COST, REVENUE, AND STRATEGIC INTERACTION

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Abstract

We provide two new examples of imperfectly competitive scenarios in which the popular reduced-form revenue test for competitive conduct cannot reliably identify the degree of market power. Second, we show that, if marginal costs change asymmetrically across firms, a change in one firm's marginal cost will affect its rival's revenue. In a related result, we show that the outcome of the revenue test depends on the response of a firm's marginal cost to changes in the marginal cost of its rival. Finally, we discuss the implications of our results for the empirical assessment of competition.

Keywords: reduced-form revenue test; Panzar-Rosse; competition; oligopoly

JEL codes: D4, L1

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1. Introduction

The response of firms' revenue to changes in marginal cost has long been a topic of intrinsic interest, as well as affording an empirical test for competitive conduct (Rosse and Panzar, 1977; Panzar and Rosse, 1987; Vesala, 1995). The linear homogeneity of marginal cost in factor prices suggests a parsimonious implementation of this model with modest data requirements, an approach that has recently seen growing popularity (e.g., Coccorese, 2009; Schaeck et al., 2009; Podpiera and Rakova, 2009; Li, 2010; Bikker et al., forthcoming). Bikker et al. (forthcoming) count 31 published applications to the banking industry alone, including 15 between 2005 and 2009. The same method has also been applied to non-banking samples by Rosse and Panzar, 1977; Sullivan, 1985; Ashenfelter and Sullivan, 1987; Wong, 1996; Fischer and Kamerschen, 2003; Tsutsui and Kamasaki, 2005; Podpiera and Rakova, 2009; Li, 2010; among others.

The test statistic in the standard reduced-form revenue model, often denoted H, is the sum of elasticities of total revenue with respect to each input price. A sample of firm-level observations in long-run competitive equilibrium would exhibit $H = 1$, while a sample of observations from a profit-maximizing monopoly yields $H < 0$ (Panzar and Rosse, 1987). For intermediate cases, the literature varies widely and remains incomplete.

Early studies emphasized the general potential for the sign of H to distinguish between competition and monopoly power, and the most recent studies have only slightly modified that interpretation.[1] Theoretical studies report H as alternately an increasing (Shaffer, 1983) or decreasing (Panzar and Rosse, 1987) function of the Lerner index for firms facing constant elasticity of demand, and several empirical studies interpret smaller values of

[1] Bikker et al. (forthcoming) characterize H as a "one-tail test of conduct."

H as indicating greater monopoly power (e.g., De Bandt and Davis, 2000; Bikker and Haaf, 2002; Coccorese, 2009; Schaeck et al., 2009).[2] Many empirical studies interpret positive estimates of H as reflecting Chamberlinian monopolist competition, though Panzar and Rosse (1987, p. 454) state that positive H could also result from conjectural variation oligopoly if firms face inelastic market demand. Some studies report empirical estimates of H > 1, which is not consistent with any previously analyzed theoretical equilibrium (De Bandt and Davis, 2000; Bikker and Haaf, 2002; Coccorese, 2009).

This article explores the equilibrium response of revenue to changes in marginal cost in three previously overlooked market settings. Our findings broaden the known causes of positive H to include two new, imperfectly competitive scenarios. We also provide the first theoretical justification of H > 1, thus establishing that such estimates can be a valid outcome and not just an artifact of econometric problems.[3] Further, we show that, if marginal costs change asymmetrically across firms, a change in one firm's marginal cost will affect its rival's revenue. In a related result, we show that parallel changes in input prices across firms will yield larger values of H, and perhaps even reverse the sign of H, compared to the case in which input prices change for only one firm. This complication can be avoided in empirical applications only by controlling for the input prices of all relevant firms. Since all prior studies have controlled only for each firm's own input prices, these results suggest profound reinterpretations of the empirical reduced-form literature.

While other recent work has noted additional weaknesses in the Panzar-Rosse model as widely implemented (Bikker et al., forthcoming), those results nevertheless suggest that a *properly specified* reduced-form revenue equation can yield a one-tail test of conduct based

[2] The disagreement between Shaffer (1983) and Panzar and Rosse (1987) is due to a simple algebraic error by the latter.

[3] Econometric problems such as model misspecification, estimation uncertainty, and measurement errors might cause spurious estimates of H > 1.

on the sign of H. By contrast, our findings here establish that, contrary to widespread belief, neither the sign nor the magnitude of H alone can reliably identify the degree of market power. Rather, in addition to new refinements of specification, independent information about the sequence of firms' actions and the relative levels of firms' costs would be required to draw valid conclusions from observed values of H.

2. Duopoly with positive H

We derive our results in the context of duopoly for clarity and brevity, as well as to emphasize that positive H can result even with a *high degree* of oligopoly power, contrary to common perceptions. Similar calculations establish corresponding generalizations to arbitrary numbers of firms.[4] We begin by analyzing a standard Stackelberg duopoly, and show that it provides the first direct example supporting a claim of Panzar and Rosse (1987) and Vesala (1995, p. 58) that $H > 0$ is possible in a conventional static oligopoly. We show:

Result 1: H can take either sign for a Stackelberg duopoly facing linear cost and demand.

Proof: Because marginal cost is homogeneous of degree 1 in all input prices, H equals the elasticity form of (and has the same sign as) $\partial TR/\partial MC$. Thus, it suffices to solve for $\partial TR/\partial MC$. Let $P = a - bq_1 - bq_2$ for firms producing q_i ($i = 1,2$) at constant marginal cost c. Firm i earns profit $p_i = q_i (P - c) = q_i (a - c - bq_1 - bq_2)$. The Stackelberg follower (firm 1) chooses output q_1 to maximize its profit, given the output selected by the other firm, with first-order condition $a - c - bq_2 - 2bq_1 = 0$, so $q_1 = (a - c - bq_2) / 2b$. The leader (firm 2) chooses output q_2 to maximize its profit, conditional on the reaction function of firm 1. Its

[4] Moreover, duopoly is often an empirically relevant case, as documented by Bresnahan and Reiss (1991).

first-order condition is $(a - c)/2 - bq_2 = 0$ or $q_2 = (a - c) / 2b$. Then $q_1 = (a - c) / 4b$ and $P = (a + 3c) / 4$.[5]

If both firms maintain the same leader-follower sequence, we can solve directly for the impact of a change in c on each firm's output, equilibrium price, and revenue: $\partial q_1/\partial c = -1/4b$, $\partial q_2/\partial c = -1/2b$, and $\partial P/\partial c = 3/4$. In general, $\partial TR_i/\partial c = P\, \partial q_i/\partial c + q_i\, \partial P/\partial c + (\partial q_i/\partial c)(\partial P/\partial c)$. Thus, $\partial TR_1/\partial c = (2a - 6c - 3) / 16b$ and $\partial TR_2/\partial c = (2a - 6c - 3) / 8b$. Both expressions have the sign of $2a - 6c - 3$, which depends on the parameter values.

For a sustainable equilibrium, the demand price must cover the cost of production, so $a > c$. Within this range, we can have either $c < a < 3c + 3/2$ (where $\partial TR/\partial c < 0$ for both firms) or $a > 3c + 3/2$ (where $\partial TR/\partial c > 0$ for both firms). By the linear homogeneity of marginal cost with respect to all input prices, this means that the Panzar-Rosse H statistic can take either sign for Stackelberg duopolists, depending on the relative magnitudes of a and c.∎

Remarks: This result broadens the known cases of $H > 0$ under imperfect competition. In this setting, both firms have the same sign of H, and the leader's value of H is twice as far from zero as that of the follower. The follower's Lerner index (relative markup of price over marginal cost) is $(a - c) / (a + 3c)$, which is positive for $a > c$, confirming the existence of market power.

3. H > 1

Next we turn to the possibility of H exceeding unity and show the following:

Result 2: $H > 1$ is possible for the low-cost firm in a homogeneous Cournot duopoly with asymmetric costs and linear demand.

[5] The second-order condition is satisfied for all $b > 0$.

Proof: Let $P = a - bx - by$ where x is the output quantity of one firm and y is the output quantity of the other firm. Let the total cost of x equal cx and total cost of y equal αcy. Assume $a > c$ to ensure non-negative profits in equilibrium. Assume $\alpha > 1$ so the first firm has lower marginal cost. We define TR_x as the first firm's total revenue and H_x as its H-statistic. By standard first-order conditions for profit maximization, $x = [a - c(2 - \alpha)] / 3b$ and $y = [a - c(2\alpha - 1)] / 3b$. Then $P = [a + c(\alpha + 1)] / 3$, $TR_x = [a^2 + ac(\alpha - 1) + c^2 (\alpha^2 - \alpha - 1)] / 9b$, and $\partial TR_x / \partial c = [a(\alpha - 1) + 2c(\alpha^2 - \alpha - 1)] / 9b$. Thus $H_x = (c / TR_x) \partial TR_x / \partial c = [ac(\alpha - 1) + 2c^2(\alpha^2 - \alpha - 1)] / [a^2 + ac(\alpha - 1) + c^2 (\alpha^2 - \alpha - 1)]$.
Because $a > c$ and $\alpha > 1$, $a^2 > c^2(1 + \alpha - \alpha^2)$ and so $H_x > 1$. ▓

Remarks: Similar calculations show $H_x > 0$ for all $\alpha > \{2c - a + [(2c + a)^2 + 16c^2]^{1/2}\} / 4c$. This is another example of how we can get $H > 0$ with imperfect competition. For instance, if $a = 2c$, then $H_x > 0$ for all $\alpha > \sqrt{2}$ and $H_x > 1$ for all $\alpha > (1 + \sqrt{21}) / 2$.

The Lerner index for the low-cost firm is $L_x = [a + c (\alpha - 2)] / [a + c (\alpha + 1)]$, which is positive for $a > c$ and $\alpha > 1$, confirming market power. Moreover, $\partial L_x / \partial\alpha = 3c^2 / [a + c (\alpha + 1)]^2 > 0$, so greater inequality in marginal cost results in a higher value of the Lerner index for the low-cost firm. Finally, this example reduces to the symmetric case if $\alpha = 1$. Then the above calculations give $H_x = -2c^2 / (a^2 - c^2) < 0$.

Critical to the above analysis is the assumption that input prices move in parallel for both firms, which might be expected if the firms compete in the same input markets. The next section relaxes this assumption by deriving the impact of a change in one firm's marginal cost on its rival's revenue when this assumption is relaxed.

4. Asymmetric Cost Changes

Prior theoretical analysis has implicitly incorporated contrasting assumptions about interfirm correlations in input prices, though without acknowledging or discussing them. For instance, Panzar and Rosse (1987) and Vesala (1995) carry out their oligopoly analysis under the implicit assumption that input prices change identically across firms, while Shaffer (1982) assumes that input prices change independently across firms.[6] Empirical applications and interpretations of the reduced-form revenue model have ignored these distinctions.

In practice, since output prices are likely to be positively – but imperfectly – correlated across rival firms, it is important to consider how a firm's revenue may respond to changes in input prices or marginal cost at other firms. Such interaction has been recognized in other settings, as in the strategy of "raising rivals' costs" (Salop and Scheffman, 1983). Here we show:

Result 3: In a homogeneous Cournot duopoly with linear demand, each firm's revenue responds positively to a change in the other firm's marginal cost.

Proof: Let $P = a - bx - by$ as above. Let the total cost of x equal cx and total cost of y equal dy. Profit maximization implies $x = (a + d - 2c) / 3b$, $y = (a + c - 2d) / 3b$, and $P = (a + c + d) / 3$. Then $TR_x = (a + d - 2c)(a + c + d) / 9b$ and $\partial TR_x / \partial d = (2a + 2d - c) / 9b > 0$, with a similar result for $\partial TR_y / \partial c$. ▓

Remarks: This result suggests that a proper implementation of the Panzar-Rosse reduced-form revenue test should control for the input prices of all firms operating in the same output market, not just each firm's own input prices. Prior empirical studies have never done so. Moreover, this requirement also implies that a proper Panzar-Rosse test requires an output market to be defined, contrary to some prior claims (e.g., Shaffer, 2004).

[6] The assumption in Panzar and Rosse (1987) and Vesala (1995) is required to justify the manner in which symmetry is imposed in their analysis, such that industry output equals firm output times the number of firms. Then output changes in parallel for all firms, which requires that input prices also must change in parallel for all firms. Shaffer (1982), by contrast, incorporates only own-firm changes in input prices.

Given that the input prices of all firms matter for the estimation of H in a homogeneous Cournot duopoly with linear demand, it is important to explore the effect on H of failing to control for rivals' prices. We do so next by comparing $\partial TR_x / \partial c$ in the polar cases where input prices move independently across firms ($\partial d / \partial c = 0$) versus together ($\partial d / \partial c = 1$), and show:

Result 4: H is larger when input prices move together across firms than when input prices move independently across firms.

Proof: In general, $\partial TR_x / \partial c = [- (a + d + 4c) + (2a - c + 2d)(\partial d / \partial c)] / 9b$. If $\partial d / \partial c = 0$, then $\partial TR_x / \partial c = - (a + d + 4c) / 9b < 0$. If $\partial d / \partial c = 1$, then $\partial TR_x / \partial c = (a + d - 5c) / 9b$, which can take either sign. The difference between these two expressions (second case minus the first case) is $(2a + 2d - c) / 9b$, which is positive since $a > c > 0$, $b > 0$, and $d > 0$.[7] Finally, multiplying $\partial TR_x / \partial c$ by c / TR_x, which is positive and independent of $\partial d / \partial c$, converts these expressions to the corresponding quantities for H_x. ▓

Remarks: In panel data with multiple firms, estimating H while neglecting to control for rivals' input prices – as in all prior PR studies – will yield estimates that suffer from an omitted variable bias if $\partial d / \partial c \neq 0$ (Greene, 2011; pp. 56-57). The bias depends on two factors: the impact of a change in the other firm's input prices on the own firm's revenue and $\partial d / \partial c$. If one of these factors is zero, there is no bias.[8] If both factors have the same sign, the bias is positive. In all other cases the bias is negative. Result 3 allows us to determine the sign of the omitted variable bias for a homogeneous Cournot duoply with linear demand. Because the influence of a change in the other firm's marginal cost on the own firm's revenue is positive (see Result 3), the sign of the bias is equal to the sign of $\partial d / \partial c$. Usually $0 < \partial d /$

[7] Equivalently, and more generally, define $K = \partial d / \partial c$. Then $(a + d - 5c) / 9b = \partial^2 TR_x / \partial c \, \partial K > 0$.

[8] If $\partial d / \partial c = \pm 1$, c and d are perfectly collinear. In these two cases there is no omitted variable bias.

$\partial c < 1$, in which case the bias turns out positive. If the revenue function contains additional control variables, the sign of the *partial* correlation between c and d (i.e., after controlling for the other covariates) determines the sign of the bias of H. Most of the time, but certainly not always, the sign of the partial correlation and the simple bivariate correlation are the same.

5. Conclusion and Implications

Motivated by a recent proliferation of the method's use, this article has analyzed the response of equilibrium revenue to changes in marginal cost under three market structure scenarios not previously explored. We identified two imperfectly competitive cases in which the sum of revenue elasticities with respect to input prices, H, can take a positive value, providing the first formal proof of a property claimed by Panzar and Rosse (1987) and Vesala (1995) and contrary to a pattern previously established for specific forms of oligopoly. We further showed that one of those cases can generate $H > 1$ if input prices vary in parallel across rival firms, potentially explaining some previous empirical results and clarifying that the perfectly competitive value of 1 is not an upper bound on H as was previously thought.

Together with prior theoretical studies, these first two results indicate that neither the sign nor the magnitude of H can unambiguously identify the degree of market power without additional information on the characteristics of cost and the sequence of actions by firms, contrary to prior belief. This conclusion calls into question the standard interpretations of dozens of existing reduced-form revenue studies and prescribes additional procedures for future estimation.

Next, we showed that a firm's equilibrium revenue will respond to a change in its rival's input prices. This result implies that, whenever it is possible for input prices to vary idiosyncratically across firms, empirical reduced-form revenue tests must control for the input prices of all firms within the relevant output market, not just for a firm's own input prices as in prior studies. Related analysis implies that the traditional specification will lead to inflated estimates of H whenever input prices change in parallel across firms, as we would commonly expect. These latter findings imply that all prior empirical studies using the reduced-form revenue model – except those on pure monopoly – have been systematically misspecified.

Overall, the implications of the analysis here are not sanguine for the extensive and growing body of empirical reduced-form revenue studies. Far from being the convenient, robust measure of market conduct as originally portrayed, the model must be augmented by considerable additional information from independent sources to take account of a wide variety of alternative outcomes.

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