International competition with non-linear pricing

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Version of March 1, 2013

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Abstract

This paper models international competition between two upstream firms, one domestic and one foreign, which both serve a domestic downstream firm. The productivity of their inputs in downstream production is private information of the downstream firm. We scrutinize the optimal non-linear pricing schemes and compare them to linear pricing. In particular, we show that a reduction in trade costs does not reduce domestic input demand if upstream firms compete with non-linear pricing schemes, contrary to the effects of trade liberalization when pricing has to be linear.
JEL-Classification: F12, D43

Keywords: Trade with intermediate inputs, outsourcing, non-linear pricing

1 Introduction

A well-known empirical observation is that a large part of world trade is not in final goods, but in intermediate goods. The empirical literature reports a substantial increase of vertical linkages in international production (see for example Egger and Egger, 2005, Feenstra, 1998, and Feenstra and Hanson, 1996). These intermediate goods are used as an input into the production process, either to produce another intermediate good in the value chain or to produce a final good. Due to an increasing integration of national economies, firms, even if they are not multinational, are able to source their inputs from different countries. In any case, trade in intermediate inputs leads to an international organization of firm activities, and these activities have also been investigated by the international trade literature.[1]

How do domestic firms respond to an increase in foreign competition due to trade liberalization? This question has been analyzed both theoretically and empirically in a number of papers. Usually, the adjustment to an increase in import competition is considered to have potentially two effects, one on the firm size of active firms (intensive margin) and the other one on exit decisions of firms (extensive margin). The evidence is mixed; for example, Gu et al (2003) find for the effects of NAFTA that it had no significant effect on Canadian firm size (but on exit decisions of manufacturing firms, see also Head and Ries, 1999). So why is it that we do not find stronger effects when international fragmentation has become so important?

In this paper, we suggest an explanation which captures some of the obvious features of vertical relationships between independent partners. Firstly, while we see a lot of outsourcing activities, all of the recent literature either considers a bilateral vertical partnership along the value chain, or assumes that independent suppliers have to apply a linear pricing scheme. Both assumptions may be inappropriate: for certain tasks there

[1] A part of this literature has focussed on the general equilibrium effects of vertical fragmentation, and whether it leads to factor prices convergence or divergence as a response to fragmentation (see Grossman and Helpman, 2003,, Helpman, 2004, and Helpman and Krugman, 1985). As for the international organization of the firm, the seminal papers by Antràs (2003) and Antràs and Helpman (2004, 2008) have discussed the trade-off between integrating an activity into firm boundaries or outsourcing it to an independent supplier and this trade-off depends on capital intensities in an environment of incomplete contracts and potential hold-up problems. Furthermore, Grossman and Rossi-Hansberg (2008) have taken this issue further by considering trade in tasks, suggesting that firms can outsource not only production but also service activities, and this may even lead to competition between workers within an international firm.

may be more than just one supplier which can do the job, but also not too many. In this case, we will expect that suppliers will compete against each other, and this should give rise to strategic interactions. Secondly, we also see that firms source from different suppliers at the same time, so potential competition does not automatically lead to a natural upstream monopoly. Thirdly, the marketing literature suggests that firms make use of non-linear pricing schemes very often (see for example Bonnet et al, 2012, and Draganska et al, 2010), and it seems that the market for intermediate inputs is much more suitable for such a marketing strategy than the market for final goods in which arbitrage and secondary markets may exist. Intermediate input suppliers are specialized and serve a few customers only, making arbitrage difficult.[2] Consequently, it is the purpose of this paper to scrutinize competition between intermediate input suppliers which compete by non-linear pricing schemes. We will show that considering the intensive margin is misleading if it is measured by firm output. In our model, the domestic firm's response to trade liberalization is not to change its size, but to reduce both the fixed fee and the per-unit price while leaving the discount scheme unchanged.

For this purpose, we use a common agency model in which two upstream firms compete against each other for input demands by a downstream firm. In the theoretical literature, common agency models have been developed both under complete and under incomplete information. The problem with models under complete information is that they may imply a multiplicity of equilibria (Bernheim and Whinston, 1986a, 1986b). We will not follow this approach for several reasons. Firstly, the common agency problem in this setup is mainly a problem of coordination, but we want to focus on the case of competition. Secondly, modern trade theory has emphasized that firm heterogeneity plays an important role in trade, but nearly all models have considered firm heterogeneity in a non-strategic setting of monopolistic competition.[3] We want to break with this tradition and derive the non-linearity of pricing schemes as an endogenous result in a model of strategic interactions. Therefore, in our model, the downstream firm has some private cost information, and the range of the possible cost realizations defines the degree of heterogeneity. Thirdly, this setting also allows to consider the effects changes in (downstream) firm heterogeneity

[2] See McCalman (2012) for a model of monopolistic competition in which firms use a two-part tariff.

[3] For the seminal paper in the monopolistic competition setting, see Melitz (2003). An exemption to the monopolistic competition approach is Long et al (2011) who consider firm heterogeneity in a trade model in which firms have private cost information and can influence the cost realization by research and development.

may play.[4]

Accordingly, the paper is organized as follows: Section 2 introduces the model and shows the results for the case of competition between a domestic and a foreign input supplier with linear pricing. Section 3 extends this model to the case of non-linear pricing schemes. Section 4 considers how trade liberalization and downstream firm heterogeneity change the pricing schemes. Section 5 concludes.

2 The model

Our model assumes a domestic downstream firm, which produces a final good y. The location of this target market is irrelevant for our analysis (except for potential welfare effects) so this could be a domestic or an international market. The inverse demand function for y is common knowledge and given by

$$p = a - \frac{b}{2}y.$$

This downstream firm needs an input q_i from at least one upstream supplier. There are two upstream firms, a domestic one labelled 1, and a foreign one, labelled 2, which sell intermediate inputs, q_1 and q_2, to the downstream firm. Without loss of generalization, we normalize the marginal cost of the domestic upstream firm to zero. The foreign upstream firm's marginal costs are also normalized to zero, but this firm has to carry a trade cost t per unit. In what follows, we confine our analysis to competition between upstream firms and thus consider only cases in which the downstream firm will always source from both upstream firms. Furthermore, producing the input itself is too costly for the downstream firm so integration of input production is not an option. Without loss of generality, we assume that one unit of input is needed for one unit of output such that $q = y$.

The cost function of the upstream firm(s) is common knowledge. The profitability of the downstream firm, however, is private information. Processing the input q_i produced by firm i requires an additional marginal cost ω_i per input unit by the downstream firm which is private information. Due to our linear structure, we may redefine the privately known parameter, denoted by θ_i, as the difference between the maximum willingness to

[4]See Stole (1991) and Martimort (1992) for early contributions the the common agency theory under adverse selection, and Martimort (2006) for an excellent overview of the common agency literature.

pay a of the inverse demand function and the marginal cost of processing the input ω_i, so that $\theta_i = a - \omega_i$.[5]

The upstream firms and the downstream firm play a two-stage game. In the case of linear pricing, each upstream firm sets a price for its intermediate input it produces in the first stage. In the second stage, the downstream firm makes its orders and produces for its target market. The downstream firm produces an output y using two inputs, q_1 and q_2, and its profit function is given by

$$\Pi = v(\theta_1, \theta_2, q_1, q_2) - p_1 q_1 - p_2 q_2$$

where $v(\theta_1, \theta_2, q_1, q_2)$ is the gross profit before paying for both inputs q_1 and q_2. The vector (θ_1, θ_2) characterizes the downstream firm's type. The downstream firm's production cost depends on the input mix. In addition to the marginal cost of processing a single input (which is private information), we allow that the two inputs are either substitutes or complements in production so that production cost is given by

$$C = \omega_1 q_1 + \omega_2 q_2 - \mu q_1 q_2.$$

The parameter μ is common knowledge and measures the degree of substitutability or complementarity in production. If $\mu > 0$, both inputs are complements in production because the synergy effect is positive. If $\mu < 0$, both inputs are substitutes in production, and due to incompatibilities, an increase in one input increases the marginal cost of the other one. Thus, the gross profits of the downstream firm are given by

$$v(\theta_1, \theta_2, q_1, q_2) = \theta_1 q_1 + \theta_2 q_2 - \frac{b}{2} q_1^2 - \frac{b}{2} q_2^2 - (b - \mu) q_1 q_2, b > \mu. \tag{1}$$

The assumption that $b > \mu$ ensures that v is concave in (q_1, q_2) and implies $b - \mu > 0$: the demand effect is always dominant and can never overcompensate a potential synergy effect so that the two inputs are substitutes when considering both demand and production effects at the same time.

We now solve the game in the backward induction fashion. Profit maximization by

[5]Incomplete information may also have other sources which we could consider at the same time. For example, the upstream firm may not exactly know the vertical quality of the final good when produced with its input. In this case, the downstream firm would have private information on the inverse demand intercept a. Considering different a_is, known to the downstream firm only, instead or in addition to marginal processing costs is strategically equivalent to our approach.

the downstream firm leads to demands for intermediate inputs which are given by

$$q_1 = \frac{b\left(\theta_1 - p_1\right) - \left(b - \mu\right)\left(\theta_2 - p_2\right)}{(2b-\mu)\mu}, q_2 = \frac{b\left(\theta_2 - p_2\right) - \left(b - \mu\right)\left(\theta_1 - p_1\right)}{(2b-\mu)\mu}. \tag{2}$$

Throughout the paper, we assume that participation constraints are not binding which means in the context of linear pricing that we confine the analysis to the case in which any type of downstream firm will always source from both suppliers, that is, $q_1, q_2 > 0$.

We now turn to the potential heterogeneity of the downstream firm. Since we deal with linear pricing in this section and the upstream firms do not know (θ_1, θ_2), they form expectations on the downstream firm's type. There is a priori no reason why the downstream firm should draw the productivity of the upstream inputs from different distributions, and hence we assume that $\theta_i \in \left[\underline{\theta}, \overline{\theta}\right], 0 \leq \underline{\theta} < \overline{\theta} < \infty$, and that the downstream firm draws the productivities (θ_1, θ_2) independently from the same c.d.f. $F(\theta_i)$; this means, that the productivities are neither correlated nor that a certain upstream firm has a natural advantage or disadvantage over the other. Furthermore, we assume that $F(\theta_i)$ is differentiable in the range $\left[\underline{\theta}, \overline{\theta}\right]$, so that $f(\theta_i) = F'(\theta_i)$ exits in this range. Consequently, let $\hat{\theta} = \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta) d\theta$ denote the common expected value of both θ_1 and θ_2. Given $\hat{\theta}$, each upstream firm is able to compute its expected demand, and these demands are given by

$$\hat{q}_1 = \frac{\hat{\theta}\mu - bp_1 + (b-\mu)p_2}{(2b-\mu)\mu}, \hat{q}_2 = \frac{\hat{\theta}\mu - bp_2 + (b-\mu)p_1}{(2b-\mu)\mu}. \tag{3}$$

Both upstream firms maximize their expected profits $p_1\hat{q}_1$ and $(p_2 - t)\hat{q}_1$ w.r.t. p_1 and p_2, respectively. This leads to optimal prices

$$p_1 = \frac{\hat{\theta}(3b-\mu)\mu + b(b-\mu)t}{(3b-\mu)(b+\mu)}, p_2 = \frac{\hat{\theta}(3b-\mu)\mu + 2b^2 t}{(3b-\mu)(b+\mu)}.$$

We are now able to discuss how trade liberalization, measured by a reduction in trade costs t for the foreign supplier, will affect the pricing behavior. We find:

$$\frac{\partial p_2}{\partial t} = \frac{2b^2}{(3b-\mu)(b+\mu)} > \frac{\partial p_1}{\partial t} = \frac{b(b-\mu)}{(3b-\mu)(b+\mu)} > 0.$$

Both prices decline with a decline in in t which is not surprising because price competition implies strategic complementarity in the sense of Bulow et al (1985). The price effect is stronger for the foreign firm, so it follows that

$$\frac{d\hat{q}_1}{dt} = \frac{b^2(b-\mu)}{(2b-\mu)\mu(3b-\mu)(b+\mu)} > 0, \tag{4}$$

$$\frac{d\hat{q}_2}{dt} = \frac{-b^2(b+\mu)}{(2b-\mu)\mu(3b-\mu)(b+\mu)} < 0, \tag{5}$$

so that the foreign upstream firm gains and domestic upstream firm loses market share. This has a clear effect for the domestic supplier's profits. Let the maximized (expected) profits of the domestic (foreign) supplier be denoted by $\pi_1^*(\pi_2^*)$. Using the envelope theorem, we find that

$$\frac{\partial \pi_1^*}{\partial t} = p_1 \frac{\partial \hat{q}_1}{\partial p_2}\frac{\partial p_2}{\partial t} = p_1 \frac{b-\mu}{(2b-\mu)\mu}\frac{2b^2}{(3b-\mu)(b+\mu)} > 0 \tag{6}$$

so that trade liberalization will unambiguously reduce the domestic supplier's profits. However, the effect on the foreign supplier is not clear:

$$\begin{aligned} \frac{\partial \pi_2^*}{\partial t} &= -\hat{q}_2 + (p_2 - t)\frac{\partial \hat{q}_2}{\partial p_1}\frac{\partial p_1}{\partial t} \\ &= -\hat{q}_2 + (p_2 - t)\frac{b-\mu}{(2b-\mu)\mu}\frac{b(b-\mu)t}{(3b-\mu)(b+\mu)}. \end{aligned} \tag{7}$$

The first effect is the direct effect: trade liberalization reduces the supplier's cost per sold unit and thus increases his profits. The second effect is the strategic effect, because the domestic supplier will decrease his price in response to trade liberalization, and this harms the foreign supplier. These effects are also well-known from strategic trade policy models with price competition (see for example the seminal paper by Eaton and Grossman, 1986).

How does the potential heterogeneity of the downstream firm affect the outcome? An increase in heterogeneity can be measured by a mean-preserving spread of θ_i. Since $\hat{\theta}$ stays constant, such a mean-preserving spread does not change the pricing behavior of firms unless the participation constraint would be violated for some downstream types. Therefore, an increase in the downstream firm's heterogeneity does not change equilibrium prices and expected profits of the two upstream firms.

3 Competition with Non-Linear Pricing Schemes

After having described the case of linear pricing, let us now turn to the case of competition with non-linear pricing schemes. Now, each firm i ($i = 1, 2$) offers a *schedule* $T_i(q_i)$ to the downstream firm which tells the downstream firm the total payment, T_i, that it must make to i if it wants to buy the quantity q_i. This is again a two-stage game: in the first stage, both upstream firms simultaneously specify a transfer scheme T dependent on the size of the order. In the second stage, the downstream firm makes its orders and produces for the target market. The demand and cost structures are the same as before, that is, the downstream firm produces an output y using two inputs, q_1 and q_2, but its profit function is now given by

$$\begin{aligned}\Pi &= v(\theta_1, \theta_2, q_1, q_2) - T_1(q_1) - T_2(q_2) \\ &= \theta_1 q_1 + \theta_2 q_2 - \frac{b}{2} q_1^2 - \frac{b}{2} q_2^2 - (b - \mu) q_1 q_2 - T_1(q_1) - T_2(q_2).\end{aligned}$$

Again, we have a situation in which both (θ_1, θ_2) are private information. Given the schedules $T_i(q_i)$, $i = 1, 2$, the downstream firm chooses the input levels $q_1(\theta_1, \theta_2)$ and $q_2(\theta_1, \theta_2)$ to maximize its profit Π. The upstream firms now choose non-cooperatively their schedules $T_1(.)$ and $T_2(.)$ to maximize their expected profits. We assume that the inputs of the rival firms cannot be observed and/or verified when the downstream firm compensates upstream firms for their inputs, so we confine the analysis to transfer schemes of firm i which can be made dependent only on the input q_i, and not on the rival input.[6] In general, firm i's optimal schedule $T_i^*(.)$ depends on what it expects $T_j^*(.)$ to be. Thus we must seek a Nash equilibrium pair of schedules (T_1^*, T_2^*). This problem is a common agency problem under adverse selection in which the upstream firms are the principals and the downstream firm is the common agent.

In general, the Revelation Principle does not apply in a common agency context. However, given the linear-quadratic structure of the problem at hand, Ivaldi and Martimort (1994) showed that the Revelation Principle can be applied after a judicious transforma-

[6]Martimort (2006) calls this setup private agency as compared to public ageny. Under public agency, contract are incomplete due to the lack of centralization because firm do not coordinate their offers. Under private agency, another source of incompleteness is that each principal contracts with the agent also on different variables.

tion of variables so that upstream firm i is behaving as if it were facing a fictitious type $z_i \in [\underline{z}, \overline{z}]$ rather than $(\theta_1, \theta_2) \in [\underline{\theta}, \overline{\theta}] \times [\underline{\theta}, \overline{\theta}]$.[7] In particular, under certain assumptions about the probability distribution function, Ivaldi and Martimort (1994) found that there exists a Nash equilibrium pair of schedules (T_1^*, T_2^*) such that T_i^* is quadratic in q_i. They showed that, among all possible replies, player i's best reply to a quadratic schedule $T_j(.)$ is a schedule $T_i(.)$ that is itself quadratic in q_i.[8] For this reason, let us now restrict attention to quadratic schedules. Suppose firm 2's schedule is linear-quadratic and given by

$$T_2(q_2) = \gamma_2 + \alpha_2 q_2 + \frac{\beta_2}{2} q_2^2.$$

This pricing schedule has three parameters: γ_2, to which we will refer to as the fixed fee. This fixed fee has to be paid by the downstream firm upfront if it wants to source inputs from the upstream firm. The parameter α_2 captures the linear part of the pricing schedule, and therefore we will refer to this part as the per unit price. Finally, β_2 is the parameter for the quadratic part of the schedule, and since we will show that $\beta_2 < 0$, we will refer to this part as the discount.

What is firm 1's best reply to this schedule? Firm 1 takes γ_2, α_2 and β_2 as given (as does the downstream firm). Now firm 1 can deduce that the downstream firm's choice of q_1 and q_2 must satisfy the following two conditions[9]

$$\theta_2 - \alpha_2 = (b + \beta_2) q_2 + (b - \mu) q_1, \tag{8}$$

$$\theta_1 = b q_1 + (b - \mu) q_2 + T_1'(q_1). \tag{9}$$

Eq. (8) gives the first-oder condition for the optimal order of the downstream firm from the foreign firm 2, assuming that the pricing schedule of the foreign firm is linear-quadratic. Eq. (9) is the first-order condition for orders from the domestic firm for the general schedule $T_1(q_1)$. We now show that $T_1(q_1)$ is also linear-quadratic, so that the specifications of the pricing schedules are mutually consistent. Substituting eq. (8) into

[7]Their analysis has been applied in the industrial organization literature to study theoretically and empirically competition in the mobile phone industry; see for example Miravete (2002).

[8]In fact they did not require that players use only quadratic schedules; their Nash equilibrium is found from a quite general space. Of course, we cannot be sure if there are other Nash equilibria that are not quadratic schedules.

[9]This is based on the assumption that the restriction that the downstream firm's demands for both inputs should be positive, that is, $q_1 \geq 0$ and $q_2 \geq 0$, is not binding.

(9), we obtain

$$\theta_1 - \frac{(b-\mu)\theta_2}{b+\beta_2} + \frac{(b-\mu)\alpha_2}{b+\beta_2} = \left(b - \frac{(b-\mu)^2}{b+\beta_2}\right) q_1 + T_1'(q_1) \tag{10}$$

Eq. (10) represents the constraint that firm 1 faces in choosing its function $T_1(.)$. Notice that firm 1 can now think of its customer as being characterized by a sufficient statistic z_1 defined by

$$z_1 \equiv \theta_1 - \frac{(b-\mu)\theta_2}{b+\beta_2}.$$

This is similar for firm 2, so we can introduce a new random variable z_i which is distributed between $\underline{z}_i$ and $\overline{z}_i$ where

$$\underline{z}_i = \underline{\theta} - \frac{b-\mu}{b+\beta_{-i}}\overline{\theta}, \overline{z}_i = \overline{\theta} - \frac{b-\mu}{b+\beta_{-i}}\underline{\theta}. \tag{11}$$

Due to overall substitutability of both inputs, that is $b-\mu > 0$, the random variable reaches its minimum for the smallest realization of the own input and the largest realization of the rival output and it maximum for the largest realization of the own input and the smallest realization of the rival output if $b + \beta_{-i} > 0$.

We can now (i) solve for the function $T_i(.)$ using the Revelation Principle, where the agent is characterized by z_i and the distribution of z_i is known, and (ii) prove that $b + \beta_{-i} > 0$ in equilibrium. The result is summarized by

Proposition 1 *If (i) z_i is uniformly distributed between $\underline{z}$ and $\overline{z}$ and no upstream firm is excluded, that is, $q_1, q_2 > 0$, equilibrium pricing schemes exist which are linear-quadratic and concave, giving a discount for larger orders. Each upstream supplier offers*

$$T_i = \gamma_i + \alpha_i q_i + \frac{\beta_i}{2} q_i^2,$$

where the equilibrium parameters are as follows:

$$\gamma_i = \frac{\left[2\underline{z} - \overline{z} + \frac{(b-\mu)\alpha_{-i}}{b+\beta} - m_i\right]^2}{4\left(b - \frac{(b-\mu)^2}{b+\beta}\right)},$$

where $m_i(m_{-i})$ denotes the marginal cost of the upstream firm i (its rival firm) which is either equal to 0 or equal to t, depending on whether it is the domestic or the foreign upstream firm,

$$\alpha_i = \frac{1}{2\delta}\left[(m_i + \overline{z} + \frac{(b-\mu)(m_{-i}+\overline{z})}{b+\beta}\right],$$

where

$$\delta = 1 - \frac{(b-\mu)^2}{(b+\beta)^2},$$

$$\beta_i = \beta = \frac{b}{4}\left[\sqrt{1+8\frac{(b-\mu)^2}{b^2}} - 3\right] < 0.$$

Furthermore, $b + \beta > 0$.

Proof: See Appendix.

Since $\beta_i = \beta$ and the probability function of z_i is uniform, the definition if the random variable becomes even simpler. The bounds $\underline{z}_i$ and $\overline{z}_i$ are identical because the θ's are drawn from the same distribution, so we may drop the subscript. Furthermore, $b+\beta > 0$ confirms the specification of the bounds as given by eq. (11). From

$$z_i = \theta_i - \frac{b-\mu}{b+\beta}\theta_{-i},$$

we find in reverse that

$$\theta_i = \frac{b+\beta}{(2b+\beta-\mu)(\beta+\mu)}\left((b+\beta)z_i + (b-\mu)z_{-i}\right). \tag{12}$$

Let us consider expectations. From the uniform probability distribution function $G(z_i) = (z_i - \underline{z})(\overline{z} - \underline{z})$, we find that the expectation of z_i is equal to[10]

$$\hat{z} = \frac{\underline{z}+\overline{z}}{2} = \frac{\beta+\mu}{b+\beta}\hat{\theta} \Leftrightarrow \hat{\theta} = \frac{b+\beta}{\beta+\mu}\hat{z}.$$

which also follows from (12) because $\hat{z} = E(z_{-i}) = E(z_i)$, so in expected terms the transformation is a very simple one.

Of course, we could feel uncomfortable with doing comparative static exercises because β is an endogenous variable when transforming the θ-distribution into the z-distribution. However, changes in the marginal costs or trade costs have no impact on β_i. Furthermore,

[10]Note carefully that the uniform density function $g(z_i)$ giving rise to the c.d.f. $G(z_i)$ is a convolution of $f(\theta_1)$ and $f(\theta_2)$. Thus, the density function $f(\theta_i)$ is not uniform.

$\beta_i = \beta < 0$ because $(b-\mu)^2/b^2 < 1$ which means that the transfer scheme is regressive in outputs so that larger demands pay a lower per unit price, and the equilibrium discount is the same for both firms irrespective of their marginal costs. The parameter β does also not depend on the bounds of the distribution, so any mean-preserving spread will have no bearing on the regressive part of the pricing scheme. Therefore, while this result is owed to the specification of the model, it has the great analytical advantage that we will be able to do comparative static exercises for changes in the marginal costs and for variations of downstream firm heterogeneity. Note that the invariance of the discount w.r.t. trade costs and the bounds of the distribution originates from the simplest linear-quadratic setup of our vertical relationship.

Let us now turn to the equilibrium orders of the downstream firm. Given the linear-quadratic pricing schedules, we use the first-order conditions and find that

$$\theta_1 - bq_1 - (b-\mu)q_2 - \alpha_1 - \beta q_1 = 0,$$

$$\theta_2 - bq_2 - (b-\mu)q_1 - \alpha_2 - \beta q_2 = 0,$$

which can be rewritten in matrix from as

$$\begin{bmatrix} -(b+\beta) & -(b-\mu) \\ -(b-\mu-) & -(b+\beta) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 - \theta_1 \\ \alpha_2 - \theta_2 \end{bmatrix},$$

and its determinant is $\Delta = (b+\beta)^2 - (b-\mu)^2 = (\mu+b)(2b+\beta-\mu) > 0$. Then, given the equilibrium transfer schemes, the downstream input demands are given by

$$\begin{aligned} q_i &= \frac{(b+\beta)z_i + (b-\mu)\alpha_{-i} - (b+\beta)\alpha_i}{(2b+\beta-\mu)(\beta+\mu)} \\ &= \frac{(b+\beta)\theta_i - (b-\mu)\theta_{-i} + (b-\mu)\alpha_{-i} - (b+\beta)\alpha_i}{\Delta} \end{aligned} \tag{13}$$

because $(b+\beta)z_i = \theta_i(b+\beta) - \theta_{-i}(b-\mu)$. If we impose $\beta = 0$ we are back to the linear pricing case.

The case of non-linear pricing features some remarkable properties. Firstly, we observe that the difference in costs between the domestic and the foreign firm has also an effect on the fixed fee. Secondly, the per unit price α_i depends positively on both firms' marginal costs, with a stronger dependence on own marginal costs. Thirdly, and most importantly, the strategic interaction in non-linear pricing schemas has a different quality than in linear pricing schemes. If the opponent's per unit price increases, the fixed fee will increase, but neither the per unit price nor the discount are modified. An increase in the opponent's per unit price implies that the firm's input supply has become more attractive, and instead of changing parts of its pricing schedule which depends on output, it prefers not to distort its demand but to adjust the fixed fee only. This is, of course, not a full-fledged equilibrium analysis as we look upon the optimal adjustment of a single firm to a changed rival's per unit price only. However, it already gives us a flavor that non-linear pricing gives firms more flexibility in adjusting to changes.

4 Trade Costs and Firm Heterogeneity

We now discuss how trade liberalization, measured by a decrease in trade cost t, and a mean-preserving spread will change the pricing schemes. Since we observe that the concave part of the pricing scheme does neither depend on the marginal cost of both rivals nor on the bounds of the distribution, both trade liberalization and an increase in downstream firm heterogeneity will affect only the fixed fee and the per unit component of the scheme, but not its regressive part.

We find for the effect of trade liberalization:

$$\frac{\partial \alpha_1}{\partial t} = \frac{1}{2\delta}\left(\frac{b-\mu}{b+\beta}\right) > 0$$

and

$$\frac{\partial \alpha_2}{\partial t} = \frac{1}{2\delta} > 0$$

Thus we conclude that trade liberalization will lead to a reduction in the per unit component of the pricing schemes. Furthermore, since

$$0 < \frac{b-\mu}{b+\beta} < 1.$$

the foreign supplier's reduction in the linear component is larger than the domestic firm's reduction.

For the effect of an increase in t on the fixed fees γ_1 and γ_2, define

$$N_i \equiv 2\underline{z} - \overline{z} + \frac{(b-\mu)\alpha_{-i}}{b+\beta} - m_i > 0$$

which should be positive for an interior solution for all types, and define

$$\Omega \equiv 2\left(b - \frac{(b-\mu)^2}{b+\beta}\right) > 0.$$

Then, we can write the fixed fees as

$$\gamma_i = \frac{N_i^2}{2\Omega}$$

and we find that

$$\frac{d\gamma_1}{dt} = \frac{N_1}{\Omega}\left(\left(\frac{b-\mu}{b+\beta}\right)\frac{\partial \alpha_2}{\partial t}\right) > 0,$$

$$\frac{d\gamma_2}{dt} = \frac{N_2}{\Omega}\left(\left(\frac{b-\mu}{b+\beta}\right)\frac{\partial \alpha_1}{\partial t} - 1\right) = \frac{N_2}{\delta\Omega}\left(\left(\frac{b-\mu}{b+\beta}\right)^2 \frac{3}{2} - 1\right) \lesseqgtr 0.$$

We have a clear result for firm 1: trade liberalization will induce the domestic upstream firm to reduce the fixed charge. This is because the downstream firm's valuation of the access to the domestic input is reduced when the foreign input becomes relatively cheaper at the margin. However, the fixed fee of the foreign upstream firm may increase or decrease, depending on whether

$$\left(\frac{b-\mu}{b+\beta}\right)^2 \lesseqgtr \frac{2}{3}.$$

Let us rewrite the square root of the LHS as a function of μ such that

$$r(\mu) \equiv \frac{b-\mu}{b+\beta} = \frac{4(b-\mu)}{b+\sqrt{b^2+8(b-\mu)^2}}.$$

The fixed fee γ_2 will increase (decrease) with trade liberalization if $r^2 > (<) 2/3$. Differentiation w.r.t. μ yields

$$\frac{dr}{d\mu} = -\frac{4b}{9b^2 - 16b\mu + 8\mu^2 + b\sqrt{9b^2 - 16b\mu + 8\mu^2}} < 0$$

because $9b^2 - 16b\mu + 8\mu^2 > 0$ and shows clearly that r declines with μ.

Let us consider the upper bound of μ. If μ is close to b, $r = r^2 = 0$ and thus less than 2/3. In this case, the two inputs are highly substitutable, and the foreign firm will decrease its fixed charge at t falls. What is the reason for the reduction in the fixed fee? Once again, there is a direct effect of trade liberalization which makes the foreign supplier more attractive, also because this supplier will reduce its per unit price of the tariff, and this should lead to an increase in the fixed fee. At the same time, however, the domestic firm will also reduce its price, and the foreign supplier is still at a disadvantage due to the trade cost. This effect is strongest if the the two inputs are highly substitutable, so the reduction in the domestic per unit price overcompensates the trade cost reduction such that also the foreign firm will reduce the fixed fee. With a decrease in μ, this effect becomes weaker. For example, if $\mu = 0$, $r = r^2 = 1 > 2/3$ and the foreign firm will increase its fixed charge at t falls. In that case, the domestic per unit price effect is not too strong.

We now turn to considering the change in input demands by the downstream firm. Somewhat surprisingly, trade liberalization does not change the demand for the domestic input q_1:

$$\frac{dq_1}{dt} = \left(\frac{1}{2(2b+\beta-\mu)(\beta+\mu)\Delta}\right)\left[(b-\mu) - (b+\beta)\left(\frac{b-\mu}{b+\beta}\right)\right] = 0. \tag{14}$$

This is in contrast with the case of linear pricing which we discussed in the last section (see eq (4)). In the case of linear pricing, trade liberalization increases the demand for the foreign intermediate input at the expense of domestic intermediate input. In the case of nonlinear pricing, only the demand for the foreign intermediate input changes:

$$\frac{dq_2}{dt} = -\frac{1}{\Delta(\beta+b)} < 0. \tag{15}$$

Why does input demand from the domestic firm stay constant? Obviously, both firms reduce the per unit price as a response to trade liberalization. At the same time, the discount scheme is not changed. Hence, the reduction in the per unit price of the domestic firm is supported by the discount scheme: for a given foreign input, the downstream firm benefits from both a lower domestic price and a larger discount due to a larger order. These two effects are sufficient to compensate for the effect that foreign input has become more attractive. In the case of linear pricing, there is no additional adjustment in input orders due to the discount scheme. While the exact offsetting of these effects is surely a result of our linear-quadratic setup, it clearly indicates that the discount scheme plays a mitigating role in the domestic adjustment process to trade liberalization. We summarize our results in

Proposition 2 *Trade liberalization does not change the regressive part of the pricing schemes, but (i) decreases the per unit part of both pricing schemes, (ii) leads to a reduction in the fixed domestic fixed fee, but the effect on the foreign fixed fee is ambiguous. Domestic input demand does not change, but foreign input demand increases.*

As a corollary, it follows immediately that trade liberalization reduces the expected profit of the domestic upstream firm. Since q_1 is unchanged for each type, the change in its expected profit of the domestic firm is

$$d\pi_1 = \left(\frac{d\gamma_1}{dt}\right) dt + \left[\left(\frac{d\alpha_1}{dt}\right) dt\right] E(q_1)$$

which is negative for $dt < 0$.

What happens to the expected profit of the foreign firm? We find that

$$d\pi_2 = \left(\frac{d\gamma_2}{dt}\right) dt + \left[\left(\frac{d\alpha_2}{dt}\right) dt\right] E(q_2) + \alpha_2 \frac{dE(q_2)}{dt} dt + \beta E(q_2) \frac{dq_2}{dt} dt.$$

The overall effect is ambiguous because the first term is ambiguous, the second term is negative, the third term is positive, and the fourth term is negative. The reason is similar to the case of linear pricing: a decrease in trade costs increases foreign profits, but at the same time, the domestic supplier reduces the per unit component of his pricing scheme. Furthermore, it depends on the substitutability as measured by μ whether the foreign supplier will increase or decrease the fixed fee.

Considering the effect of firm heterogeneity as measured by a mean-preserving spread, we also find that the regressive part of the pricing scheme does not change with $d\overline{z} = -d\underline{z} > 0$. We observe that the per unit part of the pricing scheme, α_i, depends positively on the upper bound $\overline{z}$ only, but not on the lower bound. Thus, a mean-preserving spread will unambiguously increase the per unit parts of both pricing schemes.

The fixed fees γ_i depend negatively upon the upper bound $\overline{z}$ and positively upon the lower bound $\underline{z}$, but at the same time also on α_{-i}. Let us define the function

$$\Psi(\underline{z}, \overline{z}) = 2\underline{z} - \overline{z} + \frac{b - \mu}{b + \beta}\alpha_i.$$

A mean-preserving spread has qualitatively the same effect on γ_i as on Ψ. Differentiation yields

$$\Psi_{\underline{z}} = 2, \Psi_{\overline{z}} = -1 + \frac{b - \mu}{(b - \mu)(b + \beta)} < 0,$$

which proves that a mean-preserving spread will unambiguously lead to lower fixed fees. We summarize this finding in

Proposition 3 *An increase in downstream firm heterogeneity, measured by a mean-preserving spread, does not change the regressive part of the pricing schemes, but (i) increases the per unit part of both pricing schemes (ii) and leads to a reduction in the both the domestic and the foreign fixed fee.*

What is the intuition behind these results? Here it is the principal-agent relationship which drives these results. Both firms compete for input demand, but at the same time, the combination of a discount scheme and a fixed fee allows them to reap some of the downstream firm's profits. The least productive types determine the fixed fee as they must be held back from not sourcing from a supplier at all. A mean-preserving spread makes the least productive type less productive, so this fee must fall. At the same time, since more productive types get a rent, and the rent at the high end of the distribution increases with a mean-preserving spread, making these types more important. Although the discount scheme does not change with a mean-preserving spread, now the discount scheme covers more types of firms at both ends of the distribution. In particular, the most productive types will receive a larger discount. The increase in the per unit price should compensate for the necessary reduction in the fixed fee and has the intention to appropriate a part of the increased profits of the most productive types.

5 Concluding remarks

This paper has developed a model in which a domestic and a foreign upstream firm compete for orders from a domestic downstream firm. We could show that trade liberalization will change the pricing schemes in a particular way: while the regressive part is not affected by the level of trade costs, the per unit part is unambiguously reduced. Furthermore, also the domestic fixed fee declines under an increasing competitive pressure from the foreign upstream firm. However, trade liberalization makes the foreign downstream firm's access to the domestic market easier, and it is not clear whether the mutual decline in the per unit part of the transfer scheme goes along with a decrease or an increase in the foreign fixed fee. Furthermore, we find that the demand for foreign intermediate goods will increase while it stays constant for the domestic supplier. This is is sharp contrast to the case of linear pricing for which we find that domestic input demand would unambiguously decline. Thus, the adjustment which can be done via non-linear pricing schemes allows a domestic firm to keep the effects on its sales much smaller than they would be under linear pricing.

Appendix

To compute the distribution of z_1 we make use of the joint density function $\varphi(\theta_1, \theta_2)$ and the definition of z_1. We define the joint density function for (z_1, θ_2) by

$$g(z_1, \theta_2) \equiv \varphi\left(z_1 - \frac{(b-\mu)\theta_2}{b+\beta_2}, \theta_2\right)$$

Then the density function of z_1 is

$$g(z_1) = \int_{\underline{\theta}}^{\overline{\theta}} \varphi\left(z_1 - \frac{(b-\mu)\theta_2}{b+\beta_2}, \theta_2\right) d\theta_2$$

Let us assume that $b + \beta_2 > 0$. Since $b - \mu > 0$ (i.e. the two inputs are substitutes) the highest value of z_1 is

$$\overline{z} \equiv \overline{\theta} - \frac{(b-\mu)\underline{\theta}}{b+\beta_2}$$

and the lowest possible value of z_1 is

$$\underline{z} \equiv \underline{\theta} - \frac{(b-\mu)\overline{\theta}}{b+\beta_2}$$

We will put some restrictions on $g(z_1)$ later on. Given a quadratic schedule $T_2(.)$ designed by firm 2, the upstream firm 1 faces a principal-agent problem regarding the downstream firm D. It must design a schedule $T_1(.)$ to maximize its profit subject to the downstream firm's incentive compatibility constraint and participation constraint. Ivaldi and Martimort(1994) showed that under certain assumptions on the density function $g(z_1)$, firm 1's best reply $T_1(.)$ is quadratic in q_1, i.e.

$$T_1(q_1) = \gamma_1 + \alpha_1 q_2 + \frac{\beta_1}{2} q_1^2$$

where α_1, β_1 and γ_1 depend on the parameter vector $(\alpha_2, \beta_2, \gamma_2)$ chosen by firm 2, as well as on $(b, (\mu - b))$, the marginal cost m_1, and parameters of the density function $g(z_1)$. Let the density function $g(z_1)$ be defined over $[\underline{z}, \overline{z}]$. Assume that the cumulative distribution function is

$$G(z_1) = 1 - \frac{(\overline{z} - z_1)}{(\overline{z} - \underline{z})}$$

Under this assumption, the solution of firm 1's problem using the Revelation Principle has the following properties:

1. $q_1(z_1)$ is linear and increasing in z_1,
2. $T_1(q_1(z_1))$ is quadratic in z_1 (i.e. $T_1(q_1)$ is quadratic in q_1). (See Ivaldi and Martimore, 1994, p. 88).

Assume $b - \frac{(b-\mu)^2}{b+\beta_2} > 0$ (Ivaldi and Martimort, 1994, Proposition 5, p. 89) . The best reply $T_1(q_1)$ has the following properties:

$$\alpha_1 = m_1 + \frac{1}{2}\left(\overline{z} + \frac{(b-\mu)\alpha_2}{b+\beta_2} - m_1\right) \tag{A.1}$$

$$\beta_1 = -\frac{1}{2}\left(b - \frac{(b-\mu)^2}{b+\beta_2}\right) = \beta \tag{A.2}$$

By symmetry,

$$\alpha_2 = m_2 + \frac{1}{2}\left(\overline{z} + \frac{(b-\mu)\alpha_1}{b+\beta_1} - m_2\right) \tag{A.3}$$

$$= \frac{m_2 + \left(\overline{z} + \frac{(b-\mu)\alpha_1}{b+\beta_1}\right)}{2} \tag{A.4}$$

$$\beta_2 = -\frac{1}{2}\left(b - \frac{(b-\mu)^2}{b+\beta_1}\right) = \beta \tag{A.5}$$

because β_1 and β_2 are symmetric. We also find (see Ivaldi and Martimort, 1994, Proposition 5, p. 89) concerning the fixed fees (γ_1, γ_2), because the inputs are substitutes (i.e. $(\mu - b) < 0$), the equilibrium choice of γ_1 is directly dependent on the rival's choice (α_2, β_2), but not on γ_2:

$$\gamma_1 = \frac{\left[2\underline{z} - \overline{z} + \frac{(b-\mu)\alpha_2}{b+\beta_2} - m_1\right]^2}{4\left(b + \frac{(b-\mu)^2}{b+\beta_2}\right)} > 0$$

where the denominator is positive (see Ivaldi and Martimort, 1994, Appendix 5). Note that the fixed fees, being similar to fixed costs, do not have any impact on D's production plan, they only affect D's rent. Solving for (β_1, β_2) simultaneously using (A.2) and (A.5), we obtain the Nash equilibrium parameter choice

$$\beta_1 = \beta_2 = \beta = \frac{b}{4}\left[-3 + \sqrt{1 + 8\frac{(b-\mu)^2}{b^2}}\right] < 0$$

$$\beta_1 + b = \frac{b}{4}\left[1 + \sqrt{1 + 8\frac{(b-\mu)^2}{b^2}}\right] > 0 \tag{A.6}$$

It can also be checked that

$$(b + \beta)^2 - (b - \mu)^2 > 0. \tag{A.7}$$

We can solve for (α_1, α_2) using eq (A.1) and (A.3):

$$\begin{bmatrix} 1 & -\frac{b-\mu}{b+\beta} \\ -\frac{b-\mu}{b+\beta} & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(m_1 + \overline{z}) \\ \frac{1}{2}(m_2 + \overline{z}) \end{bmatrix}$$

Let

$$\delta = 1 - \frac{(b-\mu)^2}{(b+\beta)^2} > 0.$$

Then

$$\alpha_1 = \frac{1}{2\delta}\left[(m_1 + \overline{z}) + \frac{(b-\mu)(m_2 + \overline{z})}{b_2 + \beta}\right],$$

$$\alpha_2 = \frac{1}{2\delta}\left[(m_2 + \overline{z}) + \frac{(b-\mu)(m_1 + \overline{z})}{b + \beta}\right].$$

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