A Factor Analytical Method to Interactive Effects Dynamic Panel Models with or without Unit Root

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Motivation

- One of the most well-known problems in (micro) econometrics is the presence of fixed effects/incidental parameter bias in dynamic panels.
- In fixed-N panels this bias renders OLS inconsistent.
- Allowing T to be "large" eliminates the inconsistency problem, but the asymptotic distribution of OLS is still miscentered.
- These problems have made EVERYONE use GMM.
- In fact, in certain literatures it is basically impossible to publish work that is not based on GMM.
- Main problem: GMM cannot be used if T is "large".

Motivation

- Bai (2013) recently proposed a new factor analytical (FA) method to the estimation of dynamic panel models.
- Pros:
  - Completely bias-free.
  - Can accommodate heteroskedasticity across both time and cross-section.
  - Robust to arbitrary initialization.
- Cons:
  - Restricted to fixed effects models.
  - The possibility of unit roots is excluded.

Motivation

- The contribution of the current paper:
  - Extend FA to the case of interactive effects.
  - Consider both stationary and unit root panels.
- These extensions make it possible to apply FA without knowing the order of integration of the data.
- FA is the unique in that it enables normal inference both with and without unit root.
- The common/deterministic component of the data is basically unrestricted.
- Since the interactive effects can be treated as unknown there is no need to model the deterministic component.

Model

- DGP:

$$y_{i,t} = c_{i,t} + \rho y_{i,t-1} + \varepsilon_{i,t}$$

where $\rho \in (-1, 1]$, $y_{1,0} = ... = y_{N,0} = 0$, $c_{i,t}$ is a common component and $\varepsilon_{i,t}$ is an error term.

- Two interactive effect specifications of $c_{i,t}$ are considered:

C1. $c_{i,t} = \lambda_i' F_t$

C2. $c_{i,t} = \lambda_i'(F_t - \rho F_{t-1})$

where F_t is an $m \times 1$ vector of common factors and λ_i is a vector of loading coefficients.

Model

- C1 and C2 are indistinguishable for $|\rho| < 1$.

- Since the analysis under C1 is simpler we assume that C1 holds whenever $|\rho| < 1$.

- Under $\rho = 1$,

$$y_{i,t} = \sum_{n=1}^{t} c_{i,n} + \sum_{n=1}^{t} \varepsilon_{i,n}$$

  The common component therefore has different meanings depending on whether C1 or C2 holds. Under $\rho = 1$ we therefore consider both C1 and C2.

Model in a matrix form

- Notation: Let y_i, c_i and ε_i be $T \times 1$ vectors and

$$J = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ \rho & 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \rho^{T-2} & \dots & \rho & 1 & 0 \end{bmatrix}$$

- Matrix DGP:

$$y_i = c_i + \rho J y_i + \varepsilon_i$$

- Solution for y_i:

$$y_i = \Gamma(c_i + \varepsilon_i)$$

where $\Gamma = (I_T - \rho J)^{-1} = I_T + \rho L$.

Assumptions

- $\varepsilon_{i,t}$ is iid across both i and t with $E(\varepsilon_{i,t}) = E(\varepsilon^3_{i,t}) = 0$, $E(\varepsilon^2_{i,t}) = \sigma^2 > 0$ and $E(\varepsilon^4_{i,t}) < \infty$.
- $||\lambda_i|| < \infty$ for all i and $S_\lambda = N^{-1}\sum_{i=1}^{N} \lambda_i\lambda_i' \to \Sigma_\lambda > 0$ as $N \to \infty$.
- F_t satisfies the following:
  - If $|\rho| < 1$, $T^{-1}F'F$, $T^{-1}F'L'F$, $T^{-1}F'LL'F$ and $T^{-1}F'L'LF$ converge to positive definite matrices.
  - If $\rho = 1$, $T^{-1}F'F$, $T^{-2}F'L'F$, $T^{-3}F'LL'F$ and $T^{-3}F'L'LF$ are converge to positive definite matrices.
  - $||F_t|| < \infty$ for all t.

FA with F known

- Vector of parameters: $\theta = [(\text{vech}\, S_\lambda)', \rho, \sigma^2]' = (\theta_1', \theta_2')'$, where $\theta_2 = (\rho, \sigma^2)'$.

- "Discrepancy" function:

$$Q(\theta) = \log(|\Sigma(\theta)|) + \text{tr}\,(S_y \Sigma(\theta)^{-1})$$

  where $S_y = N^{-1} \sum_{i=1}^N y_i y_i'$, $\Sigma(\theta) = \sigma^2 \Gamma \Lambda \Gamma'$ and $\Lambda = I_T + \sigma^{-2} F S_\lambda F'$.

- Objective function:

$$\ell(\theta) = -\frac{N}{2} Q(\theta)$$

- θ does not contain $\lambda_1, ..., \lambda_N$, only S_λ.

- Since the dimension of θ remains fixed as $N \to \infty$ there is no incidental parameter bias.

FA with F known

- Concentration with respect to S_λ yields

$$Q_c(\theta) = T\log(\sigma^2) + \log(|\hat{\Lambda}(\theta_2)|) + \sigma^{-2}\text{tr}\,[G\hat{\Lambda}(\theta_2)^{-1}]$$

where $G = \Gamma^{-1}S_y\Gamma^{-1\prime}$, $\hat{\Lambda} = I_T + \sigma^{-2}F\hat{S}_\lambda F'$,
$\hat{S}_\lambda = \sigma^2 F^-(\sigma^{-2}G - I_T)F^{-\prime}$ and $F^- = (F'F)^{-1}F'$.

- Concentrated objective function:

$$\ell_c(\theta_2) = -\frac{N}{2}Q_c(\theta_2)$$

- The FA estimator $\hat{\theta}_2 = (\hat{\rho}, \hat{\sigma}^2)'$ of $\theta_2^0 = (\rho_0, \sigma_0^2)'$ is obtained by minimizing $\ell_c(\theta_2)$.

- Lemma 1:

$$
\begin{aligned}
(NT)^{-1}\ell_c(\theta_2) &= -\frac{1}{2}\left(\log(\sigma^2) + \frac{\sigma_0^2}{\sigma^2}\right) - \frac{\sigma_0^2}{2\sigma^2}(\rho_0 - \rho)^2\omega_1^2 \\
&+ O_p((NT)^{-1/2}) + O_p(T^{-1}\log(T))
\end{aligned}
$$

with

$$\omega_1^2 = T^{-1}\mathrm{tr}\,(L_0L_0' + \sigma_0^{-2}S_\lambda F'L_0'M_FL_0F) \geq 0$$

where $L_0 = L(\rho_0)$, $M_F = I_T - P_F$ and $P_F = F(F'F)^{-1}F'$.

- $(NT)^{-1}\ell_c(\theta_2)$ is maximized at $\rho = \rho_0$ and $\sigma^2 = \sigma_0^2$ implying consistency.

- Consistency only requires $T \to \infty$.

Asymptotic results with $|\rho| < 1$ and F known

- Theorem 1: As $T \to \infty$ for any N, including $N \to \infty$ with $\sqrt{N}T^{-3/2} \to 0$,

$$\sqrt{NT}(\hat{\rho} - \rho_0) \sim N(0, \omega_1^{-2})$$

- Remarks:
  - There is no bias.
  - N may be fixed.
  - $\sqrt{N}T^{-3/2} \to 0$ requires $T^3 > N$, which is not very restrictive.
  - If $F_t = 1$, then $\omega_1^2 = T^{-1}\text{tr}\,(L_0 L_0') + o(1) = 1/(1 - \rho_0^2) + o(1)$, which is the same as for bias-corrected OLS.

Asymptotic results with $\rho = 1$ and F known under C1

- Theorem 2: As N, $T \to \infty$ with $\sqrt{N}T^{-3/2} \to 0$,

$$\sqrt{N}T^{3/2}(\hat{\rho} - \rho_0) \sim N(0, T^2\omega_1^{-2})$$

- Remarks:
  - Again, there is no bias.
  - FA is normal with the "same" variance for all $\rho_0 \in (-1, 1]$.
  - In contrast to before $N \to \infty$ is required.
  - The rate of consistency of $\hat{\rho}$ is $\sqrt{N}T^{3/2} >> \sqrt{NT}$.
  - $T^{-2}\omega_1^2 = T^{-3}\text{tr}\,(\sigma_0^{-2}S_\lambda F' L_0' M_F L_0 F) + o(1)$, suggesting that in this case $S_\lambda \to \Sigma_\lambda > 0$ is crucial.

Asymptotic results with $\rho = 1$ and F known under C2

- Theorem 3: As N, $T \to \infty$ with $\sqrt{N}T^{-1} \to 0$,

$$\sqrt{N}T(\hat{\rho} - \rho_0) \sim N(0, T\omega_2^{-2})$$

  where

$$\omega_2^2 = T^{-1}\text{tr}\left(L_0 L_0' + \sigma_0^{-2} S_\lambda F' \Gamma^{-1\prime} L_0' M_{\Gamma^{-1}F} L_0 \Gamma^{-1} F\right)$$

- Remarks:
  - There is no bias.
  - $T^{-1}\omega_2^2 = T^{-2}\text{tr}\left(L_0 L_0'\right) + o(1) = 1/2 + o(1)$, suggesting that in this case $\Sigma_\lambda > 0$ is no longer necessary.
  - Under C2 the variance of FA (2) is lower than the variance of bias-corrected OLS ($51/5 \approx 10$).

Asymptotic results with $\rho = 1$ and F known under C2

- It can be shown that under $|\rho_0| < 1$,

$$\sqrt{NT}(\hat{\rho} - \rho_0) \sim N(0, \omega_2^{-2})$$

- Hence, just as in C1 FA enable normal inference with the "same" variance for all $\rho_0 \in (-1, 1]$.

  - The FA-based t-statistic for testing $H_0 : \rho_0 = \rho^0$ is given by

  $$t(\rho^0) = \hat{\omega}_1 \sqrt{NT}(\hat{\rho} - \rho^0)$$

  where $\hat{\omega}_2^2$ is an estimator of ω_2^2.

  - Under H_0,

  $$t(\rho^0) \to_d N(0, 1)$$

  which holds for all values of $\rho_0 = \rho^0 \in (-1, 1]$.

Comparison of the results for C1 and C2 when $\rho = 1$

- The C2 requirement that $\sqrt{N}T^{-1} \to 0$ $(T^2 > N)$ is stronger than the C1 requitement that $\sqrt{N}T^{-3/2} \to 0$ $(T^3 > N)$.
- The rate of consistency of $\hat{\rho}$ under C1 $(\sqrt{N}T^{3/2})$ is higher than under C2 $(\sqrt{N}T)$.
- In practice we never know if our specification of F is correct.
  - The C1 requirement that $S_\lambda \to \Sigma_\lambda > 0$ means there cannot be any redundant elements in F.
  - The fact that under C2 $\Sigma_\lambda > 0$ is not needed means that we just have to pick F general enough.
- The fact that the asymptotic distribution under C2 does not depend on F is a unique and very useful property.

Asymptotic results with F unknown

- New vector of parameters: $\theta = [(\text{vech}\, S_\lambda)', \rho, \sigma^2, (\text{vec}\, F)']'$ $= (\theta_1', \theta_2')'$, where $\theta_2 = [\rho, \sigma^2, (\text{vec}\, F)']'$.
- With λ_i and F unknown there is an identification issue, which can be resolved by imposing m^2 restrictions.
- Proposition 1:

$$||\hat{F}_t - F_t^0|| = o_p(1)$$

- Since we are only interested in controlling for F_t, consistency is enough.
- The dimension m of F_t can be estimated using an information criterion.
- Since F_t can be treated as unknown there is no need to model the deterministic part of the model.

Monte Carlo results

- DGP: $\varepsilon_{i,t} \sim N(0,1)$ and $\lambda_i \sim U(1,2)$.
- Experiments:

  F1. $F_t = 1$

  F2. $F_t = (1,0)'$ if $t < \lfloor T/2 \rfloor$ and $F_t = (1,1)'$ otherwise

  F3. $F_t \sim N(0,1)$

- In F1 and F2 F_t is known, whereas in F3 it is unknown/estimated.

Table 1: Bias, RMSE and 5% size results for F1 when $\rho_0 = 0.5$.

		AHL			AHD			FA			LS			BCLS		
N	T	Bias	RMSE	Size	Bias	RMSE	Size	Bias	RMSE	Size	Bias	RMSE	Size	Bias	RMSE	Size
10	10	0.0438	26.1063	0.0	−1.3524	82.6024	7.2	−0.0042	0.0621	6.1	−0.1255	0.1539	41.4	0.0119	0.0987	5.2
10	50	0.0029	0.1271	0.0	0.0415	0.2986	9.0	−0.0009	0.0343	5.1	−0.0293	0.0485	18.1	0.0001	0.0394	5.4
10	100	0.0009	0.0693	0.0	0.0143	0.1888	9.3	−0.0003	0.0257	4.8	−0.0148	0.0311	13.2	0.0001	0.0277	5.2
10	200	0.0007	0.0433	0.0	0.0088	0.1288	8.6	0.0000	0.0187	5.0	−0.0074	0.0207	9.5	0.0001	0.0194	5.1
50	10	−0.1001	35.6017	0.0	0.1275	0.7946	8.2	−0.0008	0.0299	5.2	−0.1286	0.1349	94.4	0.0085	0.0456	4.5
50	50	0.0006	0.0511	0.0	0.0091	0.1215	8.2	−0.0004	0.0158	5.2	−0.0294	0.0341	51.3	0.0000	0.0176	5.6
50	100	−0.0001	0.0298	0.0	0.0032	0.0821	8.6	−0.0003	0.0117	5.0	−0.0151	0.0194	33.9	−0.0002	0.0124	5.2
50	200	0.0000	0.0189	0.0	0.0016	0.0558	7.4	0.0000	0.0085	4.9	−0.0074	0.0115	21.9	0.0000	0.0088	5.3
100	10	−0.3243	21.2584	0.0	0.0475	0.3848	8.2	−0.0003	0.0214	5.1	−0.1285	0.1318	99.8	0.0086	0.0329	3.9
100	50	0.0003	0.0367	0.0	0.0013	0.0851	8.4	−0.0003	0.0113	5.2	−0.0291	0.0316	77.2	0.0003	0.0126	4.7
100	100	0.0002	0.0212	0.0	0.0012	0.0577	8.1	−0.0001	0.0083	5.1	−0.0148	0.0172	52.5	0.0000	0.0089	5.1
100	200	0.0002	0.0136	0.0	0.0003	0.0399	7.6	0.0000	0.0060	5.1	−0.0074	0.0097	33.3	0.0000	0.0063	5.2
200	10	−0.1975	22.3755	0.0	0.0245	0.2235	6.7	−0.0003	0.0154	5.2	−0.1289	0.1305	100.0	0.0082	0.0237	3.1
200	50	0.0003	0.0256	0.0	0.0026	0.0595	7.5	0.0000	0.0080	5.2	−0.0290	0.0303	95.7	0.0004	0.0090	5.3
200	100	0.0002	0.0149	0.0	0.0011	0.0407	7.3	0.0000	0.0059	4.7	−0.0147	0.0159	77.6	0.0002	0.0063	5.2
200	200	0.0002	0.0096	0.0	0.0008	0.0281	7.3	0.0000	0.0042	4.5	−0.0074	0.0086	52.3	0.0001	0.0044	4.9

Table 2: Bias, RMSE and 5% size results for F1 when $\rho_0 = 0.95$.

		AHL			AHD			FA			LS			BCLS		
N	T	Bias	RMSE	Size	Bias	RMSE	Size	Bias	RMSE	Size	Bias	RMSE	Size	Bias	RMSE	Size
10	10	0.0014	0.0417	0.0	0.0014	0.0623	0.3	−0.0001	0.0184	5.5	−0.0159	0.0265	19.3	0.1775	0.1791	0.0
10	50	0.0002	0.0159	0.0	0.0020	0.0684	1.3	−0.0001	0.0036	5.0	−0.0019	0.0043	12.4	0.0371	0.0373	0.0
10	100	0.0000	0.0150	0.0	0.0058	0.1007	3.1	−0.0001	0.0027	5.1	−0.0013	0.0031	11.5	0.0182	0.0184	0.0
10	200	0.0000	0.0160	0.0	0.0101	0.1537	5.4	−0.0001	0.0023	5.4	−0.0010	0.0026	11.8	0.0087	0.0090	0.0
50	10	0.0005	0.0205	0.0	0.0008	0.0327	0.2	0.0001	0.0090	5.0	−0.0195	0.0221	58.3	0.1736	0.1739	0.0
50	50	−0.0001	0.0078	0.0	0.0004	0.0384	1.8	0.0000	0.0018	4.6	−0.0023	0.0030	32.8	0.0367	0.0367	0.0
50	100	0.0000	0.0076	0.0	0.0020	0.0570	3.4	0.0000	0.0013	4.7	−0.0015	0.0021	30.3	0.0179	0.0180	0.0
50	200	0.0001	0.0084	0.0	0.0035	0.0855	4.9	0.0000	0.0011	4.6	−0.0012	0.0017	27.0	0.0085	0.0086	0.0
100	10	−0.0002	0.0146	0.0	0.0003	0.0232	0.3	−0.0001	0.0065	5.3	−0.0200	0.0213	85.1	0.1730	0.1732	0.0
100	50	−0.0001	0.0057	0.0	0.0010	0.0277	1.8	−0.0001	0.0013	5.0	−0.0023	0.0027	53.7	0.0366	0.0366	0.0
100	100	0.0000	0.0054	0.0	0.0010	0.0407	3.3	0.0000	0.0009	4.5	−0.0015	0.0018	46.6	0.0180	0.0180	0.0
100	200	−0.0001	0.0059	0.0	0.0024	0.0623	4.7	0.0000	0.0008	4.8	−0.0012	0.0015	43.0	0.0085	0.0086	0.0
200	10	−0.0002	0.0105	0.0	0.0000	0.0169	0.4	0.0000	0.0046	4.8	−0.0203	0.0210	98.8	0.1726	0.1727	0.0
200	50	0.0000	0.0040	0.0	0.0000	0.0203	1.8	0.0000	0.0009	4.9	−0.0024	0.0026	78.1	0.0366	0.0366	0.0
200	100	0.0000	0.0039	0.0	0.0003	0.0298	3.1	0.0000	0.0007	5.1	−0.0016	0.0017	71.2	0.0179	0.0179	0.0
200	200	−0.0001	0.0043	0.0	0.0003	0.0455	4.8	0.0000	0.0006	4.4	−0.0013	0.0014	68.5	0.0085	0.0085	0.0

Table 3: Bias, RMSE and 5% size results for F2 when $|\rho_0| < 1$.

		$\rho_0 = 0$			$\rho_0 = 0.5$			$\rho_0 = 0.95$		
N	T	Bias	RMSE	Size	Bias	RMSE	Size	Bias	RMSE	Size
10	10	0.0006	0.0859	5.5	−0.0010	0.0635	6.1	−0.0002	0.0245	6.0
10	50	0.0007	0.0431	4.6	−0.0008	0.0326	5.4	0.0000	0.0042	4.5
10	100	0.0004	0.0306	4.6	−0.0003	0.0248	5.0	0.0000	0.0027	4.9
10	200	0.0004	0.0218	4.8	−0.0001	0.0181	4.7	−0.0001	0.0020	5.3
50	10	0.0008	0.0396	4.9	0.0002	0.0293	5.3	0.0001	0.0117	5.2
50	50	0.0000	0.0192	5.0	−0.0002	0.0146	5.3	0.0000	0.0020	4.8
50	100	−0.0002	0.0139	5.5	−0.0003	0.0112	4.9	0.0000	0.0013	5.5
50	200	0.0000	0.0099	7.6	−0.0001	0.0083	5.0	0.0000	0.0009	4.7
100	10	0.0003	0.0287	5.4	0.0002	0.0212	5.6	0.0000	0.0084	5.1
100	50	0.0000	0.0136	4.8	−0.0001	0.0105	4.5	0.0000	0.0014	4.9
100	100	0.0000	0.0099	6.5	−0.0001	0.0079	5.3	0.0000	0.0009	4.9
100	200	0.0001	0.0071	7.1	0.0000	0.0059	5.2	0.0000	0.0007	5.0
200	10	−0.0001	0.0201	5.0	−0.0003	0.0147	5.2	−0.0002	0.0058	5.1
200	50	0.0000	0.0098	5.2	−0.0001	0.0075	4.9	0.0000	0.0010	4.7
200	100	0.0001	0.0071	5.9	0.0000	0.0057	5.0	0.0000	0.0006	4.6
200	200	0.0001	0.0050	6.9	0.0000	0.0042	4.8	0.0000	0.0005	5.1

Table 4: Bias, RMSE and 5% size results for F3 when $|\rho_0| < 1$.

		$\rho_0 = 0$			$\rho_0 = 0.5$			$\rho_0 = 0.95$		
N	T	Bias	RMSE	Size	Bias	RMSE	Size	Bias	RMSE	Size
10	10	0.0018	0.1201	6.2	−0.0138	0.1191	6.5	−0.0463	0.1316	8.0
10	50	0.0010	0.0480	5.6	−0.0015	0.0429	5.9	−0.0063	0.0237	7.2
10	100	0.0005	0.0338	5.5	−0.0006	0.0299	5.8	−0.0031	0.0145	7.1
10	200	0.0006	0.0234	5.6	−0.0002	0.0207	5.6	−0.0015	0.0089	7.2
50	10	−0.0001	0.0517	5.5	−0.0039	0.0513	5.5	−0.0108	0.0460	3.2
50	50	−0.0004	0.0205	5.3	−0.0008	0.0184	5.5	−0.0010	0.0091	5.3
50	100	−0.0002	0.0145	5.7	−0.0003	0.0128	5.2	−0.0006	0.0059	4.7
50	200	0.0001	0.0103	7.4	0.0000	0.0090	5.2	−0.0003	0.0037	5.3
100	10	−0.0006	0.0367	5.0	−0.0024	0.0364	5.3	−0.0063	0.0307	3.5
100	50	0.0000	0.0145	4.8	−0.0002	0.0129	4.7	−0.0005	0.0063	4.8
100	100	−0.0001	0.0102	5.9	−0.0002	0.0089	5.0	−0.0004	0.0041	5.4
100	200	0.0001	0.0074	7.9	0.0000	0.0064	5.2	−0.0001	0.0026	5.3
200	10	0.0003	0.0256	4.6	−0.0012	0.0255	5.2	−0.0035	0.0207	4.1
200	50	0.0001	0.0105	5.7	−0.0001	0.0092	4.9	−0.0002	0.0045	4.5
200	100	0.0001	0.0074	6.6	0.0000	0.0064	5.1	−0.0001	0.0029	4.4
200	200	0.0001	0.0051	6.7	0.0000	0.0045	4.6	−0.0001	0.0018	4.7

Table 5: Bias, RMSE and 5% size results for F1–F3 when $\rho_0 = 1$ in C1.

		F1			F2			F3		
N	T	Bias	RMSE	Size	Bias	RMSE	Size	Bias	RMSE	Size
10	10	0.0001	0.0152	5.3	−0.0001	0.0206	5.7	−0.0543	0.1430	9.0
10	50	0.0000	0.0013	4.9	0.0000	0.0018	4.5	−0.0085	0.0230	7.6
10	100	0.0000	0.0005	5.1	0.0000	0.0006	4.9	−0.0053	0.0133	9.3
10	200	0.0000	0.0002	4.3	0.0000	0.0002	5.3	−0.0026	0.0068	9.5
50	10	0.0001	0.0074	5.1	0.0001	0.0099	5.3	−0.0123	0.0483	2.7
50	50	0.0000	0.0007	4.4	0.0000	0.0008	4.8	−0.0012	0.0065	4.0
50	100	0.0000	0.0002	5.3	0.0000	0.0003	5.1	−0.0009	0.0038	3.6
50	200	0.0000	0.0001	4.3	0.0000	0.0001	4.9	−0.0004	0.0020	3.9
100	10	0.0000	0.0054	5.6	0.0000	0.0071	5.1	−0.0066	0.0299	2.5
100	50	0.0000	0.0005	4.9	0.0000	0.0006	4.5	−0.0006	0.0043	4.0
100	100	0.0000	0.0002	4.3	0.0000	0.0002	4.7	−0.0004	0.0025	3.8
100	200	0.0000	0.0001	4.7	0.0000	0.0001	4.5	−0.0002	0.0013	3.5
200	10	0.0000	0.0038	5.1	−0.0001	0.0049	4.9	−0.0036	0.0195	3.5
200	50	0.0000	0.0003	4.7	0.0000	0.0004	4.6	−0.0002	0.0030	4.2
200	100	0.0000	0.0001	4.7	0.0000	0.0002	4.7	−0.0002	0.0016	3.9
200	200	0.0000	0.0000	4.8	0.0000	0.0001	5.2	−0.0001	0.0009	3.9

Table 6: Bias, RMSE and 5% size results for F1–F3 when $\rho_0 = 1$ in C2.

		F1			F2			F3		
N	T	Bias	RMSE	Size	Bias	RMSE	Size	Bias	RMSE	Size
10	10	−0.0186	0.0696	3.2	−0.0294	0.1002	4.5	−0.0137	0.0588	7.6
10	50	−0.0030	0.0112	6.4	−0.0032	0.0118	6.02	−0.0031	0.0117	8.14
10	100	−0.0015	0.0054	6.5	−0.0015	0.0055	6.28	−0.0016	0.0057	8.16
10	200	−0.0007	0.0027	6.36	−0.0007	0.0027	6.36	−0.0007	0.0028	7.74
50	10	−0.0037	0.0241	4.88	−0.0043	0.0278	3.48	−0.0025	0.0218	5.44
50	50	−0.0005	0.0042	5.2	−0.0005	0.0043	5.14	−0.0004	0.0041	5.42
50	100	−0.0002	0.0021	5.38	−0.0003	0.0021	5.34	−0.0003	0.0021	5.74
50	200	−0.0001	0.0010	5.48	−0.0001	0.0010	5.56	−0.0001	0.0010	5.78
100	10	−0.0019	0.0168	5.46	−0.0025	0.0191	4.4	−0.0014	0.0152	5.38
100	50	−0.0003	0.0029	5.58	−0.0003	0.0030	5.32	−0.0003	0.0029	5.58
100	100	−0.0001	0.0015	5.64	−0.0001	0.0015	5.38	−0.0001	0.0014	5.58
100	200	−0.0001	0.0007	5.18	−0.0001	0.0007	5.02	−0.0001	0.0007	5.28
200	10	−0.0006	0.0117	4.8	−0.0009	0.0131	4.08	−0.0004	0.0108	5.5
200	50	−0.0001	0.0021	5.06	−0.0001	0.0021	4.78	−0.0001	0.0020	5.42
200	100	0.0000	0.0010	4.7	0.0000	0.0010	4.86	0.0000	0.0010	4.88
200	200	0.0000	0.0005	5.1	0.0000	0.0005	5.1	0.0000	0.0005	5.28

The end

Thank you for listening!