by a factor of five or more, they are not worth paying attention to for people who place a relatively low premium on not having to wear a seatbelt when no accident occurs. For intermediate values of g_R/g_L, the optimal action conditions on X, manifested in the behavior of drivers who wear seatbelts only on the highway, at night, or conditional on some other signal of elevated risk. For all other parameter combinations, rational ignoring prevails.

4.5. *Rates of rational ignoring*

To complement Fig. 1's depiction of the large size of the rational ignoring set in the discrete-action-space case, this section reports results from a simulation whose purpose is to investigate the frequency with which the rational ignoring condition, $F(\theta) > 0$, holds. Any such simulation must rely on an auxiliary assumption regarding the distribution of θ. Given the distribution of θ, one can calculate the fraction of admissible environments in which rational ignoring occurs, which provides a simulated frequency of rational ignoring, $Pr[F(\theta) > 0]$.

The simulation reported here draws realizations of f_1, g_L and g_R under the assumption that they are independent and uniformly distributed on the unit interval. For f_0, the admissibility condition, $f_1 > f_0$, must be satisfied. Therefore, f_0 is generated uniformly on the support $[0,f_1]$. After 1,000,000 draws from θ, we find that the inequality $F(\theta) > 0$ holds in 61% of the draws. Thus, slightly more than three

[4] There is a non-trivial question of units of time. Does the decision maker decide his or her seatbelting action for the entire year? For each trip? For each mile? Or for each minute in the car? The assumption of daily seatbelt decisions strikes a balance between annual and continuous time. The odds ratios in the ignoring index do depend crucially on time normalization.

out of every five two-state, two-action environments (drawn from uniformly distributed θ) exhibit rational ignoring.

4.6. Paired comparison

Paired comparison is a frequently studied task in the experimental psychology literature. Given N objects (e.g., city populations, the salaries of a particular group of scientists, or a pool of loan applicants), subjects are presented with randomly drawn pairs (from N-choose-2 possibilities, i.e., $N(N-1)/2$) and asked to rank them (e.g., say which is larger or better among the two). See Gigerenzer, Hoffrage, and Kleinbölting (1991) and Goldstein and Gigerenzer (2002) for detailed examples. Because ordering of objects in the presentation of pairs is randomized, the event that the correct answer is listed first has an experimentally fixed probability of 1/2.

Given unconditional state probabilities of 1/2, one may investigate parameterizations in which rational ignoring is predicted to occur. To simplify, we assume that the two realized signals (values of X) condition the event $\omega = L$ symmetrically

$$f_1 = 1/2 + \sigma, \quad \text{and} \quad f_0 = 1/2 - \sigma, \tag{17}$$

where σ, $0 < \sigma < 1/2$, parameterizes the distance between conditional and unconditional probabilities, and is referred to as the strength of the signal X.

Table 1 presents a range of parameterizations in which σ and g_R/g_L both vary. When marginal payoffs are highly asymmetric (e.g., when $g_R/g_L = 0.01$ or 10), most signals, over a wide range of strengths, are ignored except for the very strongest ($\sigma = .45$). On the other hand when marginal gains are identical ($g_R/g_L = 1$) or nearly so, ignoring rarely occurs, and only does so for the very weakest (low-σ) signals. This result closely follows the intuition and formal results in Delquié (2006) showing a general

Table 1
Rational ignoring as a function of the marginal payoff ratio g_R/g_L and strength of signal σ

g_R/g_L	σ	$f_1/(1-f_1)$	$f_0/(1-f_0)$	$F(\theta)$[a]	ignoring?
0.10	0.01	1.04	0.96	0.81	yes
	0.10	1.50	0.67	0.79	yes
	0.25	3.00	0.33	0.68	yes
	0.45	19.00	0.05	−0.90	no
0.50	0.01	1.04	0.96	0.25	yes
	0.10	1.50	0.67	0.17	yes
	0.25	3.00	0.33	−0.42	no
	0.45	19.00	0.05	−8.28	no
0.90	0.01	1.04	0.96	0.01	yes
	0.10	1.50	0.67	−0.14	no
	0.25	3.00	0.33	−1.19	no
	0.45	19.00	0.05	−15.34	no
1.00	0.01	1.04	0.96	−0.00	no
	0.10	1.50	0.67	−0.17	no
	0.25	3.00	0.33	−1.33	no
	0.45	19.00	0.05	−17.05	no
2.00	0.01	1.04	0.96	1.00	yes
	0.10	1.50	0.67	0.67	yes
	0.25	3.00	0.33	−1.67	no
	0.45	19.00	0.05	−33.11	no
5.00	0.01	1.04	0.96	15.99	yes
	0.10	1.50	0.67	15.17	yes
	0.25	3.00	0.33	9.33	yes
	0.45	19.00	0.05	−69.26	no
10.00	0.01	1.04	0.96	80.98	yes
	0.10	1.50	0.67	79.33	yes
	0.25	3.00	0.33	67.67	yes
	0.45	19.00	0.05	−89.53	no

[a] Only the sign of $F(\theta)$ is needed to determine whether ignoring occurs.

inverse relation between the value of information and strength of preference. Following Delquié, the case of $\frac{g_R}{g_L} = 1$ would correspond to prior indifference regarding which action is better, which is precisely when the signal X becomes so valuable that it can never be ignored. On the other hand, one can interpret cases where $\frac{g_R}{g_L}$ is very large or small as indicating a strong prior preference for choosing one way or the other, which results in a low value of information and high likelihood of rational ignoring.

This finding makes a prediction about laboratory studies involving paired choice in which subjects have the chance to pay attention to or ignore information. If ignoring is to be understood as consistent with expected-payoff maximization, one would need to demonstrate that the minimum strength of cues at which those cues begin to be ignored is negatively correlated with asymmetry of the marginal payoffs. In the paired comparison task, asymmetric marginal payoffs seem unlikely, because one would need a theory of why subjects have strict preferences over winning one dollar for correctly ranking items listed in ascending order over winning one dollar for correctly ranking items listed in descending order. Rational ignoring as modeled above does not therefore appear to provide a promising explanation for the empirical phenomena reviewed in the first two sections of this paper unless payoff asymmetry can be established. The results of this section imply the more payoff asymmetry, the broader is the range of signals that can be rationally ignored. Asymmetry may arise, however, even when monetary payoffs are completely symmetric. For example, if subjects receive greater psychic payoff from correctly answering challenging questions than for easy ones, then total payoffs (after aggregating monetary and psychic components) will be asymmetric when experimenters pay a constant quantity of experimental currency units for each correct answer.

Two frequently cited experiments in which prior probabilities are ignored are Kahneman and Tversky's (1973) engineer–lawyer problem and Tversky and Kahneman's (1982) taxi cab problem. In both studies, respondents were asked to produce posterior probabilities given experimenter-controlled prior probabilities, hit rates, and false positive rates. These studies showed a surprising lack of sensitivity to changes in base rates, known as base-rate neglect or the base-rate fallacy. To the extent that experimental subjects perceive asymmetric payoffs (e.g., guessing $\omega = 1$ correctly is better than correctly guessing $\omega = 0$), it is possible that base-rate neglect is consistent with expected-payoff maximization. It may therefore be worthwhile to investigate changes in behavior resulting from treatments in which signals of varying strength are tested, just as it would be interesting to collect post-experiment survey data that could reveal subjective asymmetry in the evaluation of equal monetary payoffs depending on contextual variables like item difficulty.

5. Environments where action is a continuous variable

This section provides an existence proof of the rational ignoring set for the case where the decision maker's choice set is a continuum. The continuous action case reveals an intuitive cancellation principle that illuminates how the structure of information and payoffs in the environment interact to produce rational ignoring. Comparing the continuous and discrete-choice cases analyzed in the previous section, a key difference emerges. Although the rational ignoring set continues to exist in the continuous case, it does so only on a measure-zero subset of the model's parameter space, in contrast with the large rational ignoring sets encountered in the discrete-choice case. Psychological and cognitive realism suggests that human decision makers discretize continuum choice sets, as has been analyzed in the just-noticeable-differences literature. We introduce an example below providing detail into how a small amount of discretization in perceptible units of action (or in payoffs or probabilities) leads once again to positive-measure rational ignoring sets, which can be quite large. Further examples introduce continuous cues and make concrete the reality that ordinary, non-pathological stochastic payoff structures are, under certain conditions, consistent with rationally ignoring cues that significantly condition future payoffs.

5.1. Cancellation principle

Consider a family of environments parameterized by θ, with K cues that non-trivially enter both the payoff and conditional density functions

$$E = \{\{f_{\omega|X}(\omega, X, a;\theta), \pi(\omega, X, a;\theta)\}|\theta \in \Theta\}, \tag{18}$$

where Θ represents the universe of admissible values of θ, assumed to be non-empty and non-singleton.

To guarantee a well-behaved expected-payoff objective function (one for which a unique maximizer in a exists, with straightforward first-order conditions characterized by calculus derivatives that can be moved inside and outside the expectations operator), regularity conditions on π and $f_{\omega|X}$ are assumed to hold (see, for example, Billingsley, 1995):

A4 (Regularity). *For every value of X, there exists a unique maximizer of $E[\pi(\omega,X,a)|X]$ with respect to a on the interior of $\mathscr{A}$, respecting all conditions required for the Theorem of the Maximum and the Implicit Function Theorem.*

Rather than defending this assumption as realistic or general, its restrictiveness serves to strengthen the argument. Showing that rational ignoring can occur even under the most favorable conditions for constrained optimization implies that the rational ignoring result does not depend on obscure technical problems involving computation of extrema. Even under conditions ideal for optimization, unboundedly rational expected-payoff maximizers sometimes ignore relevant predictors. Incorrect beliefs about probabilities and other cognitive limitations, as well as challenging technical issues in the solution of the constrained optimization problem (which the assumptions above rule out), only make it more likely that decision makers might reasonably ignore information.

Provided $f_{\omega|X}(\omega,X,a)$ and $\pi(\omega,X,a)$ respect assumptions A1–A4, the expected-payoff-maximizing decision rule ignores relevant information if and only if there exists a cue X_k, such that the function $a^*(x)$, implicitly defined by $\frac{\partial E[\pi(\omega,X,a)|X]}{\partial a} = 0$, is constant with respect to X_k over its entire support. In case $f_{\omega|X}(\omega,X,a)$ and $\pi(\omega,X,a)$ are differentiable with respect to X_k, a necessary and sufficient condition for rational ignoring is

$$\int_\Omega \left[\frac{\partial^2\pi(\omega,X,a)}{\partial a\partial X_k} f_{\omega|X}(\omega,X,a) + \frac{\partial\pi(\omega,X,a)}{\partial a}\frac{\partial f_{\omega|X}(\omega,X,a)}{\partial X_k} + \frac{\partial\pi(\omega,X,a)}{\partial X_k}\frac{\partial f_{\omega|X}(\omega,X,a)}{\partial a} + \pi(\omega,X,a)\frac{\partial^2 f_{\omega|X}(\omega,X,a)}{\partial a\partial X_k}\right] d\Omega = 0. \tag{19}$$

The proof is implicit differentiation of $\frac{\partial E[\pi(\omega,X,a)|X]}{\partial a} = 0$ with respect to X_k.

Eq. (19) provides a complete characterization of the set of ignoring environments with continuous action space and continuously valued cues satisfying A1–A4. Thus, the ignoring set I is defined as

$$I = \{\{f_{\omega|X}, \pi\} \in E \quad | \quad \text{Eq. (19) holds}\}. \tag{20}$$

A strict subset of I is the set of environments in which cancellation of payoffs and probabilities occurs

$$C = \{\{f_{\omega|X}, \pi\} \in I \quad | \quad \pi f_{\omega|X} \quad \text{is independent of } X_k \quad \text{for some } k = 1,\ldots,K\}. \tag{21}$$

The set C reveals an intuition about inversely related probabilities and payoffs that cancel under the expectations operator and lead to rational ignoring.

An obvious limitation of the cancellation set C and ignoring set I is that they both occur on a measure-zero subset of E. The virtue of C, however, is that its members possess a recognizable feature of the decision-making environment and therefore offer hope that rational ignoring environments might be identified in the laboratory and the field. The next example illustrates the cancellation principle and shows how just-noticeable differences facilitate the re-emergence of a rational ignoring set with strictly positive measure in E.

5.2. Berry gathering with diminishing nutritional returns

To highlight the importance of continuous versus discrete actions (rather than continuous versus discrete cues or states of nature), this example features a continuously valued action variable a chosen from the non-negative real line, while maintaining the binary signals and binary states of nature from previous models in Section 4. To fix ideas, suppose signals and states of nature are interpreted as variables indicating large versus small harvests from strawberry and raspberry gathering, respectively. Suppose that strawberries share with raspberries a common set of weather conditions and precede raspberries in the gathering season. Therefore, the quantity of strawberries harvested is observed first and provides a signal about the states of nature defined by the sizes of raspberry harvests later in the season.

The signal $X = 1$ represents large strawberry harvests, and $X = 0$ represents small strawberry harvests. This cue provides a statistically valid signal for forecasting raspberry harvests, represented by $\omega = L$ for large raspberry harvests and $\omega = R$ for small raspberry harvests. Assume that a large strawberry harvest implies a greater than 50% chance of a large raspberry harvest, and a small strawberry harvest implies a lower than 50% chance of a large raspberry harvest:

$$Pr(\omega = L|X = 1) \equiv f_1 > 1/2, \tag{22}$$
$$Pr(\omega = L|X = 0) \equiv f_0 < 1/2. \tag{23}$$

The decision variable a, $a \geqslant 0$, represents the amount of time allocated to raspberry gathering.[5] One naturally conjectures that the decision maker should choose higher values of a whenever X (which is positively correlated with ω) is observed to be high. However, larger than average strawberry harvests earlier in the season leave berry gatherers more nutritionally fortified, with extra stored energy and, consequently, reduced marginal payoffs from additional berry consumption. This is formalized with the following stylized payoff function:

$$\pi(\omega, X, a) = \alpha \log(1 + X + a\mathbf{1}(\omega = L)) - ca, \tag{24}$$

where $\alpha > 0$ scales the marginal nutritional benefit of berry consumption, $c > 0$ is the marginal cost of time, and $\mathbf{1}(\cdot)$ is an indicator function returning 1 when its argument is a true statement. According to the functional specification above, the berry gatherer's season-long accumulation of nutrition is represented by the sum of an initial allocation of 1, the cue value X which contributes 0 or 1 depending on the size of the strawberry harvest, and $a\mathbf{1}$ $(\omega = L)$ from raspberry consumption. The term $a\mathbf{1}(\omega = L)$ is equal to a (the time allocated to raspberry gathering) if the large-harvest state of nature prevails and zero otherwise. The sum representing accumulated nutrition is transformed by the concave natural logarithm function to reflect diminishing marginal payoffs from additional berry consumption.

The expected-payoff conditional on X is

$$E[\pi(\omega, X, a)|X] = \begin{cases} f_1 \alpha \log(2 + a) + (1 - f_1)\alpha \log(2) - ca & \text{if } X = 1, \\ f_0 \alpha \log(1 + a) - ca & \text{if } X = 0. \end{cases} \tag{25}$$

Assuming the parameters satisfy the following admissibility conditions that guarantee an interior maximizer $a^* > 0$ (away from the boundary $a = 0$)

$$f_1 > 2c/\alpha \quad \text{and } f_0 > c/\alpha, \tag{26}$$

it is straightforward to show that the expected-payoff-maximizing action is

$$a^*(X) = \begin{cases} -2 + \alpha f_1/c & \text{if } X = 1, \\ -1 + \alpha f_0/c & \text{if } X = 0. \end{cases} \tag{27}$$

The relevant condition for rational ignoring identifies cases where a^* is independent of X, which is satisfied whenever $a^*(1) = a^*(0)$, that is, whenever:

$$f_1 - f_0 = c/\alpha. \tag{28}$$

Thus, ignoring occurs when the informational benefit $f_1 - f_0$ (i.e., change in likelihood of large raspberry harvests with respect to different observed levels of strawberry harvests) is exactly offset by the benefit-scaled cost of effort c/α that is required to make use of the information provided by the cue. We emphasize that rational ignoring occurs in this example without any cognitive bounds or informational limitations. However, in contrast with previous models in which the action variable was discrete, the rational ignoring set in this model is small. Because a is continuously valued, the ignoring set has measure zero in the universe of admissible values of $\theta = [f_0, f_1, \alpha, c]$.

[5] The decision maker's information about raspberry harvests is gathered by paying attention to strawberry harvests earlier in the growing season, whereas the action variable a represents the amount of time allocated to raspberry gathering – two distinct senses of "gathering."

5.3. Berry gathering example continued with just-noticeable differences

Now suppose that time is subjectively perceived in discrete units such that, if a_0 is the current choice of a, then nearby values within $\epsilon > 0$ units of a_0 are regarded as equivalent

$$a \in (a_0 - \epsilon, a_0 + \epsilon) \ \Rightarrow \ a \text{ is equivalent to } a_0, \tag{29}$$

where ϵ represents the minimum change in action that is perceptibly different from the current action, and the ϵ-interval about a_0 is an equivalence class. Therefore, if change in some component of X never moves $a^*(X)$ more than ϵ, then changes in that signal will imply no perceptible change in action and, consequently, the signal can be ignored. We investigate the size of the ignoring set relative to the admissible parameter space as a function of the perceptual limit ϵ.

For the case of just-noticeable differences in a with perceptual limit ϵ applied to the berry gathering example, the following inequality represents the condition for rational ignoring: Ignore X if $|a^*(X=1) - a^*(X=0)| < \epsilon$. Using the formula for $a^*(X)$ from (27), rational ignoring occurs whenever the exogenous parameters fall within the bounds:

$$\left|\frac{\alpha}{c}(f_1 - f_0) - 1\right| < \epsilon, \tag{30}$$

or equivalently

$$f_1 - \frac{c}{\alpha}(1+\epsilon) < f_0 < f_1 - \frac{c}{\alpha}(1-\epsilon). \tag{31}$$

Inequality (31) defines the rational ignoring set as a subspace comprised of pairs (f_1, f_0) analogous to Fig. 1, which plotted the rational ignoring set in coordinate pairs of the form $\left(\frac{f_1}{1-f_1}, \frac{f_0}{1-f_0}\right)$ for the case of discrete action. The berry gathering model's admissible parameter space is defined by the four inequalities: $f_1 \leqslant 1$, $f_0 < f_1$, $2c/\alpha < f_1$, and $c/\alpha < f_0$.

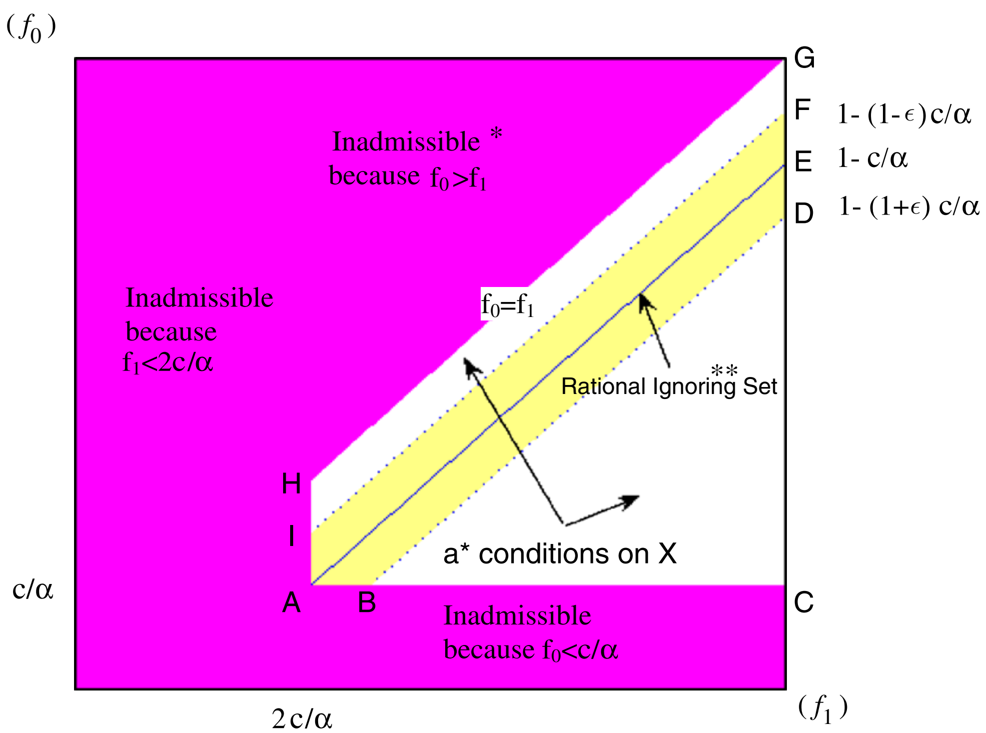

Fig. 2. Rational ignoring with continuous action and just noticeable differences. *The admissible parameter space is given by the trapezoid ACGH, and the rational ignoring set is given by segment AE whose equation is given by: $f_0 = f_1 - c/\alpha$. **Just noticeable differences makes an equivalence class of all values within ε units of the current level of a, and the rational ignoring set expands around the zero-measure segment AE to the positive-measure polygon ABDEFI.

Fig. 2 depicts the model's admissible parameter space as the trapezoid ACGH, and the measure-zero rational ignoring set as the segment AE. With ϵ-noticeable-differences in a, the rational ignoring set grows to the shaded polygon ABDFI. The following is an analytic expression for the ratio of the area of ABDFI to the area of ACGH can be computed, which gives the rate of rational ignoring r (under the assumption of uniformly distributed f_1 and f_0):

$$r(\epsilon) = (4\epsilon c/\alpha)[1 - (\epsilon c/\alpha)/(1 - 2c/\alpha)], \tag{32}$$

for $0 \leqslant \epsilon < 1$ and $c/\alpha < 1/2$. Thus, the rate of rational ignoring is approximately proportional to ϵ. Double the coarseness and the size of the rational ignoring set roughly doubles as well. Numerical simulations allow one to explore this parameter space further under different distributional assumptions, and one consistently finds that just-noticeable differences grow the ignoring set to sizable measurable proportions even for modest coarseness of the perceived action space or discretizing social conventions (e.g., telling time to the nearest quarter of an hour, etc.).

5.4. Example with continuous outcomes and cues

This section provides two final examples of the continuous action model – this time with continuous outcomes and cues. This helps illustrate how ordinary and non-pathological functional forms can give rise to rational ignoring.

Suppose the joint pdf of ω and X is

$$f_{\omega X}(\omega, X, a) = \omega + X \quad \text{for } \omega, \quad X \in [0,1], \quad \text{and 0 otherwise.} \tag{33}$$

Computing the marginal density f_X and the ratio $f_{\omega X}/f_X$, it is easy to show that the conditional pdf of ω given X is

$$f_{\omega|X} = \frac{\omega + X}{\frac{1}{2} + X} \quad \text{for } \omega, \quad X \in [0,1], \quad \text{and 0 otherwise.} \tag{34}$$

The cue is relevant because the conditional expectation of ω given X depends non-trivially on X

$$E[\omega|X] = \int_0^1 \omega \frac{\omega + X}{\frac{1}{2} + X} \mathrm{d}\omega = \frac{1}{\frac{1}{2} + X}\left(\frac{1}{3} + \frac{X}{2}\right), \quad X \in [0,1], \tag{35}$$

which obviously has a non-zero derivative with respect to X.

To illustrate the cancellation principle, suppose the payoff function π is given by the product of the reciprocal of $f_{\omega|X}$ and a simple function of a with a global maximum

$$\pi(\omega, X, a) = \frac{\frac{1}{2} + X}{\omega + X}(a - a^2). \tag{36}$$

The expected payoff simplifies to

$$E[\pi(\omega, X, a)|X] = a - a^2, \tag{37}$$

which is independent of X, and $a^* = \frac{1}{2}$ for every possible observed value of X, despite the fact that both $f_{\omega|X}$andπ depend non-trivially on X.

5.5. Ignoring one of two continuous cues

In the previous continuous-cue environment, the conditional pdf of ω was independent of action a; the payoff function was quasi-concave in a, building in a well-defined interior maximum; and the optimal decision rule was constant. The following example demonstrates that none of these features is necessary for the ignoring condition (19) to hold.

Consider an extension of the previous example, this time with two cues, X_1 and X_2, and joint pdf:

$$f_{\omega,X_1,X_2}(\omega, X_1, X_2) = a(\omega + 2X_1X_2), \quad X_1 \in [0,1], \; X_2 \in [0,1], \; \omega \in [0, \alpha], \tag{38}$$

where the upper bound on ω is the constant $\alpha \equiv -\frac{1}{2} + \left(\frac{2}{a} + \frac{1}{4}\right)^{1/2}$, and $a > 0$. Integrating out ω, the joint pdf of X_1 and X_2 (on its support) is

$$f_{X_1,X_2}(X_1,X_2) = \int_0^{\alpha} a(\omega + 2X_1X_2)\mathrm{d}\omega = a\left(\frac{1}{2}\alpha^2 + 2\alpha X_1X_2\right), \tag{39}$$

and the conditional pdf of ω with respect to (X_1,X_2) (on its support) is

$$f_{\omega|X_1,X_2}(\omega,X_1,X_2) = \frac{\omega + 2X_1X_2}{\frac{1}{2}\alpha^2 + 2\alpha X_1X_2}, \tag{40}$$

which depends on a non-trivially through α.

Now consider the payoff function:

$$\pi(\omega,X_1,X_2) = \frac{\frac{1}{2}\alpha^2 + 2\alpha X_1X_2}{\omega + 2X_1X_2}(X_2 - 2\omega). \tag{41}$$

In contrast to the previous example, the payoff function here is monotonic in a with no built-in interior maximum. The expected payoff conditional on X_1 and X_2 is

$$\int_0^{\alpha} \pi(\omega,X_1,X_2)f_{\omega|X_1,X_2}(\omega,X_1,X_2)\mathrm{d}\omega = \int_0^{\alpha}(X_2 - 2\omega)\mathrm{d}\omega = X_2\alpha - \alpha^2. \tag{42}$$

The condition $\alpha = X_2/2$ implicitly defines the optimal action

$$a^* = 8/(X_2^2 + 2X_2). \tag{43}$$

The non-constant decision rule a^* ignores the cue X_1 while making use of X_2, even though both cues are payoff relevant.

6. Conclusion

Ignoring is, in many cases, manifestly detrimental to the well being of decision makers and the societies they populate. One need only think of global warming as an example in which failing to pay attention to initially subtle signals of future outcomes can have drastic consequences. The ubiquity of ignoring, even when the stakes are very high, prompts the question of why humans, who can be exceedingly sensitive to subtle signals in some contexts, systematically ignore predictive information. This paper demonstrates that throwing away information is, contrary to intuition, consistent with expected utility maximization. Nothing pathological is required. It requires only sufficient asymmetry in payoffs to rationalize ignoring, consistent with recent developments showing that the value of information falls as prior preferences become more intense or decisive (Delquié, 2006).

This paper avoids making categorical claims about the benefits of ignoring. From a descriptive point of view, it shows that ignoring is not always detrimental, and that it is wrong to automatically label decision makers who throw away information as irrational. Expected-payoff-maximizing decision rules in a variety of stochastic payoff environments are shown to ignore information even though that information unambiguously helps predict payoff-relevant events in the future. The normative implications of ignoring are therefore ambiguous and must be analyzed on a case-by-case or context-specific basis. Rather than automatically concluding that subjects are irrational, experimental studies reporting evidence of ignoring would benefit from analysis of the match between those decision processes that led to ignoring and the contexts in which they are used. This would require analyzing the extent to which ignoring provides a shortcut to achieving high levels of performance, one marker of which is negatively correlated marginal effects of the signal on payoffs and probabilities.

This paper attempts to go as far as possible in studying ignoring without introducing decision costs, cognitive limitations or other forms of bounded rationality. The motivation for this is to isolate structure in the stochastic payoff environment that is friendly to decision strategies which ignore information. Thus, the theory of ecological or environment-based ignoring is both distinct from, and complementary to, bounded rationality, suggesting an enlarged set of joint explanations for the success of information-frugal decision rules.

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