Abstract

This article proposes that technological progress in financial intermediation can make the economy more fragile if it is not accompanied by a proportional degree of technological progress in the real sector. In the model economy discussed, firms are operated by heterogeneous managers who differ in their ability to run a successful project. Systemic risk is defined by the characteristics of the most risky firms. If financial development outpaces technological advances in the real sector, the price of financial services decreases, allowing riskier producers to enter the market. It is concluded that commensurate rates of technological development in both the financial and real sectors are necessary for stable and balanced economic growth.

Keywords: financial development; technological progress; firm size.

JEL Classification Numbers: G01; O11; O16.
1 Introduction

In most theories that aim to explain the role of financial development in economic growth, greater financial development is considered inevitably better. It reduces agency costs and information asymmetries, facilitates risk sharing, and allocates resources to more efficient users. Recently, the negative aspects of financial innovation, such as increased fragility in the economic system, have been gaining attention in the literature (see, for example, Allen and Gale, 2004; Allen and Carletti, 2006; Wagner, 2007; Gennaioli, Shleifer, and Vishny 2012).

This paper proposes a simple model whereby financial development increases systemic risk if it is not accompanied by a proportional degree of development in the real sector of the economy. The outcome of this theoretical model embraces recent discussions of the trade-offs between growth and fragility by explicitly relating technological progress in the financial and real sectors of the economy to systemic risk and economic growth. The paper uses the standard approach to model financial intermediation, as postulated by Townsend (1979) and Williamson (1986): the financial intermediaries arise to screen and monitor the borrowers. As in Greenwood, Sanchez, and Wang (2010), the outcome of monitoring is random. The probability of detecting fraud depends on the amount of resources invested and on the state of monitoring technology. The novel element of this paper is the attention paid to the risk-taking resulting from the relative efficiency of the financial sector and the real sector, that is, the sector that uses the financial intermediaries’ services.

In the model, the relative technological progress in the financial and real sectors defines the set of firms that are active in the markets for funds and final goods. Financial development makes monitoring more efficient and increases the probability of detecting fraud, thus reducing the costs of borrowing and allowing less efficient firms to participate in the funds market. Technological development in the real sector increases the demand for funds, thus increasing their price, and reduces the number of active firms. (In particular, technological development prevents the least efficient firms that face a higher cost of borrowing from entering the markets.)

The main assumption underlying the results is that all firms operating in either the financial or real sector of the economy exhibit decreasing returns due to the span of control (Lucas, 1978) and differ in their probability of successful production. Riskier firms face higher borrowing costs, and if active, are smaller in size. The role of such heterogeneity as a source of system fragility is explored.

The measure of systemic risk is defined by the aggregate probability of financial crisis, which is a function of the probability of default by the most risky firm operating in the market. Such a
schematic representation of system fragility reflects the recent findings on the role of individual institutions and their networks in systemic risk and financial contagion (see, for example, Allen and Gale, 2000; Cont and Moussa, 2009; Bluhm and Krahnen, 2012; Anand, Gai, and Marsili, 2012). The network is not considered explicitly, but it is taken as given that such a small shock (coming from a default by the most risky firm) can lead to a failure of the whole system. Systemic risk is not internalized by the banks because of the limited liability, as discussed by Allen and Gale (2004).

Recent studies provide empirical support for the positive association between financial development (proxied by the amount of credit to GDP) and system fragility. Rajan (2005) discussed how financial development can encourage the intermediaries to undertake more risks. Mendoza and Terrones (2012) found that credit booms lead to an increased probability of financial crises. Beck et al. (2012) discovered that financial innovation can be associated with higher growth volatility and higher idiosyncratic bank fragility. Gourinchas and Obstfeld (2011) suggested that domestic credit expansion is one of the most robust and significant predictors of financial crises. Credit is usually measured in terms of the ratio to GDP. In this paper, credit expansion is considered possible when the technological progress in the financial sector exceeds that in the real sector. There is evidence that such credit expansion has taken place recently. The slowdown of technological progress in the 1980s and 1990s was accompanied by a credit boom in the U.S. (see Figure 1).

![Figure 1. Total factor productivity (total industries) and domestic credit to private sector (% of GDP) in the U.S.A.; sources: EU-KLEMS and World Bank.](image)

Many empirical studies support the assumption of the paper that small firms are more risky (see, for example, Mansfield, 1962; Evans, 1987; Mata and Portugal, 1994; Persson, 1994;
Audretsch, 1995; Guimaraes, Mata, and Portugal, 1995; Geroski, 1995). Indeed, if both riskier and more stable firms maintain constant size or grow over time, riskier firms are, on average, more likely to shut down before reaching a sufficiently large size.

Finally, it is a stylized fact that financial intermediaries do not internalize systemic risk, due to different kinds of externalities (as, for example, in Beale et al., 2011) or moral hazard (as, for example, in Bhattacharya, 1982). The simple model considered in this paper abstracts from the issues of capital requirements and insurance of deposits. As shown by Rochet (1992) and Hellmann, Murdock, and Stiglitz (2000), the regulation of banks’ reserve requirements alone cannot solve the moral hazard problem faced by the banks; neither is the insurance of deposits a panacea (Smith 1984; Allen and Gale, 2004).

The paper proceeds as follows. In Section 2, the basic overlapping generation model with heterogeneous firms operating in the real and financial sectors is described; the existence and uniqueness of the competitive equilibrium for the model economy is stated; and balanced and unbalanced growth paths are defined. Section 3 discusses the role of firms’ heterogeneity in the model for systemic risk, growth, and stability. Section 4 summarizes and concludes the paper. All proofs are in the appendix.

2 The Model

The model economy consists of overlapping generations of heterogeneous agents. Each generation lives for two periods and is composed of two groups of agents, each group being of measure one. The first group represents “potential producers,” the individuals who are able to run a firm that produces output of the final good. The second group consists of “potential bankers,” the individuals who are able to run a financial intermediary institution. The individuals from both groups can be hired as workers in either the productive or financial sector. The groups do not have common members.

All agents are risk-neutral, born with zero assets, and work only in the first period of their life. They may save out of their first-period income to consume during the second period of their life. The only source of heterogeneity across agents within a group is their ability to be a successful entrepreneur: to run a firm in the final goods sector if the individual belongs to group one, or to run an intermediary institution if the individual belongs to group two. The distribution of abilities in each group is time-invariant and characterized by cumulative distribution function $F(z)$ and probability density function $f(z)$ for $z \in [z, \bar{z}]$.

The final good is assumed to be perfectly storable and transformable into capital at zero
cost.

At the beginning of the first period of their life, the individuals decide whether to become an entrepreneur in their group or to be hired in the labor market as a worker. Those who decide to become entrepreneurs have a span of control to operate a decreasing returns-to-scale technology and to choose the optimal amount of capital and labor inputs to hire, given expectations about the output that they can produce. The output is uncertain, with the probability of success depending on the ability of the entrepreneur. The members of the two groups interact in the competitive markets. The “bankers” intermediate transfers of capital from savers to “producers,” and all entrepreneurs hire labor. The financial intermediaries arise to mitigate information asymmetries and to screen entrepreneurs—the tasks the savers cannot perform. All agents receive their income and decide on savings at the end of the first period of their life.

The problem of the individuals in each group, the role of abilities, and the markets are described in more detail below.

The problem of a “potential producer”

Each individual from the group of potential producers decides whether to run a firm and produce output in the form of final goods, or to be hired as a worker in the labor market. The decision is made based on the expected payoffs of these occupational choices.

The technology that a potential producer can operate has the following form:

\[ A(k^a l^{1-a})^q, \]

where \( k \) and \( l \) are capital and labor hired by the entrepreneur; \( a \in (0, 1) \) is the capital share; \( q \in (0, 1) \) is the span of control parameter; and \( A > 0 \) represents the real sector’s state of technology.

Given that entrepreneurs start life with zero assets, they have to borrow capital to run their firms. The borrowing is complicated by two factors: the ultimate success of the entrepreneur’s project is uncertain, and the entrepreneur can hide the final outcome of his production project.

The probability of success of the project \( \pi(z) \) depends on the ability of the potential producer \( z \). In particular,

\[ \pi(z) = 1 - z^{-v}, \quad v > 0. \]

The financial intermediaries (individuals from the second group who chose to become entrepreneurs) are able to identify the ability of the producers, and thus, to estimate \( \pi(z) \) correctly.
They issue loans at the risk-adjusted competitive interest rate \( r_e/\pi(z) \).

The producer borrows capital from the financial intermediaries and hires labor at a competitive wage \( w \), before he knows if his project is successful. Once the firm’s inputs are employed, a random draw from uniform distribution on \([0,1]\) determines if the project is successful, with probability of success being \( \pi(z) \). The entrepreneur can hide the successful realization of his project with probability \( 1 - P \), which depends on the ability of his financial intermediaries and is defined below. If the project is successful, the entrepreneur produces final goods according to the technology (1) and repays the loan conditional on successful monitoring by the intermediaries. If the project is unsuccessful, the entrepreneur announces bankruptcy and does not repay the loan to the financial intermediaries. For simplicity, the liquidation value of the bankrupt firm is zero.

The maximization problem of the producer is the following:

\[
\max_{k,l} \text{E}\Pi_e = \pi(z) \left[ (k^a l^{1-a})^a A - \frac{r_e P}{\pi(z)} k \right] - wl. \tag{3}
\]

The first order conditions are:

\[
[k] : aqk^{aq-1}l^{(1-a)q} A = \frac{r_e P}{\pi(z)}. \tag{4}
\]

\[
[l] : \pi(z)(1 - a)qk^{aq}l^{(1-a)q-1} A = w. \tag{5}
\]

The ratio \( h_e \) of capital to labor demand is a function of the entrepreneur’s ability and prices:

\[
h_e = \frac{k}{l} = \frac{aw}{(1-a)r_e P}. \tag{6}
\]

If the project is successful, the entrepreneur hires labor according to the labor demand:

\[
l(z) = L_e \pi(z)^{\frac{1}{1-a}}, \tag{7}
\]

where

\[
L_e = \left( \frac{w^{1-aq}r_e^{aq} P^{aq}}{qA(1-a)^{1-aq^{aq}}} \right)^{\frac{1}{q-1}}. \tag{8}
\]

The expected profits of the potential entrepreneur can be expressed as follows:

\[
\text{E}\Pi_e(z) = \pi(z)^{\frac{1}{1-a}} \frac{wL_e}{(1-a)} \left( \frac{1}{q} - 1 \right). \tag{9}
\]
Each potential producer decides whether to undertake an entrepreneurial project with expected payoff $E\Pi_e$ or to become a worker with expected payoff $w$. Given that the expected profits are monotone increasing in ability, there is a threshold ability $z_e^*$ above which all potential producers undertake an entrepreneurial project, and below which all potential producers become workers:

$$w = E\Pi_e(z_e^*),$$  \hspace{1cm} (10)$$

$$z_e^* = \left(1 - L_e^{-1} \left(\frac{\frac{1}{q} - 1}{1 - a}\right)^{q-1}\right)^{-1/v}. \hspace{1cm} (11)$$

The following inverse relationship will be useful below:

$$L_e = \pi(z_e^*)^{-\frac{1}{1-q}} \frac{1 - a}{z_e^* - 1}. \hspace{1cm} (12)$$

Given $z_e^*$, the total expected profits of all operating producers are given by:

$$TotalE\Pi_e = \frac{L_e w}{(1 - a)} \left(\frac{1}{q} - 1\right) \int_{z_e^*}^{\tilde{z}} \pi(z)^{-\frac{1}{1-q}} f(z) d(z). \hspace{1cm} (13)$$

The total supply of labor from the group of potential producers is given by:

$$\int_{z_e^*}^{\tilde{z}} f(z) dz. \hspace{1cm} (14)$$

The problem of a “potential banker”

Each individual from the group of potential bankers decides whether to run a financial intermediary institution or to be hired as a worker in the labor market. The decision is made based on the expected payoffs of these occupational choices. If the potential banker runs a financial intermediary institution, he can make profits by intermediating the funds from savers to borrowers, the operating producers. The bankers buy deposits $d$ on the deposits market at a competitive deposit interest rate $r_b$ and sell loans to the producers at the competitive loan interest rate as defined above. Each potential banker can operate a common to the financial sector technology, which allows him to correctly identify the ability of the borrower to run a firm and to monitor borrowers to reduce the probability $1 - P$ of their hiding the successful realizations of projects.

Monitoring requires labor input; therefore the bankers also hire workers in the labor market. The success of the monitoring depends positively on the banker’s ability $z$ and labor input $x$. 

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and depends negatively on the volume of intermediated funds $d$. In particular (similar to Greenwood, Sanchez, and Wang, 2010),

$$P = \begin{cases} 1 - \left( \frac{1}{(zT/d)^\psi} \right)^{1/\gamma}, & \frac{T\gamma}{d} > \frac{1}{z}, \\ 0, & \frac{T\gamma}{d} \leq \frac{1}{z}, \end{cases} \text{ with } \psi, \gamma \in (0, 1), \quad (15)$$

where $T > 0$ represents the financial sector’s state of technology. The inequalities imply that the technology-augmented labor effort for monitoring, adjusted for the amount of resources monitored, must exceed the inverse of the ability of the banker to insure a positive probability of successful monitoring.

Note that the technology of the financial sector $T$ includes the factors that make the financial monitoring more efficient, and as formulated, is incomparable with the Solow residual of the constant returns-to-scale production function, commonly reported as an estimate of the sector technology. Possible sources of growth in $T$ will be discussed in the next section.

The maximization problem of the banker is the following:

$$\max_{d, x} E\Pi_b = \left( 1 - \frac{1}{(zT/d)^\psi} \right) r_e d - r_b d - wx, \quad (16)$$

$$\text{s.t.: } Tx^\gamma/d > 1/z.$$  

The first order conditions are (the constraint is not binding):

$$[d] : (1 + \psi)(zTx^\gamma)^{-\psi} r_e d^\psi = r_e - r_b, \quad (17)$$
$$[x] : \psi^\gamma(zT)^{-\psi} r_e d^{\psi+1} x^{-\psi-1} = w. \quad (18)$$

The ratio $h_b$ of deposits to labor demand is a function of the banker’s ability and prices:

$$h_b = \frac{d}{x} = \frac{(1 + \psi)w}{\psi^\gamma(r_e - r_b)}. \quad (19)$$

The banker hires labor according to the labor demand:

$$x(z) = L_b z^{1/\gamma}, \quad (20)$$

where

$$L_b = \left( \frac{w r_e^{\frac{1}{\psi}} (1 + \psi)^{1/\psi+1}}{\psi^\gamma T (r_e - r_b)^{1/\psi+1}} \right)^{\frac{1}{\gamma-1}}. \quad (21)$$

The expected profits of the potential banker can be expressed as:
Each potential banker decides whether to run an intermediary institution with expected payoff \( E\Pi_b \) or to become a worker with expected payoff \( w \). There is a threshold ability \( z^*_b \) above which all potential bankers run a financial intermediary institution, and below which all potential bankers become workers:

\[
E\Pi_b(z) = z^{\frac{1}{\gamma}} L_b w \left( \frac{1}{\gamma} - 1 \right). 
\]  

(22)

The following inverse relationship will be useful below:

\[
z^*_b = L_b^{\gamma-1} \left( \frac{1}{\gamma} - 1 \right)^{\gamma-1}. 
\]  

(24)

The total expected profits of all operating bankers are given by:

\[
Total E\Pi_b = L_b w \left( \frac{1}{\gamma} - 1 \right) \int_{\bar{z}}^{\tilde{z}} z^{\frac{1}{\gamma}} f(z) dz. 
\]  

(26)

For positive interest rate spread, \( r_e - r_b \), (28) is bounded between zero and one.

Therefore, at optimum, the probability of successful monitoring is the same across all active financial intermediaries. The intermediaries with less ability to monitor borrowers will optimally choose to intermediate fewer funds.

The common probability of successful monitoring makes all financial intermediaries identical from the point of view of both savers and borrowers. The set of active financial intermediaries represents a homogeneous financial system that accepts deposits and issues loans, performing screening and monitoring along the way. Depositors can invest in, and producers can borrow from, several financial intermediaries within a period.
The saving decision

At the end of the first period of their life, all individuals in the economy decide whether to save some of their income for the second period of life. Assume the discount rate is 1. Given that the individuals are risk-neutral, they will save all of their income if the expected rate of return is positive. Assuming this is the case, the total supply of funds in the deposit market at a given period of time is given by the sum of total realized profits (in expectation equal to the total expected profits) of all the workers, bankers, and producers, operating during the preceding period of time.

Assumptions

To facilitate the characterization of equilibrium in this economy, several assumptions must be imposed. The first assumption imposes a particular distribution of abilities. The second assumption ensures that, given the assumed distribution, all expected profits are finite and positive. The third assumption will be useful in the discussion of the uniqueness of equilibrium in the model economy.

Assumption 1: Abilities in each group follow Pareto distribution of the following form:
\[ F(z) = 1 - z^{-\psi}, f(z) = \psi z^{-\psi - 1}, \bar{z} = 1, \bar{z} = \infty. \]

Assumption 2: \(-\psi + 1/(1 - \gamma) < 0, v(1 - a)/(v - a) > 1,\)

Assumption 3: \(1/\psi + 1 - 1/v - 1/(1 - aq) > 0, (1 - aq)^2(v - 1) - va \gamma q > 0, (1 - aq)/(1 - aq)(1/\psi + 1 - 1/v - aq) > 1.\)

The equilibrium

In equilibrium, all markets clear. In particular, at the edge of every two periods the total amount of deposits collected by the newly born financial intermediaries is equal to the total realized profits of all agents who earned labor or entrepreneurial income and are moving to the second period of their life:

\[ \text{Deposits Market :} \int_{z_b}^{\bar{z}} d(z) f(z) dz = \text{Total} \Pi_c + \text{Total} \Pi_b + \text{Total} \Pi_w. \tag{29} \]

During each period, the loans market clear: the total amount of loans offered by financial intermediaries is equal to the total demand for capital by the producers:

\[ \text{Loans Market :} \int_{z^*_e}^{\bar{z}} k(z) f(z) dz = \int_{z_b}^{\bar{z}} d(z) f(z) dz. \tag{30} \]
The labor market clears: the total demand of labor by producers and financial intermediaries is equal to the total supply of labor:

\[
\text{Labor Market: } \int_{z_e}^{\bar{z}} l(z) f(z) dz + \int_{z_b}^{\bar{z}} x(z) f(z) dz = \int_{z}^{\bar{z}} f(z) dz + \int_{z}^{\bar{z}} f(z) dz. \tag{31}
\]

The market-clearing conditions define the prices \( r_b, r_e, \) and \( w. \)

More formally, a competitive equilibrium is defined as follows.

**Definition:** A competitive equilibrium given \( A \) and \( T \) is described by the thresholds \( z^*_e, z^*_b, \) allocations \( \{k(z), l(z)\}^{\bar{z}}_{z_e}, \{d(z), x(z)\}^{\bar{z}}_{z_b}, \) wages \( w, \) and interest rates \( r_e, r_b, \) such that,
- given \( w, r_e, r_b, \) all agents maximize their utility by choosing their occupation and savings;
- given \( w, r_e, r_b, \) all producers and bankers maximize their profits; and
- the markets for capital, labor, and deposits clear.

Given the assumption of the distribution of abilities, the market equilibrium conditions can be rewritten in a more compact form. The labor demand in each sector is given by the following:

\[
L(z^*_e) = \int_{z_e}^{\bar{z}} l(z) f(z) dz = \frac{(1 - a)q}{2 - q} \left( \pi(z^*_e)^{1-q} - \pi(z^*_e) \right). \tag{32}
\]

\[
X(z^*_b) = \int_{z_b}^{\bar{z}} x(z) f(z) dz = \frac{\gamma w}{v(1 - \gamma) - 1} z^*_{e}^{\gamma - v}. \tag{33}
\]

The demand for loans and deposits is proportional to labor demand in both sectors:

\[
\int_{z_e}^{\bar{z}} k(z) f(z) dz = \frac{aw(1 + \psi)}{(1 - a)(\psi r_e + r_b)} L(z^*_e). \tag{34}
\]

\[
\int_{z_b}^{\bar{z}} d(z) f(z) dz = \frac{(1 + \psi)w}{\psi \gamma (r_e - r_b)} X(z^*_b). \tag{35}
\]

The expected entrepreneurs’ profits are also proportional to labor demand:

\[
TotalE\Pi_e = \frac{1}{q - 1} w L(z^*_e), \tag{36}
\]

\[
TotalE\Pi_b = w \left( \frac{1}{\gamma} - 1 \right) X(z^*_b). \tag{37}
\]

The workers’ profits and labor supply can be rewritten in a similar fashion:

\[
W(z^*_e, z^*_b) = 2 - z^*_{e}^{v} - z^*_{b}^{v}. \tag{38}
\]

\[
TotalE\Pi_w = w W(z^*_e, z^*_b). \tag{39}
\]
The market-clearing conditions simplify as follows:

**Deposits Market**:
\[
\frac{(1 + \psi)X(z_b^*)}{\psi\gamma(r_e - r_b)} = \Pi(z_e^*, z_b^*),
\]
(40)

**Loans Market**:
\[
\frac{aL(z_e^*)}{(1 - a)(\psi\gamma(r_e - r_b))} = \frac{X(z_e^*)}{\psi\gamma(r_e - r_b)},
\]
(41)

**Labor market**:
\[
L(z_e^*) + X(z_b^*) = W(z_e^*, z_b^*),
\]
(42)

where
\[
\Pi(z_e^*, z_b^*) = \frac{1}{1 - a} L(z_e^*) + \left(1 - \frac{1}{\gamma} \right) X(z_e^*) + W(z_e^*, z_b^*).
\]
(43)

The function \(\Pi(z_e^*, z_b^*)\) can be viewed as a total “volume” of expected profits in terms of population shares. \(\Pi(z_e^*, z_b^*)\) multiplied by wage \(w\) represents the total expected profits.

The total expected output in this economy is given by the sum of all profits, or by total expected output in the real sector plus the interest accumulated on savings. Using (1), (7), (12) and simplifying:

\[
Y = w \left( \pi(z_e^*)^\frac{1}{1-\gamma} - 1 \right) + r_b w \Pi(z_e^*, z_b^*).
\]
(44)

It is possible to show that there exists a unique equilibrium with positive interest rates in the described economy.

**Proposition 1**: Given \(A\), for all \(T > \left(\psi \frac{\gamma}{\gamma - 1} \right)^{\frac{1}{q}} \left[ \frac{q A^{1+q}}{1-q} + A^{1-q} \right]^{\frac{1}{1-q}} \left( \frac{1}{1-q} - \frac{1}{1-\gamma} \right)^{\frac{1}{1-\gamma}}\), the equilibrium with positive interest rates and positive interest rate spread exists and is unique.

From now on, only the equilibrium with positive interest rates and positive interest rate spread will be analyzed. Given that there exists a solution for the economy at a given point in time, it is possible to construct a growth path using this solution. Depending on the relative pace of technological progress in the real and financial sectors, the growth path may be balanced or unbalanced. The propositions below characterize the corresponding behavior of economic variables over time.

**Balanced growth**

**Proposition 2**: Let \(T\) grow at rate \(g\) and \(A\) grow at rate \((1 + g)^{1 - aq} - 1\). There exists a balanced growth path where the wages, output, capital demand, and deposits all grow at rate \(g\). The thresholds \(z_e^*\) and \(z_b^*\), labor demand and supply remain constant.

This result is similar to the conclusion of Greenwood, Sanchez, and Wang (2012) that a balanced development of the real and financial sectors does not make the financial sector more
efficient. The probability of catching the firm that misrepresents its earnings is constant over time. The number of active firms in both sectors does not change over time.

Unbalanced growth

An unbalanced growth path occurs whenever technology in either sector outpaces the balanced growth of the other sector’s technology. Intuitively, faster technological progress in the financial sector makes it relatively more efficient in comparison to the real sector. The relative cost of monitoring producers drops, leading to relatively higher competition for deposits, higher interest rates for deposits, and crowding out of the least efficient financial intermediaries. At the same time, a greater supply of funds makes borrowing affordable to less efficient producers. This intuition is formalized in the following proposition.

Proposition 3: In equilibrium, given \( A(T) \), \( z_e^* \) decreases (increases) and \( z_b^* \) increases (decreases) with a rise in \( T(A) \); labor demand in the real sector increases (decreases) in \( T(A) \); labor demand in the financial sector decreases (increases) in \( T(A) \); the interest rate on deposits increases (decreases) and the interest rate spread shrinks (expands) with a rise in \( T(A) \).

Corollary 1: Let \( A \) grow at rate \((1 + g)^{1-qa} - 1\), and \( T \) grow at rate \( g' (> (<) g) \) and \( T \), \( g' \) being such that the equilibrium exists for at least \( n \) periods. The threshold \( z_e^* \) increases (decreases) over time, and the threshold \( z_b^* \) and interest rate spread decrease (increase) over time from period 1 to \( n \).

Whenever one sector’s technology growth outpaces that of the other sector’s, the economy experiences unbalanced growth, with an increasing number of riskier entrepreneurs in the slower growing sector and a decreasing number of entrepreneurs in the faster growing sector.

The effect of such unbalanced growth on output is uncertain. From (44), the effect of change in technologies on output can be split into two parts: the change in output due to the change in productive possibilities in the real sector and the change in the interest accumulated on profits. It can be shown that function \( \Pi(z_e^*, z_b^*) \) given by (43) is increasing in \( z_e^* \) for a plausible set of parameters for which \( qa < 1 +qv - v \). The interest rate on savings \( r_b \) is decreasing in \( z_e^* \). The product of \( \Pi(z_e^*, z_b^*) \), \( r_b \), and \( w \) defines the total interest on savings, which can increase or decrease with a change in \( z_e^* \). The derivative of the first term in (44), with respect to \( z_e^* \), is also of ambiguous sign.

Figure 2 plots an example of the equilibrium \( Y \) as a function of \( A \) and \( T \).
Section 3 introduces the notion of systemic risk to the model and discusses the role of the individual firms and the thresholds $z_e^*$ and $z_b^*$ in the growth of sector technologies and financial stability.

3 Firms as a Source of Growth and Instability

The economy described in the previous section had no aggregate uncertainty, and all technological progress was exogenous. This section discusses the thresholds $z_e^*$, $z_b^*$ that define the set of active firms and the interconnections among the active firms as possible sources of growth in the sectors and as a source of aggregate uncertainty.

Growth

Human capital is generally accepted as a source of growth in technology. In line with the literature, the growth in $A$ and $T$ in the model considered in this paper can also be thought to be generated by human capital. Consider each active firm in each sector as a contributor of a unit of knowledge to the total innovations in the given sector during a given period of time. Then, the contributions by all acting firms in sector $i$ at period $t$ is given by $\int_{z_{e,t}^*}^{z_{b,t}^*} f(z) dz, i = \{e, b\}$. There may also be some relationship between innovations in the financial and real sectors (for example, a newly invented computer increases productivity of both sectors’ firms). The growth rates $g_{A,t}$ and $g_{T,t}$ of technologies $A$ and $T$ can be schematically represented as follows:

$$g_{i,t} = G_i\left(z_{e,t}^*, z_{b,t}^*\right), i = \{A, T\}. \quad (45)$$

The functions $G_i()$ may be such that the economy experiences a balanced or unbalanced growth path.

Financial Networks
In accordance with recent studies on the role of networking in the growth and stability of the financial sector, technology $T$ can be viewed as an outcome of the networking of financial intermediaries. The interconnections across the active financial intermediaries facilitate the flow of funds through temporary interbank loans, and, other things being equal, improve monitoring efficiency by releasing resources from other activities, such as searching for liquidity. Temporary interbank loans do not add to the total amount of loans intermediated to the real-sector firms and do not influence the equilibrium conditions, except through $T$.

Schematically, the importance of networks in financial intermediation can be acknowledged in the model through the addition of networks as a part of the financial sector technology. For example, $T' = T(1 + g_{T,t}) + T_N(N)$, where $N$ represents the financial network.

**Instability**

The individual firms and their interconnections may influence the stability of the whole economy.

In the model, thresholds $z^*_e, z^*_b$ define the probability of success of the entrepreneurial projects of the marginal entrepreneur, in both the real and financial sectors. A lower threshold implies that the firms operating in the sector are, on average, riskier. Note that the meaning of riskiness differs across sectors. Lower $z^*_b$ reflects, on average, a lower probability of successful monitoring of the borrowers, given the state of financial sector technology $T$. A lower probability of successful monitoring may result in the redistribution of funds from lenders to borrowers. Lower $z^*_e$ reflects, on average, a greater riskiness of the investments made by the intermediaries. Riskier investments may result in the loss of invested funds. Therefore, intuitively, and as illustrated in Proposition 3, a faster rate of technology growth in the financial sector compared to that in the real sector leads to a supply of funds to riskier firms. Allowing the riskier borrowers access to the loans generated by the interconnected financial system may make the whole system more fragile.

Anand, Gai, and Marsili (2012) demonstrated how the networks of financial intermediaries allow for growth in their balance sheets, while at the same time magnifying the risk of the contagion across the whole financial system in the case of negative news about one participant in the network. In the model, the aforementioned interconnections across the financial intermediaries imply that the greater riskiness of an individual borrower may spread across the entire system. Thus, the systemic risk can be schematically represented as the probability of whole system failure, which is a function of $z^*_e$:

$$
Systemic\ Risk = 1 - P(z^*_e) \in [0, 1], P_{z^*_e} > 0,
$$

(46)
where $P(z_e^*)$ is the probability that the crisis will not occur. The function $P(z_e^*)$ can be viewed as a measure of system stability.

The probability $P(z_e^*)$ evolves together with $z_e^*$ and depends on the law of motion of technological progress in both sectors.

An example of probability $P(z_e^*)$ is presented in Figure 3, where $P(z_e^*) = 1/(1 + e^{p_1-p_2z_e^*})$, $p_1, p_2 > 0$. This function is increasing in the ability of the marginal operating producer, convex from 1 up to the inflection point $p_1/p_2$ and concave on the interval $(p_1/p_2, \infty)$. For this particular example of the measure of systemic risk, a very stable economy may suddenly become fragile if very rapid financial development leads to the entry of many sufficiently risky firms.

![Figure 3. Example of the measure of stability (probability of no-crisis); $P(z_e) = 1/(1 + e^{770-200z_e})$.](image)

To complete the introduction of aggregate uncertainty in the model, assume that crisis leads to a destruction of $1 - \xi$ of total deposits, $\xi \in (0, 1)$. Given that all financial intermediaries are viewed as a homogeneous financial system by the lenders and borrowers, the crisis mechanism can be viewed as follows. The firms and financial intermediaries pre-agree on the loans at the beginning of the period. The loans are issued continuously during the period; given that all financial intermediaries are involved in the interbank loans, if one firm-borrower announces bankruptcy, the financial intermediaries must readjust their balance sheets. If several firms fail, there is a chance that many of the interconnected financial intermediaries will fail and the rest of the “bankers” will have to cut loans, reducing their amount by $\xi$. The total expected output is a function of total loans and will be reduced by factor $\xi^{aq}$.

After the crisis, some (or most) of the interbank links may be lost, so that the financial network becomes more sparse (Anand, Gai, and Marsili, 2012). Given the role of financial
interconnections in financial sector efficiency, a breakdown of financial networks will lead to a decrease in financial sector technology, say by factor $\eta \in (0,1)$.\footnote{The probability of whole system failure, and the parameters $\xi$ and $\zeta$, are implicitly defined by $N$.}

With systemic risk introduced to the economy, the expected total profits of all young individuals are given by

$$((1 - P(z_e^*))\xi + P(z_e^*)\Pi(z_e^*, z_b^*) = (\xi + P(z_e^*)(1 - \xi))\Pi(z_e^*, z_b^*).$$

(47)

The systemic risk is not internalized by the banks (similar to Allen and Gale, 2004). Their maximization problem adjusts as follows:

$$\max_{d,x} E\Pi_b = (\xi + P(z_e^*)(1 - \xi)) \left[ \left(1 - \frac{1}{(T x^d)^{\psi}}\right) r_c d - r_b d - wx \right],$$

(48)

$$s.t. : \ T x^d > 1/z.$$

Therefore, at a given period, the optimal balance of deposits and labor inputs by the financial intermediaries is not affected by the presence of systemic risk. Neither does $P(z_e^*)$ affect the optimal choice of the producers from the real sector within a period, the thresholds $z_e^*, z_b^*$, or the decision of risk-neutral individuals to save at a given period, as long as the expected return on deposits is positive.

The probability $P(z_e^*)$ affects the state of the economy over time and subsequently the evolution of thresholds $z_e^*, z_b^*$ over time. The total output, thresholds, and probability of crisis are now random variables, which depend on the history of realizations of $P(z_e^*), z_e^*, z_b^*$ and the history of crises starting from the initial period of economy life.

**Crises and Cycles**

The model economy with aggregate uncertainty is characterized by a richer dynamic of economic development. Aside from balanced or unbalanced growth paths, the economy may experience crisis episodes. A crisis is defined as follows.

**Definition:** A crisis in the considered economy occurs when the financial sector of the economy collapses: only a fraction $\xi$ of the deposits is intermediated and repaid; only a fraction $\xi^{aq}$ of potential output is realized; and financial sector technology shrinks by factor $\eta$.

The equilibrium in the considered economy with aggregate uncertainty is defined as follows.

**Definition:** A time-$t$ competitive equilibrium in the economy with aggregate uncertainty, given the available technologies $A, T$ and the level of available savings $S(w_{t-1}\Pi(z_{e,t-1}^*, z_{b,t-1}^*))$,
is described by allocations \( \{k(z), l(z)\}_{z<z^*}, \{d(z), x(z)\}_{z>z^*} \), wages \( w_t \), and interest rates \( r_{e,t}, r_{b,t} \), such that
- given \( w_t, r_{e,t}, r_{b,t}, S \), all agents maximize their utility by choosing their occupation and savings;
- given \( w_t, r_{e,t}, r_{b,t}, S \), all producers and bankers maximize their profits;
- the markets for capital, labor, and deposits clear; and
- the crisis occurs with probability \( 1 - P(z^*_e) \).

Note that if the equilibrium exists for a given \( T \) at a given period of time, there is a range of values \( \xi \in [\xi; \bar{\xi}] \subset (0, 1) \) for which the equilibrium with uncertainty exists for at least \( n(\xi, T) \) subsequent crisis-periods.

After the crisis, there are two counteracting forces that affect the number of active firms in the subsequent period. On one hand, a decline in financial sector technology and a fall in available savings (and thus deposits and loans) make loans relatively more expensive and crowds out less efficient producers, increasing the threshold \( z^*_e \). On the other hand, if a fall in savings caused by the crisis is much larger than a fall in financial sector technology, competition among the financial intermediaries can attract more active producers immediately following the crisis, decreasing the threshold \( z^*_e \).

**Proposition 4:** For any \( \eta \in (0, 1) \) there exists a range of \( \xi(\eta) \in (0, 1) \), such that, after the crisis, the number of operating firms in the real sector decreases.

Assume \( \eta \) and \( \xi \) such that, after the crisis, the number of firms in the real sector decreases.

**Corollary 2:** If \( T \) grows at a rate greater than \( g \), where \( (1 + g)^{1-aq} - 1 \) is the growth rate of \( A \), the evolution of the economy is characterized by increasing systemic risk over time; the number of operating firms in the real sector increases over time. If \( A \) grows at a rate greater than \( (1 + g)^{1-aq} - 1 \), where \( g \) is the growth rate of \( T \), the evolution of the economy is characterized by increasing stability; the number of operating firms in the real sector decreases over time.

Figure 4 shows a time plot of output (logarithmic scale) for the case of an economy in which \( v = 1.5; \psi = 0.3; \gamma = 0.3; a = 0.3; q = 0.9; \xi = \eta = 0.5 \); the probability of no crisis from Figure 3; and unbalanced growth of technology \( g_T = 0.1 \int_{z^*_e}^{\bar{\xi}} f(z)dz \), \( g_A = (1 + 0.6g_T)^{1-aq} - 1 \). As constructed, the crisis does not affect the long run growth rate in this economy, causing only temporary decline in output.
Figure 4. Total output.

Figure 5 represents the pattern of the measure of stability of the system over time, which is proportional to the threshold $z_e^*$. The crisis is more probable when the number of operating firms (in particular, small firms) in the real sector increases.

Figure 5. The measure of stability (probability of no crisis).

Figure 6 illustrates the time plot of the interest rate spread for the sample economy. During economic expansions, caused by a disproportionately rapid growth in financial intermediation, the stores of value become more demanded and their price goes up, while the cost of borrowing declines. After the financial system collapse and shrinkage of the total output, deposit interest rates soar but by less than lending rates.
4 Conclusions

Financial development should not be considered as a separate necessary and important determinant of economic growth. As this article has demonstrated, its positive effect on economic performance is possible only in conjunction with coherent development in other spheres of economic activities.

The framework developed in this article can be extended to a more detailed equilibrium model and used to measure the relative importance of technological progress in different industries for economic growth and stability.

One complication that arises in the calibration exercise is the measurement of financial sector technology. The difficulties in accounting for the sources of financial improvement are overcome in the model by an abstract notion of growth due to human capital and networking. The usual measure of total factor productivity (TFP) in the data, the Solow residual, should be modified to be comparable with the measure of technology used in the model.

The questions raised in the article call for government intervention in financial intermediation activities whenever growth in the financial sector significantly exceeds growth in the other (productive) sectors of the economy.
References


Appendix

Proof of Proposition 1. First it will be shown that the equilibrium exists; second, that it is unique.

Express the interest rates from equations (40), (41) defining the equilibrium conditions:

\[ r_e - r_b = \frac{(1+\psi)X(z_b^*)}{\Pi(z_e^*, z_b^*)}, \tag{49} \]
\[ \psi r_e + r_b = \frac{a(1+\psi)L(z_e^*)}{\Pi(z_e^*, z_b^*)}, \]
\[ r_e = \frac{aL(z_e^*) + X(z_e^*)}{(1-a)} \frac{1}{\Pi(z_e^*, z_b^*)}, r_b = \frac{aL(z_b^*) - X(z_e^*)}{(1-a)} \frac{1}{\Pi(z_e^*, z_b^*)}. \tag{50} \]

Using the expressions for interest rates and the expressions for thresholds (11) and (24), the equilibrium conditions can be simplified as follows:

\[ L(z_e^*) + X(z_b^*) = 2 - z_e^{*-\gamma} - z_b^{*-\gamma}, \tag{51} \]
\[ \pi(z_e^*) = \frac{w^{1-aq} \left( \frac{L(z_e^*)}{\Pi(z_e^*, z_b^*)} \right)^{aq} \left( \frac{1}{q} - 1 \right)^{-a}}{qA(1-a)}, \tag{52} \]
\[ z_b^* = \frac{w \left( \frac{a\psi L(z_e^*)}{1-a} + X(z_b^*) \right)^{1/\psi} \Pi(z_e^*, z_b^*)}{TX(z_b^*)^{1/\psi+1}} \left( \frac{1}{\gamma} - 1 \right)^{-1}. \tag{53} \]

The last expression for \( z_b^* \) defines wage as a function of \( z_e^*, z_b^* \):

\[ w = z_b^*/ \left( \frac{ \left( \frac{a\psi L(z_e^*)}{1-a} + X(z_b^*) \right)^{1/\psi} \Pi(z_e^*, z_b^*)}{TX(z_b^*)^{1/\psi+1}} \right)^{1-\gamma} \left( \frac{1}{\gamma} - 1 \right)^{-1}. \tag{54} \]

The labor market clearing condition (51) gives the solution for \( z_b^* \) as a function of \( z_e^* \):

\[ z_b^* = \left( \frac{v - 1}{v(1-\gamma) - 1} \right)^{1/v} \left[ 1 + \pi(z_e^*) - L(z_e^*) \right]^{-1/v}. \tag{55} \]

or

\[ X(z_b^*) = \frac{\gamma v}{v-1} \left[ 1 + \pi(z_e^*) - L(z_e^*) \right]. \tag{56} \]

Plugging \( w \) from (54) and \( z_b^* \) from (55) into (52) which defines \( \pi(z_e^*) \), the equilibrium conditions can be summarized in one equation in terms of \( z_e^* \):

or
\[ F(z_e^*) = \pi(z_e^*) - \frac{\bar{C} \left( 1 + \pi(z_e^*) - L(z_e^*) \right)^{(1-aq)(\frac{1}{\gamma}+1)} L(z_e^*)^{aq}}{\left[ \frac{\alpha}{1-a} - \frac{\gamma v}{v-1} \right] L(z_e^*) + \frac{\gamma v}{v-1} \left[ 1 + \pi(z_e^*) \right]^{(1-aq)/\psi} \left( \frac{a v}{v-1} \left[ 1 + \pi(z_e^*) \right] + \frac{\frac{1}{a} - \frac{1}{v-1}}{L(z_e^*)} \right)} = 0, \]

where

\[ \bar{C} = T^{1-aq} \left( \frac{v-1}{v(1-\gamma) - 1} \right)^{1-aq} \left( \frac{\gamma v}{v-1} \right)^{(1-aq)(\frac{1}{\gamma}+1)} \left( \frac{1-\gamma}{\gamma} \right)^{(1-\gamma)/(1-aq)} \left( \frac{1}{q - 1} \right)^{-1} / (qA(1-a)^q). \]

The equations (50), (54), and (55) uniquely define prices \( r_e, r_b, w \) and the threshold \( z_b^* \), given the solution for \( z_e^* \) from (57).

The interval in which equilibrium with positive interest rates and positive interest rate spread can exist is given by the following:

\[ r_e - r_b \geq 0, r_b \geq 0, \]

or

\[ X(z_b^*) \geq 0, a\gamma L(z_e^*) - (1-a)X(z_b^*) \geq 0. \]

The expressions for \( r_e, r_b \) imply the following restrictions on the range of \( z_e^* \):

\[ 1 + \pi(z_e^*) - L(z_e^*) \geq 0, \quad (58) \]

\[ 1 + \pi(z_e^*) - L(z_e^*) \leq a\gamma L(z_e^*)/(1-a), \quad (59) \]

or

\[ L(z_e^*) \leq 1 + \pi(z_e^*) \leq \left( \frac{a\gamma}{1-a} + 1 \right) L(z_e^*). \]

Denote the maximum and minimum positive real solutions to the equalities corresponding to (58) and (59) as \( z_2 \) and \( z_1 \). The function \( F(z_e^*) \) is continuous on \([z_1, z_2]\).

Consider the value of \( F(z_e^*) \) on the borders \( z_1 \) and \( z_2 \):

1. \( L(z_e^*) = 1 + \pi(z_e^*). \) Then the equation defining equilibrium (57) becomes

\[ F(z_e^*|r_e - r_b = 0) = \pi(z_e^*) - 0 > 0. \]

2. \( 1 + \pi(z_e^*) = \left( \frac{a\gamma}{1-a} + 1 \right) L(z_e^*). \) Then the equation defining equilibrium (57) becomes:

\footnote{It is easy to show that such solutions exist. Consider the closed interval starting at \( z_e^* = 1 \) and ending at some very large value, say \( z_e^* = 10000. \) The value of (58) and (59) at \( z_e^* = 1 \) is positive; at \( z_e^* = 10000, \) it is negative; both equalities define continuous functions on \([1,10000]\). By the Intermediate Value theorem, there are real \( z_1, z_2 \in [1,10000] \) that solve the inequalities corresponding to (58) and (59).}
Denote the product of all constant terms as $C$:

$$F(z_e^*|r_b = 0) = \pi(z_e^*) - \frac{C}{1-a} \left( \frac{\frac{a}{v-1} - \frac{\gamma v}{v-1}}{1-a} \right)^{(1-aq)/\psi} \left( \frac{\frac{\gamma}{v-1} [1 + \pi(z_e^*)] (1-aq)/\psi}{\frac{v}{v-1} [1 + \pi(z_e^*)] + \left[ \frac{\frac{1-a}{1-a} - \frac{1}{v-1} \right] L(z_e^*)} \right).$$

Note that, in equilibrium the following inequality must hold:

$$-CL(z_e^*)^{(aq-1)/v} \leq -C (1 + \pi(z_e^*)))^{(aq-1)/v},$$

So that

$$\pi(z_e^*) - CL(z_e^*)^{(aq-1)/v} \leq \pi(z_e^*) - C (1 + \pi(z_e^*))^{(aq-1)/v}.$$

Given that $1 + \pi(z_e^*)$ is bounded between 1 and 2,

$$2^{(aq-1)/v} \leq (1 + \pi(z_e^*))^{(aq-1)/v} \leq 1,$$

$$F(z_e^*|r_b = 0) \leq \pi(z_e^*) - C (1 + \pi(z_e^*))^{(aq-1)/v} \leq \pi(z_e^*) - C \leq 1 - C \leq 0,$$

given that $T$ is such that $C > 1$.

Therefore, the function $F(z_e^*)$ alternates in sign on the interval $[z_1, z_2]$. According to the Intermediate Value theorem, there is a value of $z_e^* \in [z_1, z_2]$ such that $F(z_e^*) = 0$. So, the equilibrium with positive interest rates and positive interest rate spread exists.

To show that the equilibrium is unique, consider the derivative of the function $F(z_e^*)$ with respect to $z_e^*$, and conclude that this derivative is always negative on the interval $[z_1, z_2]$, so that the function is strictly monotone on that interval.

The function $F(z_e^*)$ can be schematically rewritten as:

$$F(z_e^*) = \frac{\pi(z_e^*)(..) - (.)}{(..)} = 0,$$

where

$$(..) = \left[ \frac{\frac{a}{v-1} - \frac{\gamma v}{v-1}}{1-a} \right] L(z_e^*) + \frac{\gamma}{v-1} [1 + \pi(z_e^*)] (1-aq)/\psi \left( \frac{\gamma}{v-1} [1 + \pi(z_e^*)] + \left[ \frac{\frac{1-a}{1-a} - \frac{1}{v-1} \right] L(z_e^*) \right);$$

$$(.) = \bar{C} (1 + \pi(z_e^*) - L(z_e^*))^{(1-aq)(1/\psi+1-1/v)} L(z_e^*)^{aq}.$$
The derivative is:
\[
\frac{dF(z^*_e)}{dz^*_e} = \frac{\pi'(z^*_e) + \pi(z^*_e)\pi'(z^*_e) - (\pi'(z^*_e))}{(\pi'(z^*_e))^2} = \pi'(z^*_e) - \pi \left[ (*) - (**) \right],
\]
where
\[
(**) = \frac{(1 - aq) \psi \left[ \frac{\alpha \gamma}{1 - \alpha} - \frac{\gamma v}{v - 1} L' \pi + \frac{\gamma v}{v - 1} \pi' \right]}{L(z^*_e) + \frac{\gamma v}{v - 1} \left[ 1 + \pi(z^*_e) \right]} + \frac{\frac{\gamma v}{v - 1} \pi' \left[ \frac{1}{1 - a} \right] L'(z^*_e)}{L(z^*_e)}.
\]
\[
(*) = \frac{(1 - aq)(1/\psi + 1 - 1/v) \left[ \pi'(z^*_e) - L' \pi' \right]}{1 + \pi(z^*_e) - L(z^*_e)} + \frac{aq \pi' \left[ \pi' \right]}{L(z^*_e)}.
\]

Using the inequalities defining the region in which equilibrium exists, obtain:
\[
\frac{dF(z^*_e)}{dz^*_e} < \pi'(z^*_e) \left( 1 - \frac{\pi(z^*_e)}{L(z^*_e)} \right) \left[ \left[ -L(z^*_e) \pi' \right] \left[ ** \right] + \left[ * \right] \right],
\]
where
\[
[***] = \frac{(1 - aq)(1/\psi + 1 - 1/v) - \frac{a \gamma v}{1 - a}}{\pi' \left[ \frac{\alpha \gamma}{1 - \alpha} - \frac{\gamma v}{v - 1} L' \pi + \frac{\gamma v}{v - 1} \pi' \right]} + \frac{\frac{\gamma v}{v - 1} \pi' \left[ \frac{1}{1 - a} \right] L'(z^*_e)}{L(z^*_e)} > 1,
\]
\[
[****] = \frac{(1 - aq)(1/\psi + 1 - 1/v) - \frac{a \gamma v}{1 - a}}{\pi' \left[ \frac{\alpha \gamma}{1 - \alpha} - \frac{\gamma v}{v - 1} L' \pi + \frac{\gamma v}{v - 1} \pi' \right]} - \frac{\frac{\gamma v}{v - 1} \pi' \left[ \frac{1}{1 - a} \right] L'(z^*_e)}{L(z^*_e)} > 0,
\]
under assumption 3. Then,
\[
\frac{dF(z^*_e)}{dz^*_e} < \pi'(z^*_e) \left( 1 + \frac{\pi(z^*_e)}{L(z^*_e)} \left[ * \right] - \frac{\pi(z^*_e)}{L(z^*_e)} \left[ *** \right] \right) = \pi'(z^*_e) \left( 1 + \frac{1}{1 - a} \left[ 1 - q \right] \pi(z^*_e) \left[ * \right] - \frac{1}{1 - a} \pi(z^*_e) \left[ *** \right] \right) = \pi'(z^*_e) \left( 1 + \frac{1}{1 - a} \left[ 1 - q \right] \pi(z^*_e) \left[ * \right] - \frac{1}{1 - a} \pi(z^*_e) \left[ *** \right] \right) = \pi'(z^*_e) \left( 1 - \frac{1}{1 - a} \left[ 1 - q \right] \pi(z^*_e) \left[ * \right] - \frac{1}{1 - a} \pi(z^*_e) \left[ *** \right] \right) < 0.
\]

Thus, the function \( F(z^*_e) \) is strictly monotone decreasing on the interval \([z1, z2]\), and thus, the equilibrium is unique.

Proof of Proposition 2. Let \( A \) grow at rate \( g^{1-aq} \), and \( T \) grow at rate \( g \). If there exist a solution, there exist \( w, r_e, r_b \) that clear the markets. Conjecture that along a balanced growth
path wages, \( w \), grow at rate \( g \), and interest rates, \( r_c, r_b \), are constant. Then \( L_c \) and \( L_b \) are constant. From (11) and (24) it means that the thresholds \( z^*_c \) and \( z^*_b \) are constant. Therefore, given (6) and (8) \( d(z) \) grows at rate \( g \). This implies that probability \( P(z) \) is constant.

Given that \( L_c, L_b, z^*_c \) and \( z^*_b \) are constant over time, labor demand functions \( l(z), x(z) \) are constant over time. From (6) and (8), the capital demand is growing at rate \( g \), same rate as capital supply. Finally, output from (44) is proportional to wages and grows at rate \( g \). Therefore, the conjectured solution for the rates of growth of \( w, r_c, r_b \) was correct. ■

**Proof of Proposition 3.** Consider \( F(z^*_e) = 0 \). From the proof of Proposition 1, \( dF(z^*_e)/dz^*_e < 0 \) on the interval where the equilibrium can exist. Moreover, \( dF(z^*_e)/dT < 0, dF(z^*_e)/dA > 0 \). By the Implicit Function theorem, \( dz^*_e/dT < 0, dz^*_e/dA > 0 \). Taking derivatives with respect to \( z^*_e \): from (55), \( dz^*_b/dz^*_e < 0 \), so \( z^*_e \) decreases (increases) and \( z^*_b \) increases (decreases) with a rise in \( T \) (\( A \)) given \( A (T) \); from (32), (33), and (56), \( dL(z^*_e)/dz^*_e = \frac{(1-a)q}{a} \left( \frac{1}{q-1} \pi(z^*_e) \frac{1}{v} - 1 \right) vz^*_e^{v-1} < 0 \), \( dX(z^*_e)/dz^*_e = \frac{\gamma v}{v-1} vz^*_e^{v-1} \left[ 1 - dL(z^*_e)/dz^*_e \right] > 0 \), so labor demand in the real (financial) sector is increasing (decreasing) in \( T \) (\( A \)).

Simplifying (43), and taking derivative of the interest rates spread, given by equation (49) with respect to \( z^*_e \), using (32), (56), (55):

\[
\Pi(z^*_e, z^*_b) = \left[ \frac{1}{q} - 1 \right] \left( \frac{1}{1 - a} - \frac{1}{v - 1} \right) L(z^*_e) + \frac{v}{v - 1} \pi(z^*_e) + \frac{v}{v - 1};
\]

\[
d \left( r_e - r_b \right) = \frac{1}{\Pi(z^*_e, z^*_b)^2} \left( 1 + \psi \right) v \frac{1}{q} - 1 \left[ \pi z_b L - L \pi z^*_e \pi - L \pi z^*_e \right] > 0.
\]

Therefore, \( d \left( r_e - r_b \right)/dT < 0, d \left( r_e - r_b \right)/dA > 0 \).

Taking derivative of the interest rates on deposits \( r_b \) with respect to \( z^*_e \):

\[
dr_b/dz^*_e = \frac{\psi(a + 1/q - 1)}{(1 - a)^2} \left[ L \pi z^*_e - L \pi z^*_e \pi - L \pi z^*_e \right] < 0.
\]

Therefore, \( dr_b/dT > 0, dr_b/dA < 0 \). ■

**Proof of Corollary 1.** Follows from the proof of Propositions 1 and 3. ■

**Proof of Proposition 4.** In equilibrium, \( z^*_e \) can be expressed as an implicit function of \( S \) and \( T \). Consider the total differential:

\[
dz^*_e = \frac{\partial z^*_e}{\partial T} dT + \frac{\partial z^*_e}{\partial S} dS.
\]

It has been shown that \( z^*_e \) is decreasing in \( T \). If \( z^*_e \) is decreasing in \( S \), the number of firms in the real sector decreases after the crisis. If \( z^*_e \) is increasing in \( S \), consider \( dT = 1 - \eta \) and
\[ dS = 1 - \xi, \] where \( \xi \) is such that \( \frac{\partial \xi}{\partial T} dT + \frac{\partial \xi}{\partial S} dS < 0. \) For \( 1 > \xi \geq \xi > 1 + \frac{\partial \xi}{\partial T} / \frac{\partial \xi}{\partial S} (1 - \eta), \) the number of firms in the real sector decreases after the crisis. ■

**Proof of Corollary 2.** Follows from the proof of Propositions 1, 3, and 4. ■