Sightings of marked animals that were not individually identified

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Contents

Introduction

These notes clarify the formulation of Efford and Hunter (2018).

Spatial capture–mark–resight

Animals appear in the marked sample with location-dependent probability $p(\mathbf{s}_i; \boldsymbol{\theta}) = 1 - \exp\{-H(\mathbf{s}_i; \boldsymbol{\theta})\}$, where $H(\mathbf{s}_i) = \sum_{j \in M} \sum_k H_{jk}(\mathbf{s}_i)$.

The expected number of unidentified marked individuals is

$$\mu'_{jk} = (1 - p_I) \sum_{i=1}^{n} \delta^{-}_{ij} \int_{\mathbb{R}^2} f(\mathbf{s}_i | \boldsymbol{\omega}_i) \, H_{jk}(\mathbf{s}) \, d\mathbf{s},$$

where $f(\mathbf{s}_i)$ is the probability density for the location of animal i, and δ^{-}_{ij} is an indicator for capture before j.

Using $t(\boldsymbol{\omega}_i, \mathbf{s}) = \Pr(\boldsymbol{\omega}_i | \mathbf{s}) \lambda(\mathbf{s})$ and $f(\mathbf{s}_i | \boldsymbol{\omega}_i) = t(\boldsymbol{\omega}_i, \mathbf{s}) / \int_{\mathbb{R}^2} t(\boldsymbol{\omega}_i, \mathbf{s}) \, d\mathbf{s}$ (cf. Borchers and Efford 2008 p. 380), we have

$$\mu'_{jk} = (1 - p_I) \sum_{i=1}^{n} \delta^{-}_{ij} \int_{\mathbb{R}^2} t(\boldsymbol{\omega}_i, \mathbf{s}) \, H_{jk}(\mathbf{s}) \; d\mathbf{s} \; / \int_{\mathbb{R}^2} t(\boldsymbol{\omega}_i, \mathbf{s}) \, d\mathbf{s}. \quad (8)$$

If $p_I = 1$ then all marked animals are identified and this component drops out of the likelihood.

Sighting only, number of marked animals known (n_0)

The expected number of sightings of unmarked animals at detector k on occasion j is

$$\mu_{jk} = \int_{\mathbb{R}^2} \{\lambda(\mathbf{s}) - n_0 \pi(\mathbf{s})\} H_{jk}(\mathbf{s}) \; d\mathbf{s}, \;\; \lambda(\mathbf{s}) \geq n_0 \pi(\mathbf{s}), \quad (11)$$

where $H_{jk}(\mathbf{s})$ is the occasion- and detector-specific expected number of sightings of an individual at $\mathbf{s}$, as before.

Sightings of marked animals that were not individually identified ($\mathbf{T}'$; Tm in **secr**) relate to n individuals for which we have some observed locations (allowing us to apply (8)) and $n_0 - n$ other individuals that were marked and never sighted. Overall

$$\mu'_{jk} = (1 - p_I) \sum_{i=1}^{n_0} \int_{\mathbb{R}^2} \Pr(\omega_i|\mathbf{s})\pi(\mathbf{s})H_{jk}(\mathbf{s})\, d\mathbf{s} \;/ \int_{\mathbb{R}^2} \Pr(\omega_i|\mathbf{s})\pi(\mathbf{s})\, d\mathbf{s}. \quad (12)$$

For the n sighted individuals

$$S_1 = \sum_{i=1}^{n} \int_{\mathbb{R}^2} \Pr(\omega_i|\mathbf{s})\pi(\mathbf{s})H_{jk}(\mathbf{s})\, d\mathbf{s} \;/ \int_{\mathbb{R}^2} \Pr(\omega_i|\mathbf{s})\pi(\mathbf{s})\, d\mathbf{s}.$$

For all $n_0 - n$ unsighted individuals $\Pr(\omega_i|\mathbf{s}) = \exp\{-H(\mathbf{s})\}$, allowing these to be aggregated in the term

$$S_2 = (n_0 - n) \int_{\mathbb{R}^2} \exp\{-H(\mathbf{s})\}\pi(\mathbf{s})H_{jk}(\mathbf{s})\, d\mathbf{s} / \int_{\mathbb{R}^2} \exp\{-H(\mathbf{s})\}\pi(\mathbf{s})\, d\mathbf{s},$$

and hence

$$\mu'_{jk} = (1 - p_I)(S_1 + S_2)$$

In the C code of **secr** this data type is identified by *like = 5; $\exp\{-H(\mathbf{s})\}$ is represented by pdt for each mask point, indexed by m instead of $\mathbf{s}$.

Sighting only, number of marked animals unknown

When the number of marked animals is unknown it must be estimated.

We assume that the marked animals present at the time of sampling remain distributed as $\pi(\mathbf{s})$. Previously marked animals appear in the sighting sample with site-dependent probability $p(\mathbf{s})$ as before. We define an effective sampling area for the population of marked animals as $a_0(\boldsymbol{\theta}) = \int_{\mathbb{R}^2} \pi(\mathbf{s})p(\mathbf{s})d\mathbf{s}$. Then

$$L_S(\boldsymbol{\theta}|n, \boldsymbol{\omega}) = \binom{n}{n_1, ..., n_c} a_0(\boldsymbol{\theta})^{-n} \prod_{i=1}^{n} \int_{\mathbb{R}^2} \pi(\mathbf{s}) \Pr(\boldsymbol{\omega}_i|\mathbf{s})\, d\mathbf{s}. \quad (13)$$

For the expected numbers of sightings we substitute n/a_0 for n_0 in (11) and (12). No adjustment is required for animals not yet marked at time j because marking is completed before $j = 1$.

Testing

```
library(secr)
```

```
## This is secr 3.2.0 pre-release. For overview type ?secr
```

Adapting the function posted by mateo 2018-12-28

```
pctID.foo <- function(seed = 1, sight.sessions = 5, g0 = 0.2, pctID = 1, ...) {
    # some parameters
    set.seed(seed)
    sig <- 8
```

```r
    D <- 46.3
    g0.capture <- g0
    g0.resight <- g0

    # sim grid and animals
    grid <- make.grid(nx = 10, ny = 10, spacex = 20, spacey = 20, spacing = NULL,
                      originxy = c(0,0), hollow = F, ID = "alphay", markocc = NULL)
    # 150 animals (9*20)^2 * 46.3 / 10000; or use Nbuffer = 150
    pop2 <- sim.popn(grid, D = D, buffer = 0, Ndist = "fixed")

    ## with marking occasion
    markocc(grid) <- c(1, rep(0, sight.sessions-1))
    detector(grid) <- c("multi", rep("count",sight.sessions-1))
    g0mr <- c(g0.capture, rep(g0.resight, sight.sessions-1))
    dropLast <- sim.resight(grid, detectpar=list(g0 = g0mr, sigma = sig),
                            popn = pop2, pID = pctID)

    ## without marking occasion
    markocc(grid) <- rep(0, sight.sessions)
    detector(grid) <- rep("count",sight.sessions)
    g0mr <- rep(g0.resight, sight.sessions)
    dropFirst <- sim.resight(grid, detectpar=list(g0 = g0mr, sigma = sig),
                             popn = pop2, pID = pctID)

    ## fit models
    mask <- make.mask(grid, nx = 32, buffer = 0, type = 'traprect')
    baseargs <- list(capthist = dropLast, model = g0 ~ 1, mask = mask)
    args1 <- args2 <- c(baseargs, list(...))
    args2$capthist <- dropFirst
    par.secr.fit(list(withmark = args1, withoutmark = args2), ncores = 2)
}

works <- pctID.foo(seed = 1, sight.sessions = 5, g0 = 0.2, pctID = 1, trace = FALSE)
```

```
## Completed in 7.832 minutes at 10:47:01 05 Jan 2019
```

```r
low.pctID <- pctID.foo(seed = 1, sight.sessions = 5, g0 = 0.2, pctID = 0.8, trace = FALSE)
```

```
## Completed in 4.832 minutes at 10:51:51 05 Jan 2019
```

```r
high.sessions <- pctID.foo(seed = 1, sight.sessions = 10, g0 = 0.2, pctID = 0.8, trace = FALSE)
```

```
## Completed in 10.792 minutes at 11:02:39 05 Jan 2019
```

```r
high.g0 <- pctID.foo(seed = 1, sight.sessions = 5, g0 = 0.4, pctID = 0.8, trace = FALSE)
```

```
## Completed in 6.227 minutes at 11:08:53 05 Jan 2019
```

```r
lapply(works, predict)
```

```
## $fit.withmark
##        link   estimate  SE.estimate         lcl        ucl
## D       log 47.5597907 8.683971e+00 33.34963306 67.8248449
## g0    logit  0.1661889 4.393218e-02  0.09671518  0.2706166
## sigma   log  8.9954149 8.982925e-01  7.39997089 10.9348389
## pID   logit  1.0000000 4.702830e-17  0.99999996  1.0000000
##
```

```
## $fit.withoutmark
##        link   estimate  SE.estimate       lcl       ucl
## D       log 61.3378506 6.601491e+00 49.702879 75.696458
## g0    logit  0.1695525 4.079858e-02  0.103706  0.264853
## sigma   log  7.8903974 8.572671e-01  6.381013  9.756816
## pID   logit  1.0000000 7.310595e-16  1.000000  1.000000
```

```r
lapply(low.pctID, predict)
```

```
## $fit.withmark
##        link   estimate SE.estimate         lcl        ucl
## D       log 47.5790310  8.68841342 33.36187838 67.8548181
## g0    logit  0.1596558  0.04313092  0.09188194  0.2629460
## sigma   log  9.1862952  0.95209422  7.50161815 11.2493090
## pID   logit  0.8400214  0.07331084  0.64317578  0.9386357
##
## $fit.withoutmark
##        link   estimate SE.estimate        lcl        ucl
## D       log 64.8739644  7.24543024 52.1553305 80.6941730
## g0    logit  0.1669202  0.04631695  0.0944534  0.2779216
## sigma   log  7.6285519  0.98648355  5.9268689  9.8188109
## pID   logit  0.8250337  0.04427027  0.7210646  0.8958475
```

```r
lapply(high.sessions, predict)
```

```
## $fit.withmark
##        link   estimate SE.estimate        lcl        ucl
## D       log 53.9525036  7.11435936 41.7111982 69.7863589
## g0    logit  0.1710970  0.03316021  0.1154604  0.2460846
## sigma   log  8.2834120  0.61450640  7.1638973  9.5778753
## pID   logit  0.8164443  0.05524831  0.6834955  0.9015883
##
## $fit.withoutmark
##        link   estimate SE.estimate        lcl        ucl
## D       log 46.9477944  3.07310470 41.3006477 53.3670904
## g0    logit  0.1751897  0.02648030  0.1291539  0.2332398
## sigma   log  8.6853112  0.58584194  7.6108866  9.9114117
## pID   logit  0.7988914  0.03310783  0.7262129  0.8560999
```

```r
lapply(high.g0, predict)
```

```
## $fit.withmark
##        link   estimate SE.estimate        lcl        ucl
## D       log 47.9298175  4.59524516 39.7359771 57.8132858
## g0    logit  0.3870928  0.04984324  0.2949819  0.4880551
## sigma   log  7.7740949  0.42195045  6.9901033  8.6460170
## pID   logit  0.7953982  0.04290126  0.6986956  0.8669735
##
## $fit.withoutmark
##        link   estimate SE.estimate        lcl        ucl
## D       log 49.4199708  3.05494275 43.7859533 55.7789275
## g0    logit  0.3535006  0.04497270  0.2710351  0.4457156
## sigma   log  8.0978768  0.46706519  7.2329715  9.0662058
## pID   logit  0.8563862  0.02693198  0.7951744  0.9015693
```

```r
lapply(works, region.N)
```

```
## $fit.withmark
##     estimate SE.estimate      lcl      ucl  n
## E.N 154.0937    28.13607 108.0528 219.7525 28
## R.N 151.7183    25.24964 111.2687 211.8173 28
##
## $fit.withoutmark
##     estimate SE.estimate      lcl      ucl  n
## E.N 198.7346    21.38883 161.0373 245.2565 67
## R.N 151.0100    16.08563 124.9180 188.8563 67
```

```r
lapply(low.pctID, region.N)
```

```
## $fit.withmark
##     estimate SE.estimate      lcl      ucl  n
## E.N 154.1561    28.15046 108.0925 219.8496 28
## R.N 151.7837    25.26445 111.3108 211.9185 28
##
## $fit.withoutmark
##     estimate SE.estimate      lcl      ucl  n
## E.N 210.1916    23.47519 168.9833 261.4491 67
## R.N 162.3134    18.46329 132.4308 205.8434 67
```

```r
lapply(high.sessions, region.N)
```

```
## $fit.withmark
##     estimate SE.estimate      lcl      ucl  n
## E.N 174.8061    23.05052 135.1443 226.1078 28
## R.N 172.2295    18.88175 139.7101 214.2154 28
##
## $fit.withoutmark
##      estimate SE.estimate      lcl      ucl  n
## E.N 152.11085    9.956859 133.8141 172.9094 63
## R.N  80.58925         NaN      NaN      NaN 63
```

```r
lapply(high.g0, region.N)
```

```
## $fit.withmark
##     estimate SE.estimate      lcl      ucl  n
## E.N 155.2926   14.888594 128.7446 187.3150 53
## R.N 156.2168    8.147247 141.4436 173.4576 53
##
## $fit.withoutmark
##      estimate SE.estimate      lcl      ucl  n
## E.N 160.12071    9.898014 141.8665 180.7237 72
## R.N  94.04094         NaN      NaN      NaN 72
```

```r
summary(works[[2]]$capthist)
```

```
## Object class       capthist
## Detector type      count (5)
## Detector number    100
## Average spacing    20 m
## x-range            0 180 m
## y-range            0 180 m
##
```

```
## Marking occasions
##  1 2 3 4 5
##  0 0 0 0 0
##
## Counts by occasion
##                     1   2   3   4   5 Total
## n                  16  11  11   6   9    53
## u                  16   9   6   3   6    40
## f                  29   9   2   0   0    40
## M(t+1)             16  25  31  34  40    40
## losses              0   0   0   0   0     0
## detections         18  12  12   7   9    58
## detectors visited  15  11  11   6   8    51
## detectors used    100 100 100 100 100   500
##
## Empty histories :  27
##
## Sightings by occasion
##           1  2  3  4  5 Total
## ID       18 12 12  7  9    58
## Not ID    0  0  0  0  0     0
## Unmarked  26 28 18 22 20   114
## Uncertain  0  0  0  0  0     0
## Total     44 40 30 29 29   172
```