FISCAL FORESIGHT: ANALYTICS AND ECONOMETRICS

ERIC M. LEEPER, TODD B. WALKER, AND SHU-CHUN SUSAN YANG

Abstract. Fiscal foresight—the phenomenon that legislative and implementation lags ensure that private agents receive clear signals about the tax rates they face in the future—is intrinsic to the tax policy process. This paper develops an analytical framework to study the econometric implications of fiscal foresight. Simple theoretical examples show that foresight produces equilibrium time series with a non-invertible moving average component, which misaligns the agents’ and the econometrician’s information sets in estimated VARs. Economically meaningful shocks to taxes, therefore, cannot be extracted from statistical innovations in conventional ways. Econometric analyses that fail to align agents’ and the econometrician’s information sets can produce distorted inferences about the effects of tax policies. Because non-invertibility arises as a natural outgrowth of the fact that agents’ optimal decisions discount future tax obligations, it is likely to be endemic to the study of fiscal policy. In light of the implications of the analytical framework, we evaluate two existing empirical approaches to quantifying the impacts of fiscal foresight. The paper also offers a formal interpretation of the narrative approach to identifying fiscal policy.

Date: May 6, 2008. Prepared for the Bank of Korea Conference “Recognizing and Coping with Macroeconomic Model Uncertainty in Designing Monetary Policy,” May 26-27, 2008. Department of Economics, Indiana University and NBER, eleeper@indiana.edu; Department of Economics, Indiana University, walkertb@indiana.edu; Institute of Economics, Academia Sinica, ssyang@econ.sinica.edu.tw. Leeper acknowledges support of NSF Grant SES-0452599. We thank Karel Mertens and Morten Ravn for sharing their code and providing additional explanations of their work. We also acknowledge comments by Troy Davig, Dale Henderson, Beth Klee, Ricardo Nunes, Morten Ravn, Rob Vigfusson, and participants at workshops at the Congressional Budget Office and the Federal Reserve Board. We are particularly grateful to Jim Nason and Harald Uhlig for helpful comments.
FISCAL FORESIGHT: ANALYTICS AND ECONOMETRICS

1. Introduction

Fiscal policy presents researchers with a unique empirical challenge: how to identify and quantify the impacts of foreseen “shocks” to taxes. The challenge posed by taxes is unique because few economic phenomena provide economic agents with such clear signals about how important margins will change in the future. Intrinsic to the process of changing taxes are two kinds of lags: the legislative lag between when new tax law is proposed and when it is passed and the implementation lag between when the legislation is signed into law and when it actually takes effect. Estimates of the total lag range from a couple of months to two years or more, depending on the particular legislation being considered.

Public finance economists recognize the possibility of fiscal foresight and have accumulated empirical evidence of its importance using a variety of econometric and event-study techniques.\(^1\) Macroeconomists sometimes acknowledge the possibility of fiscal foresight in empirical work and occasionally study it in theoretical models, but the empirics are typically not grounded in theory. This paper is the first analytical study of the econometric implications of fiscal foresight. Theory suggests that fiscal foresight poses a substantial challenge to econometric analysis of fiscal policy.

Two lines of attack on fiscal foresight appear in the empirical macro literature. The first estimates conventional VARs, identified in a variety of creative ways to isolate “anticipated taxes,” and then examines the impacts of fiscal foresight \textit{ex-post} [Sims (1988), Blanchard and Perotti (2002), Yang (2007b), Mountford and Uhlig (2008)]. A second line rejects VAR identification schemes \textit{ex-ante}, arguing that they cannot adequately measure the impacts of foreseen changes in fiscal policy, and takes a different—narrative—approach to identification that brings fresh data to bear on the problem [Ramey and Shapiro (1998), Edelberg, Eichenbaum, and Fisher (1999), Ramey (2007), Romer and Romer (2007a), Mertens and Ravn (2008)]. \textit{Ex-post} and \textit{ex-ante} approaches share the aim of finding instruments for news about future tax changes.

This paper argues that fiscal foresight cannot be confronted \textit{ex-post}. Even very creative identification schemes are unlikely to correctly extract the tax news in agents’ information sets from the information embedded in conventional VARs. In addition, \textit{ex-ante} approaches, while correctly skeptical of the efficacy of VAR methods for this application, tend to achieve identification through a variety of heroic—and often implicit—identifying assumptions. Once those assumptions are laid bare, it is easy to be equally skeptical of both empirical approaches to fiscal foresight.

\(^1\)Evidence of foresight leading up to the Tax Reform Act of 1986 is documented in Auerbach and Slemrod (1997) and Burman, Clausing, and O’Hare (1994).
Fiscal foresight poses a formidable challenge because, as Yang (2005) shows, it generates an equilibrium with a non-invertible VARMA representation. Non-invertibility, in turn, implies that the fundamental shocks to tax policy cannot be recovered from current and past observable data, a central assumption of conventional econometric methods. This difficulty was pointed out in the early rational expectations econometrics literature by Hansen and Sargent (1980, 1991) and recently emphasized by Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007).

To the extent that fiscal foresight is an intrinsic feature of the tax policy process, conventional fiscal VARs ascribe to the econometrician an information set that is strictly smaller than the information set on which agents base their decisions. A smaller information set can lead the econometrician to label as “tax shocks” objects that are linear combinations of all the exogenous disturbances at various leads and lags. This mislabeling undermines efforts to quantify how tax policy changes affect the macro economy.

We present simple analytical examples of how fiscal foresight affects equilibrium time series and use the examples to examine the nature of the problems that foresight creates for econometric analysis. As the examples make clear, the non-invertibility that arises in the presence of fiscal foresight is a natural by-product of the fact that agents’ optimal intertemporal decisions discount future tax obligations. Private agents discount recent news more heavily because it informs about taxes in the more distant future. The econometrician, in contrast, discounts in the usual way, down weighting older news relative to recent news. Agents and the econometrician employ different discounting patterns because the econometrician’s information set lags the agents’. These differences are the source of the mistaken inferences the econometrician draws. Confronting foresight, therefore, is a necessary step toward detecting fiscal effects in macro time series.

To be more precise, let \( \varepsilon_t \) denote the vector of exogenous disturbances that the agents observe, with \( \varepsilon_{t,t} \) the tax element of the shock vector. At time \( t \), the agents’ information set is \( \Omega_t \), the span of \( \{ \varepsilon_t, \varepsilon_{t-1}, \ldots \} \). An econometrician who estimates a conventional VAR with macro variables identifies exogenous shocks \( \varepsilon^*_{t,t} \), with the associated tax disturbance \( \varepsilon^*_{t,t} \). The econometrician’s information set at \( t \) is \( \Omega^*_t \), the span of \( \{ \varepsilon^*_{t,t}, \varepsilon^*_{t-1}, \ldots \} \). In the presence of tax foresight, \( \Omega^*_t \) is strictly smaller than \( \Omega_t \). Analytical examples, moreover, show that typical VAR specifications are likely to obtain measures of \( \varepsilon^*_{t,t} \) that are linear combinations of current

---


3. A closely related line of work examines the conditions under which a finite-order VAR can adequately capture agents’ information sets when state variables are excluded from the VAR system [for example, Cooley and Dwyer (1995), Fry and Pagan (2005), Kapetanios, Pagan, and Scott (2005), Giannone, Reichlin, and Sala (2006), Giannone and Reichlin (2006), Chung and Leeper (2007), Dungey and Fry (2007)]. This work argues that expanding the VAR to include important state variables can solve the invertibility problem. Faust, Rogers, Swanson, and Wright (2003) and Faust, Swanson, and Wright (2004) do not directly address non-invertibility, but they use high-frequency financial data to expand the econometrician’s information set to aid in identifying monetary policy effects. Fiscal foresight is distinctive because data on the missing state variable—anticipated future tax rates—are not easily available.
and past realizations of all the exogenous shocks in $\varepsilon$. Foresight about tax changes can lead to econometric estimates of tax disturbances that confound taxes with other sources of disturbance and treat as “news” old information to which rational agents have already reacted. Foresight can also lead to identification of non-policy disturbances that are convolutions of all the underlying exogenous shocks.

Misalignment of agents’ and the econometrician’s information sets has disturbing implications for the econometric analyses that macroeconomists typically conduct. Impulse response functions and variance decompositions can be profoundly wrong. Granger-causal orderings can be reversed. An econometrician may infer that cross-equation restrictions and present-value relations do not hold, even when they hold when information sets are correctly aligned. In sum, failure to address fiscal foresight can seriously distort many of the inferences that macroeconomists draw from empirical work.

The analysis leads to a framework to explain why neither the ex-post nor the ex-ante empirical approach can resolve the foresight issue when estimating tax effects, despite the fact that the approaches appear to yield plausible results. Difficulties in handling fiscal foresight may explain why no consensus has emerged about whether foresight matters. Existing empirical work concludes that an anticipated cut in taxes may have little or no effect [Poterba (1988), Blanchard and Perotti (2002), Romer and Romer (2007a)], may be expansionary in the short run [Mountford and Uhlig (2008)], or may be contractionary in the short run [Branson, Fraga, and Johnson (1986), House and Shapiro (2006), Mertens and Ravn (2008)].

This paper focuses exclusively on foresight about taxes, but misalignment of agents’ and the econometrician’s information sets, and the possibility of non-invertible equilibrium representations, is a widespread problem in macroeconomics. A prominent example of non-invertibility is Quah’s (1990) resolution of Deaton’s (1987) paradox; the resolution relies on agents forecasting future income using strictly more information than the econometrician does. Fernandez-Villaverde, Rubio-Ramirez, and Sargent (2005) contains several examples of DSGE models that in some regions of the parameter space exhibit an invertibility problem. Non-invertibility is likely to present itself in several areas of research that are now receiving attention: news about future technological improvement [Beaudry and Portier (2006), Christiano, Ilut, Motto, and Rostagno (2007), Jaimovich and Rebelo (2008)]; foresight about large government spending run ups [Ramey (2007)]; recent moves by several inflation-targeting central banks—Norges Bank, Reserve Bank of New Zealand, Sveriges Riksbank—to publish paths of future policy interest rates [Holmsen, Qvigstad, and Roisland (2007), Blattner, Catenaro, Ehrmann, Strauch, and Turunen (2008), Laseen, Linde, and Svensson (2008)]. Any of these applications, when studied in a rational expectations model,  

---

4The problems associated with fiscal foresight apply with equal force to estimated dynamic stochastic general equilibrium models of fiscal policy, such as Braun (1994), McGrattan (1994), Coenen and Straub (2004), Forni, Monforte, and Sessa (2006), or Kamps (2007).
would lend themselves to the type of analysis that we conduct. Our paper raises a warning flag about econometric work on these topics.

Companion papers develop the econometric approach that the analytical work implies and then execute empirical analyses using a variety of techniques to handle fiscal foresight.

2. Evidence of Fiscal Foresight

Plenty of reduced-form and anecdotal evidence, mostly in the form of case studies of one or more tax episodes, suggests that private agents possess and react to expected changes in taxes. Reactions to tax foresight are most evident when policy changes are phased in, incorporate sunset provisions, or include a delay between enactment and effective dates.

Steigerwald and Stuart (1997) develop a residual-based test statistic, which allows comparison of predictions errors for econometric models under different assumptions about the period of tax foresight. By studying investment behavior around major tax legislation that altered corporate income tax rates, investment tax credits, or the deductions for depreciation allowances, Steigerwald and Stuart conclude that firms had 4.5-month foresight for tax changes in 1954, 1962, and 1981, and 16.5-month foresight for the tax reform of 1986.

In contrast to Steigerwald-Stuart, Poterba's (1988) study of consumption responses to tax announcements associated with legislation in 1964, 1968, 1975, 1981, and 1986, finds little evidence of anticipatory behavior. He dates expectational changes as occurring the month the legislation passes Congress. Poterba regards his findings as “tentative” because his event study “is likely to have relatively low power against the alternative hypothesis that consumers gradually revise their expectations of future tax policy and adjust consumption accordingly” [p. 416].

The possibility that fiscal foresight can produce a “purely anticipatory recession” was first put forth by Branson, Fraga, and Johnson (1986) in an exploration of President Reagan’s tax cuts. The Economic Recovery Tax Act, enacted in March 1981, announced a three-stage tax cut to be phased in from 1982 to 1984. Branson, Fraga, and Johnson argue that in an open-economy IS-LM model, anticipated tax cuts can reconcile three salient features of the U.S. macro experience of the early 1980s: an inverted and steep real yield curve, real appreciation of the dollar, and a severe recession—all of which occurred in 1981 and 1982, before the tax cuts were fully realized.

The Tax Reform Act of 1986 offered public finance economists a rich laboratory in which to test hypotheses that economic agents may respond to news about tax changes before the changes are effective. The legislation, which was enacted in October 1986, repealed the capital gains exclusion from ordinary income, raising the maximum capital gains tax rate beginning in 1987 from an effective rate of 20 percent to 28 percent. During the lag between
enactment and effective dates, capital gains realizations jumped from $172 billion in 1985 to $328 billion in 1986 in anticipation of the tax hike [Congressional Budget Office (2002, Table 1, p. 3)].

Auerbach and Slemrod (1997) thoroughly review the economic effects of the 1986 reform. They also point out the jump in long-term capital gains realizations that predated the effective date of the reform [see also Burman, Clausing, and O’Hare (1994)]. Auerbach and Slemrod argue that the 1986 Act “established once and for all that the timing of capital gains realizations behavior can be enormously sensitive to anticipated changes in the rate of taxation” [p. 605]. The act also reduced corporate tax rates from a maximum of 46 percent in 1986 to a maximum of 34 percent over 1987 and 1988. Scholes, Willson, and Wolfson (1992) find that the corporate tax rate reduction induced firms to act in anticipation to defer revenue recognition or accelerate expense recognition.

Figure 1 documents two instances in which anticipations of the tax code changes associated with the Tax Reform Act of 1986 spilled over to affect monetary policy behavior. At the end of both 1985 and 1986, the daily federal funds rate spiked as traders shifted their portfolios to get ahead of expected restrictive tax legislation. On December 31, 1985 the funds rate rose almost 4.5 percentage points when, after the House of Representatives passed the legislation on December 17, traders placed higher probability that tax reform would take effect during 1986. As it happened, the bill was not enacted until October 22, 1986, to be effective on January 1, 1987. This is why the funds rate more than doubled on December 30, 1986, and remained high on the last day of 1986.\(^5\) Evidently, the trading desk at the New York Fed underestimated the extent to which money markets would react to foresight about tax policy changes.

A well-documented example of behavior due to foresight underlies the controversy related to estimating the taxable income elasticity with respect to the tax hike for the high-income group that was embedded in the Omnibus Budget Reconciliation Act of 1993. Feldstein and Feenberg (1996) examine the changes in income and tax data from 1992 to 1993 and find a large taxable income elasticity for the high-income group. Their finding implies that an increase in the marginal tax rate has a strong negative effect on the tax base. Goolsbee (2000), on the other hand, finds this elasticity drops to 0.4 if the data are extended to 1995. As the share of time-shiftable compensation (such as one-time bonuses, exercising stock options, and so forth) rose dramatically with income from 1991 to 1992, the contrasting estimates of a high short-run and low long-run elasticity arise because high-income taxpayers in 1992 acted upon their foresight about anticipated higher income tax rates.

House and Shapiro (2006) use a neoclassical growth model with fiscal foresight to calibrate the macroeconomic effects of President Bush’s phased-in cuts in capital and labor tax rates

\(^5\)Behavior in the reserves market is detailed in Federal Reserve Bank of New York (1986, 1987).
beginning in 2001. They argue that the slow recovery from the 2001 recession, especially in the job market, can be attributed to the law’s phasing-in provisions, which created incentives for workers and firms to delay production. Elimination of the phase-ins in 2003, moreover, accounted for about half of the rebound in GDP in the middle of 2003. In recent work, House and Shapiro (2008) estimate that investment in qualified types of capital—capital with a tax recovery period of up to 20 years—rose in anticipation of the bonus depreciation provision of the law. Investment began to rise after the House of Representatives passed the bill in October 2001, before President Bush signed the legislation in March 2002.

Evidence of the impacts of fiscal foresight extends well beyond a single piece of legislation. Yang (2007b) compares responses to a typical tax innovation from VAR systems with and without interest rates and prices. She finds that adding the three-month Treasury bill rate and the GDP price deflator (or commodity prices) to a VAR substantially reduces the responses of labor, investment, and output to a tax innovation. Since financial markets tend to be relatively sensitive to news in the economy, the result is consistent with the interpretation that information contained in financial variables reflects agents’ knowledge of future tax policy changes. Sims (1988) finds similar results for government spending multipliers.

Fiscal foresight may also apply to government spending: anticipated shifts in spending may trigger changes in economic behavior before the spending shift is realized. Recent efforts by Ramey (2007) to reconcile the discrepancies in the effects of government spending using the narrative approach based on war dummies and VAR approaches lead her to conclude
that the VAR-based innovations are indeed anticipated.\textsuperscript{6} Since the war dates are identified based on the timing when the media reported future defense buildups, the news reports predate the actual rises in defense spending. Ramey finds that the war dates Granger-cause defense spending and aggregate government spending, but not vice versa. This supports the existence of foresight about government spending policy.

There is a line of work that is often interpreted as providing evidence that consumers do not adjust savings in response to anticipated changes in taxes and that, therefore, fiscal foresight is unlikely to be important in time series data. The work, which uses natural tax experiments to test the permanent income hypothesis, finds that agents respond strongly to predictable changes in tax liabilities and to current disposable income, a result that is taken as evidence against the hypothesis.\textsuperscript{7} Because these papers focus on responses at the time of or after the implementation of the policy change, they are silent about how behavior changes during the foresight period. In fact, in a neoclassical growth model, where the permanent income hypothesis holds, consumption responds both at the time that news about a future change in capital or labor tax rates arrives \textit{and} after the implementation of the tax rate change [see Yang (2005)].

The evidence of fiscal foresight recounted here, although not decisive, does support the view that some adjustment in behavior occurs before tax changes are implemented. The evidence, however, is typically rather casual, relying on case studies, and it is often not tightly connected to theory. We now turn to an illustrative model in which some of the econometric implications of fiscal foresight can be derived explicitly.

\section*{3. Analytical Example}

In this section, we introduce fiscal foresight into a simple economic environment so that the econometric issues can be exposited analytically. The results and conclusions reached below extend to more general setups, as discussed in section 4.2.


Consider a standard growth model with log preferences, inelastic labor supply, and complete depreciation of capital. A proportional tax is levied against income. The equilibrium conditions are well known and given by

\[
\frac{1}{C_t} = \alpha \beta E_t (1 - \tau_{t+1}) \frac{1}{C_{t+1}} \frac{Y_{t+1}}{K_t}
\]

\[
C_t + K_t = Y_t = A_t K_t^{\alpha}
\]

where \(C_t, K_t, \) and \(Y_t\) denote time-\(t\) consumption, capital, and output, respectively, and \(\{A_t\}\) is an exogenous technology shock. As usual, \(0 < \alpha < 1\) and \(0 < \beta < 1\). The government sets the tax rate according to a time-invariant rule and then adjusts lump-sum transfers to satisfy the constraint, \(T_t = \tau_t Y_t\). Government spending is identically zero.

After log linearizing equations (1)–(2), the equilibrium is characterized by a second-order difference equation in capital

\[
E_t k_{t+1} - (\theta^{-1} + \alpha) k_t + \alpha \theta^{-1} k_{t-1} = E_t [a_{t+1} - \theta^{-1} a_t] + \left\{ \theta^{-1} (1 - \theta) \left( \frac{\tau}{1 - \tau} \right) E_t \hat{\tau}_{t+1} \right\}
\]

where \(\theta = \alpha \beta (1 - \tau)\) is a particularly important constant in the analysis and \(0 \leq \tau < 1\) is the steady state tax rate. The solution to (3) satisfies the well-known saddlepath property and is given by

\[
k_t = \alpha k_{t-1} + a_t - (1 - \theta) \left( \frac{\tau}{1 - \tau} \right) \sum_{i=0}^{\infty} \theta^i E_t \hat{\tau}_{t+i+1}.
\]

Equilibrium investment depends negatively on the expected discounted present value of future tax rates, a well-known result [Lucas (1976), Abel (1982), Judd (1985), Auerbach (1989)]. Of course, more distant tax rates are discounted relative to more recent rates.

To model foresight, we must specify how news about taxes signals future tax rates. For many of the points we wish to make, it suffices to assume that tax information flows take a particularly simple form: agents at \(t\) receive a signal that tells them exactly what tax rate they will face in period \(t + q\). The tax rule is \(\tau_t = \hat{\tau} e^{\hat{\varepsilon}_{\tau,t-q}}\), or in log-linearized form

\[
\hat{\tau}_t = \varepsilon_{\tau,t-q}
\]

where \(\hat{\tau}\) denotes percentage deviations of the tax rate from its steady state value.\(^8\) We assume the technology and tax shocks are \(i.i.d.\) and the representative agent’s information set at date \(t\) consists of variables dated \(t\) and earlier, including the shocks, \(\{\varepsilon_{A,t}, \varepsilon_{\tau,t}\}\). Given the tax rule in (5), this implies that the agent at \(t\) has (in this case, perfect) knowledge of \(\{\hat{\tau}_{t+q}, \hat{\tau}_{t+q-1}, \ldots\}\).

It is useful to replace the tax rates in (4) with the news about taxes and display the equilibrium for various degrees of fiscal foresight.

\(^8\)Jaimovich and Rebelo (2008) assume an analogous process for the arrival of news about total factor productivity or investment-specific shocks.
$q = 0$ implies:

$$k_t = \alpha k_{t-1} + \varepsilon_{A,t}$$  (6)

$q = 1$ implies:

$$k_t = \alpha k_{t-1} + \varepsilon_{A,t} - (1 - \theta)\left(\frac{\tau}{1 - \tau}\right)\varepsilon_{\tau,t}$$  (7)

$q = 2$ implies:

$$k_t = \alpha k_{t-1} + \varepsilon_{A,t} - (1 - \theta)\left(\frac{\tau}{1 - \tau}\right)\left\{\varepsilon_{\tau,t-1} + \theta\varepsilon_{\tau,t}\right\}$$  (8)

$q = 3$ implies:

$$k_t = \alpha k_{t-1} + \varepsilon_{A,t} - (1 - \theta)\left(\frac{\tau}{1 - \tau}\right)\left\{\varepsilon_{\tau,t-2} + \theta\varepsilon_{\tau,t-1} + \theta^2\varepsilon_{\tau,t}\right\}$$  (9)

If there is no foresight, $q = 0$, we get the usual result that i.i.d. shocks to tax rates have no effect on capital accumulation. When there is some degree of tax foresight ($q > 0$), we obtain the unusual implication that even serially uncorrelated tax hikes reduce capital accumulation.

In this model with inelastic labor supply, i.i.d. tax shocks should not affect capital accumulation. As expression (7) makes clear, when there is one period of foresight, tax shocks appear to affect capital contemporaneously. Moving average terms and, therefore, potential invertibility problems appear to arise only when the foresight horizon is greater than one period. This result is not general and stems from the assumption of fixed labor. When labor choice is elastic, even one period of foresight can produce an equilibrium with a non-invertible moving average component.

An interesting, though seemingly perverse, implication of (8) and (9) is that more recent news is discounted (by $\theta = \alpha\beta(1 - \tau) < 1$) relative to more distant news. This is because with two-quarter foresight, $\varepsilon_{\tau,t-1}$ affects $\hat{\tau}_{t+1}$, while $\varepsilon_{\tau,t}$ affects $\hat{\tau}_{t+2}$, so the news that affects tax rates farther into the future receives the heaviest discount. Tax rates are discounted in the usual way, while tax news is discounted in reverse order. We now explore the broader ramifications of this discounting result.

4. The Econometrics of Foresight

We now pose the following question that underlies the premise of this section: What erroneous conclusions might be drawn by an econometrician who ignores foresight and proceeds with the usual vector autoregression analysis?\footnote{Much of the analysis that follows is studied in a more general setting by Hansen and Sargent (1991).}
4.1. Misalignment of Information Sets and Improper Discounting. With fiscal foresight, the information set of private agents may be larger than the econometrician’s information set. The Wold representation theorem tells us that the econometrician’s information set from estimating a VAR will be the sigma algebra generated by current and past observable variables. In our setup, foresight implies that agents will observe tax news before the tax rates are realized. Information sets of private agents will not coincide with the information generated by observable variables, which the econometrician possesses. For example, suppose agents have two-quarter foresight and the econometrician estimates a VAR employing current and past capital and technology, \( \{k_{t-j}, a_{t-j}\}_{j=0}^{\infty} \),

\[
\begin{bmatrix}
a_t \\
k_t
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-\kappa(L+\theta) & 1-\alpha L
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{t,t} \\
\varepsilon_{A,t}
\end{bmatrix}
\]

\[y_t = M(L)\epsilon_t,
\]

where \( \kappa = (1-\theta)(\tau/(1-\tau)) \), \( \theta = \alpha\beta(1-\tau) \) and \( L \) is a lag operator (i.e., \( L x_t = x_{t-s} \)). If this system is invertible in nonnegative powers of \( L \), then \( \epsilon_t \) may be obtained as a square-summable linear combination of current and past \( y_t \). This suggests that knowledge of past and present \( y \) is equivalent to knowledge of past and present \( \epsilon \), and forecast errors generated by the econometrician’s VAR will coincide with the agents’. A necessary condition for (10) to be a Wold (invertible) representation is that the determinant of \( M(z) \) be analytic with no zeros inside the open unit disk.\(^\text{11}\) By direct calculation, \( \det M(z) = \frac{\kappa}{1-\alpha z}(z+\theta) \), so this representation has a zero inside the unit circle (at \( z = -\theta \)). Fiscal foresight implies the information set generated by \( \{y_t, y_{t-1}, y_{t-2}, \ldots\} \) is smaller than the information set generated by \( \{\epsilon_t, \epsilon_{t-1}, \epsilon_{t-2}, \ldots\} \).

It is useful to think about this invertibility condition for each shock process individually. By observing the process for technology, the econometrician will be able to recover current and past technology shocks \( \{\varepsilon_{A,t}, \varepsilon_{A,t-1}, \ldots\} \). However, conditioning on current and past capital will not reveal current and past tax shocks. The reason is that while the technology process is an invertible representation, the process for capital is not. The solution for capital with two-quarter foresight is given by

\[
(1-\alpha L)k_t = -\kappa(L+\theta)\varepsilon_{\tau,t}
\]

Invertibility of the stochastic process requires \(|\theta| > 1\), so that

\[
\begin{bmatrix}
1-\alpha L \\
1+\theta^{-1} L
\end{bmatrix} k_t = -\kappa\theta\varepsilon_{\tau,t}
\]

\(^{10}\)More specifically, the Hilbert space generated by \( \{y_{t-j}\}_{j=0}^{\infty} \) is equivalent (in mean-square norm) to the Hilbert space generated by \( \{\epsilon_{t-j}\}_{j=0}^{\infty} \).

\(^{11}\)See Appendix A for further exposition.
is a convergent sequence. Because \( \theta = \alpha \beta (1 - \tau) < 1 \), the process is not invertible in current and past capital. Of course, the representation is invertible in current and future capital:

\[
\left[ \frac{1 - \alpha L}{1 + \theta L} \right] k_t = -\kappa \varepsilon_{\tau,t-1} \\
k_t = (\alpha^{-1} + \theta)k_{t+1} - \theta(\alpha^{-1} + \theta)k_{t+2} + \theta^2(\alpha^{-1} + \theta)k_{t+3} - \cdots + \alpha^{-1} \kappa \varepsilon_{\tau,t}.
\]

(11)

Therefore the only way in which the econometrician can recover the true innovations seen by agents at time \( t \) in a VAR setting is if they have knowledge of future variables.\(^{12}\)

Foresight concerning fiscal policy introduces a non-invertible moving average representation into the equilibrium. This non-invertibility implies the econometrician will not be able to condition on current and past \( \varepsilon_{\tau,t} \). In order to determine the econometrician’s information set, we need to derive the one-step-ahead forecast errors associated with predicting \( y_t \) conditional only on its past values. This is achieved by flipping the root of the moving average representation from inside the unit circle to outside the unit circle via the Blaschke factor, \([(L + \theta)/(1 + \theta L)]\).\(^{13}\)

The Wold representation for system (10) is given by

\[
\left[ \begin{array}{c} a_t \\ k_t \end{array} \right] = \left[ \begin{array}{cc} 0 & 1 \\ -\kappa(L + \theta) & 1 - \alpha L \end{array} \right] \left[ \begin{array}{cc} 1 + \theta & 0 \\ L + \theta & 0 \\ L + \theta & 0 \\ 1 + \theta L & 0 \end{array} \right] \left[ \begin{array}{c} \varepsilon_{\tau,t} \\ \varepsilon_{A,t} \end{array} \right]
\]

\( y_t = \mathcal{M}^*(L) \left[ \begin{array}{c} \varepsilon_t^* \end{array} \right] \)

(12)

which yields the Wold representation for capital

\[
(1 - \alpha L)k_t = -\kappa(L + \theta)\left[ \frac{1 + \theta L}{L + \theta} \right] \frac{L + \theta}{1 + \theta L} \varepsilon_{\tau,t} \\
= -\kappa(1 + \theta L) \varepsilon_{\tau,t}^* \\
= -(1 - \theta)\left( \frac{\tau}{1 - \tau} \right) \left\{ \theta \varepsilon_{\tau,t-1}^* + \varepsilon_{\tau,t}^* \right\}.
\]

(13)

Note that by observing current and past capital, the econometrician recovers current and past \( \varepsilon_{\tau,t}^* \), which are not the news that private agents observe, \( \varepsilon_{\tau} \). The econometrician’s innovations are the statistical shocks of the VAR, which turn out to represent information that is “old news” to the agents of the economy. To see this more clearly note that

\[
\varepsilon_{\tau,t}^* = \left[ \frac{L + \theta}{1 + \theta L} \right] \varepsilon_{\tau,t} = (L + \theta) \sum_{j=0}^{\infty} -\theta^j \varepsilon_{\tau,t-j} \\
= \theta \varepsilon_{\tau,t} + (1 - \theta^2) \varepsilon_{\tau,t-1} - \theta(1 - \theta^2) \varepsilon_{\tau,t-2} + \theta^2(1 - \theta^2) \varepsilon_{\tau,t-3} + \cdots
\]

(14)

\(^{12}\)Instrumental variables has been suggested as one line of attack in dealing with fiscal foresight [Blanchard and Perotti (2002)]. This representation demonstrates clearly the properties necessary for valid instruments when confronting foresight. Sections 5 and 6 develop this point further.

\(^{13}\)Lippi and Reichlin (1994, 2003) provide a nice overview of Blaschke factors and their use in deriving fundamental representations.
The mapping in (14) shows that what the econometrician believes to be the tax innovation at time $t$, $\varepsilon_t^*$, is actually a discounted sum of the tax news observed by the agents at date $t$ and earlier.

An important implication is that the econometrician who ignores foresight will discount the innovations incorrectly. Comparing (13) with (8), according to the econometrician’s VAR, yesterday’s innovation has less effect than today’s innovation (note the terms $\theta \varepsilon_{\tau, t-1} + \varepsilon_{\tau, t}$ in (13)). Agents with foresight, in contrast, discount news according to $\varepsilon_{\tau, t-1} + \theta \varepsilon_{\tau, t}$ because yesterday’s news has a larger effect on capital accumulation than today’s news. Differences in discounting patterns applied by the econometrician and the agents lead to a variety of econometric problems.

4.2. Generality of Non-Invertibility. It is important to check whether the non-invertibility produced by tax foresight that we displayed above is a general implication of foresight. We examine the robustness of non-invertibility in a real business cycle style model with capital and labor taxes and many of the rigidities now routinely introduced into DSGE models. Given the information flows implied by a tax rule of the form

$$\hat{\tau}_t^Z = \rho \hat{\tau}_{t-1}^Z + \varphi \hat{y}_t + \varepsilon_{\tau, t-q}^Z$$

for $Z = K, L$ representing capital and labor tax rates and for $q = 4$, we have checked Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson’s (2007) condition for invertibility of the moving-average representation in a wide variety of models calibrated to match key moments in U.S. macro time series. Model variations include: elastic labor supply, variable utilization rates for capital and labor inputs, durable and non-durable consumption, habit formation in non-durable consumption, investment adjustment costs, and deliberation costs for durable goods. For the calibrated parameters and some special cases of this general model, whenever the vector of observables does not include the tax news, $\varepsilon^K$ and $\varepsilon^L$, the model produces a non-invertible moving-average representation. While a more systematic analysis is called for, these results suggest that the invertibility problem may be robust to model variations of the kind commonly used to match data.

Specification (15) is a highly stylized model of the flow of news about tax changes. It treats the foresight period, $q$, as the sum of the legislative and implementation lags. A more sophisticated model would allow agents to continuously update their beliefs about the entire path of future taxes during the legislative period and then have the dispersion of future taxes, at least over the near future, narrow sharply upon enactment. Many other variants on information flows are possible as well. Richer specifications of information flows may be important for invertibility problems, an issue that deserves further study.

4.3. Econometric Implications. By employing VAR analysis and ignoring foresight, the econometrician has unknowingly conditioned on a smaller information set and misspecified
the true dynamics of the equilibrium. Not surprisingly, the consequence of this misspecification is that inference provided by VARs may be quite misleading. The size of the approximation error depends upon several factors—the complexity of the tax rule (5), the conditioning variables of the VAR, the structural parameters, the degree of foresight, and so forth. We show below how these factors confound the analysis in the present setting.

Impulse response functions are a widely used tool for conveying how agents dynamically respond to innovations, but response functions based upon the econometrician’s information set \( \{ \varepsilon^*_{\tau,t} \} \) will not capture these responses. Consider the response functions generated by (10) and (12). Figure 2 plots the responses of capital and output assuming two-quarter foresight (setting \( \alpha = 0.36, \beta = 0.99, \tau = 0.25 \)). With foresight, agents know exactly when the innovation in fiscal policy is going to translate into changes in the tax rate. This creates the sharp decline in capital one quarter after the shock and in output two quarters out (because the tax is levied on output which depends on last quarter’s capital stock). The econometrician’s VAR, on the other hand, discounts the innovations incorrectly and gives a very different response to the tax shock. The biggest decline in capital will occur on impact of the innovation suggesting that foresight does not exist. The difference between the response functions can be quite dramatic, especially at short horizons. Note that the drop in output for the econometrician’s VAR is nearly four times as large as the true response one quarter after the innovation.

Figure 2 shows that the econometrician will infer that the tax shock is unanticipated. Obviously, not all shocks that affect fiscal policy will be knowable several quarters in advance. Consider a tax rate process, \( \tilde{\tau}_t = \varepsilon^u_{\tau,t} + \varepsilon_{\tau,t-\varrho} \), that allows for shocks that are anticipated
several quarters in advance ($\varepsilon$) and for surprises that are unanticipated ($\varepsilon_u$) at time $t$. If these shocks are orthogonal at all leads and lags, then the equilibrium dynamics of (3) will not change because i.i.d. tax shocks will not alter the dynamics of capital. The econometrician, who does not account for fiscal foresight and proceeds with estimating (12), will attribute all of the dynamics associated with the anticipated component of the tax rate to the unanticipated component. This suggests that researchers interested in the dynamic effects of fiscal policy—whether the interest is in anticipated or unanticipated changes in policy—must explicitly account for foresight in order to avoid spurious results.

The figure also highlights an important difference between foresight about taxes and the work by Christiano, Ilut, Motto, and Rostagno (2007) and Jaimovich and Rebelo (2008) on news about technology. News about total factor productivity tends to have its biggest impact in the period when technology directly affects output. News about taxes, in contrast, has its biggest impact in the period before the tax rate is realized, although, as the figure suggests, the econometrician will infer that the biggest impact occurs at the time the news arrives.

The econometric problems associated with fiscal foresight become even more pernicious if we make the more plausible assumption that the econometrician cannot perfectly identify technology shocks and, therefore, cannot condition on $a_t$. It is more natural for the econometrician to condition on observable taxes (or some function of taxes, such as revenues), rather than the technology process. Consider the case with two-quarter foresight and suppose the econometrician estimates the following VAR

$$
\begin{bmatrix}
\tilde{\tau}_t \\
\tilde{k}_t
\end{bmatrix} =
\begin{bmatrix}
\frac{L^2}{1-\alpha L} & 0 \\
-k(L+\theta) & \frac{1}{1-\alpha L}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{\tau,t} \\
\varepsilon_{A,t}
\end{bmatrix}
\begin{bmatrix}
x_t = \mathcal{H}(L)\varepsilon_t.
\end{bmatrix}
$$

(16)

As in the previous case, foresight creates a zero inside the unit circle (now at $z = 0$), implying that the information set generated by $\{x_t, x_{t-1}, x_{t-2}, \ldots\}$ is smaller than the information set generated by $\{\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\}$. The corresponding Wold representation for (16) is derived by flipping the zeroes outside the unit circle as in the previous example,

$$
\begin{bmatrix}
\tilde{\tau}_t \\
\tilde{k}_t
\end{bmatrix} =
\begin{bmatrix}
\frac{L^2}{1-\alpha L} & 0 \\
-k(L+\theta) & \frac{1}{1-\alpha L}
\end{bmatrix}
W\mathcal{B}(L)\tilde{W}B(L)B(L^{-1})\tilde{W}B(L^{-1})W'\begin{bmatrix}
\varepsilon_{\tau,t} \\
\varepsilon_{A,t}
\end{bmatrix}
\begin{bmatrix}
x_t = \mathcal{H}^\ast(L)\varepsilon_t.
\end{bmatrix}
$$

(17)

\footnote{Christiano, Ilut, Motto, and Rostagno (2007) posit a process like this for a technology shock and interpret realizations of the $\varepsilon$ terms as news.}
where

\[
W = \begin{bmatrix}
\frac{1}{\sqrt{1+(\theta\kappa)^2}} & \frac{-\kappa\theta}{\sqrt{1+(\theta\kappa)^2}} \\
\frac{\kappa\theta}{\sqrt{1+(\theta\kappa)^2}} & \frac{1}{\sqrt{1+(\theta\kappa)^2}}
\end{bmatrix}, \quad \tilde{W} = \begin{bmatrix}
\Delta(1 + \kappa^2\theta^2) & -\Delta\kappa \\
\Delta\kappa & \Delta(1 + \kappa^2\theta^2)
\end{bmatrix}, \quad \mathcal{B}(L) = \begin{bmatrix}
L^{-1} & 0 \\
0 & 1
\end{bmatrix}
\]

and \(\Delta = [(1 + \kappa^2\theta^2)^2 + \kappa^2]^{-1/2}\). In this case, the orthonormal \(W\) matrix must be employed to ensure the representation remains causal (i.e., preserves the assumption that the econometrician does not observe future values of the variables).

Now the econometric problems are more severe. First, the econometrician who ignores foresight and proceeds with VAR analysis using (17) will obtain an impulse response function in which foresight does not appear to exist in the data. Figure 3a gives the response of capital to a tax shock for the agent and econometrician \((\alpha = 0.36, \beta = 0.99, \tau = 0.25)\). This figure shows that the path of capital is nearly flat when conditioning on the econometrician’s information set. Because the tax shock is assumed to be \(i.i.d\), without foresight there would be no response of capital to a tax shock, so the econometrician is likely to infer that the identification of an unanticipated increase in taxes is good. While these results are meant to be heuristic, we believe this sounds a note of caution; by completely ignoring foresight, one may achieve a “self-fulfilling prophesy” and wrongly conclude that foresight is not an issue.

Second, the statistical shocks recovered by the VAR analysis are now linear combinations of both technology and tax shocks. From (17), the statistical shocks can be written as

\[
\varepsilon_{\tau,t}^* = a_1\varepsilon_{\tau,t-1} + a_2\varepsilon_{\tau,t-2} + a_3\varepsilon_{A,t-1} + a_4\varepsilon_{A,t-2}, \quad \varepsilon_{A,t}^* = b_1\varepsilon_{\tau,t} + b_2\varepsilon_{\tau,t-1} + b_3\varepsilon_{A,t} + b_4\varepsilon_{A,t-1}
\]

(18)
where the \( a \)'s, and \( b \)'s are complicated functions of \( \alpha, \beta \) and \( \tau \). The econometrician will not only misinterpret the response to a tax shock but also the response to a technology shock.

Comparing (18) with (14), one sees that the statistical shocks generated by (17) are not convolutions of the entire history of the structural shocks but only depend upon structural shocks dated \( t - 2 \) and earlier. This is due entirely to the simple tax rule \( \hat{\tau}_t = \varepsilon_{\tau,t-2} \). Making the more realistic assumption that tax rates adjust automatically to technology (as a proxy for output), to reflect progressivity in the tax code that operates through automatic stabilizers,

\[
\hat{\tau}_t = \varphi a_t + \varepsilon_{\tau,t-2},
\]

implies that agents no longer have perfect foresight due to the error in forecasting technology. This additional noise in the tax rule makes the econometrician’s inference especially poor with respect to technology. Figure 3b plots the response function for capital given the tax rule (19) and assuming \( a_t = \rho a_{t-1} + \varepsilon_{A,t} \). The parameter settings are the same as before with \( \varphi = 1 \) and \( \rho = 0.01 \), so the technology shock is nearly i.i.d.. The econometrician would infer a very large and persistent response of capital to a technology shock, when in fact the true dynamics are relatively negligible. This example has a key message: foresight impinges on the econometrician’s ability to correctly identify all shocks in the VAR.

Finally, as also emphasized by Hansen and Sargent (1991), variance decompositions in this environment can also lead to spurious conclusions.\(^{15}\) Let

\[
E(x_t - E^*_t\mid x_t)(x_t - E^*_t\mid x_t)' = \sum_{k=0}^{j-1} H_k^* \Sigma^* H_k'^*
\]

denote the \( j \)-step ahead prediction error variance associated with the econometrician’s information set, where \( \Sigma^* \) is the variance-covariance matrix associated with \( (\varepsilon^*_{\tau,t}, \varepsilon^*_A)_t \)' . Like impulse response functions, variance decompositions are based upon conditional expectations and hence the discrepancy in the information sets implies the coefficients generated by \( H^*(L) \) will lead to a misallocation of the variance across the structural shocks. Figure 3a suggests that the econometrician will treat the tax shock as i.i.d. and infer that none of the variation in capital (and hence output) can be attributed to tax innovations. Consequently, all of the variation will be attributed to the technology shock. This result would continue to hold even if the tax shock explained nearly all of the variation in capital (for example, by setting the variance of the technology shock, \( \sigma^2_A \), arbitrarily small).

---

\(^{15}\)This result holds even though the statistical shocks of the VAR remain uncorrelated. While the unconditional second moments of the VAR system remain the same due to the orthogonal nature of the Blaschke and \( W \) matrices (\( WW' = \bar{W} \bar{W}' = I \) and \( B(L)B(L^{-1}) = I \), the conditional moments will be certainly be different.
4.4. Testing Economic Theory. An obvious extension of the econometric implications is that tests of economic theory will also be misspecified. One such example pertaining to fiscal policy is the testing of the government’s present-value constraint, which links the value of government debt to the expected discounted value of future primary surpluses. A widely-used approach to test present-value restrictions estimates a VAR with debt and surpluses and then tests for the cross-equation restrictions that the present-value condition imposes on the model [Campbell and Shiller (1987)]. As we have shown, fiscal foresight implies the VAR obtained by the econometrician will not yield the true dynamics and hence will not impose the correct cross-equation restrictions.

As a simple example of how foresight will lead to type I error in present-value tests, consider an endowment economy with lump sum taxes, a constant equilibrium real interest rate, and one-quarter foresight with respect to innovations in surpluses (receipts less expenditures net of interest payments on the government’s debt). Taking expectations conditional on information at time \( t - 1 \) of the government’s flow budget constraint yields

\[
E(b_t|\Omega_{t-1}) = \beta^{-1}b_{t-1} - E(s_t|\Omega_{t-1}),
\]

where \( s_t \) is the primary surplus, \( b_t \) is one-period debt outstanding, and \( \beta^{-1} = (1 + r) \) is the constant gross rate of return between time \( t \) and \( t + 1 \). Fiscal sustainability is ensured by a policy rule that makes future surpluses rise with debt. Two exogenous disturbances—for revenues and spending—drive surpluses and agents have one period of foresight over both components of the surpluses. The policy rule is

\[
s_t = \gamma b_{t-1} + \frac{\varepsilon_{1,t-1}}{1 - \rho_1 L} + \frac{\varepsilon_{2,t-1}}{1 - \rho_2 L},
\]

where \( \gamma \) is set to ensure that the agent’s transversality condition for debt is satisfied and \( 0 < \rho_1, \rho_2 < 1 \) determine the serial correlation properties of the driving processes. The expectations are taken with respect to the agents’ information set, which is assumed to be, \( \Omega_{t-1} = \{\varepsilon_{1,t-j}, \varepsilon_{2,t-j}\}_{j=1}^{\infty} \). If this process holds for \( t = 0, 1, ... T \), then imposing the transversality condition on government debt,

\[
\lim_{N \to \infty} \beta^N E(b_{t+N}|\Omega_{t-1}) = 0
\]

implies the present-value restriction that the current value of outstanding debt equals future discounted surpluses,

\[
b_t = \sum_{j=1}^{\infty} \beta^j E(s_{t+j}|\Omega_{t-1}).
\]
Following Hansen, Roberds, and Sargent (1991) and Roberds (1991), the cross-equation restrictions that satisfy (22) are given by

\[
\begin{bmatrix}
s_t \\
b_t
\end{bmatrix} = \begin{bmatrix}
LA(L) \\
\beta^2A(L) - \beta^2A(\beta) \\
\beta L^2 C(L) - \beta^2 C(\beta)
\end{bmatrix} \begin{bmatrix}
\varepsilon_{1,t} \\
\varepsilon_{2,t}
\end{bmatrix}
\]

where \(A(L) = \frac{\beta^{-1} - \gamma}{(1 - \rho_1 L)(1 - \gamma L)}\), and \(C(L) = \frac{\beta^{-1}(1 - \rho_2 L)(1 - \gamma L)}{(1 - \rho_2 L)(1 - \gamma L)}\). Two observations spring from (23). First, foresight implies that (23) is not an invertible representation (due to the zero at \(L = 0\)). Second, the cross-equation restrictions imposed on the moving-average representation are nonlinear.

In light of the second observation, Campbell and Shiller (1987) derive the present-value restrictions on the VAR representation instead of the moving-average representation. This simplification makes the present-value constraint easy to test, as it amounts to restrictions on the coefficients of the VAR. Denote the invertible representation of (23) by \(P^*(L)\) and write the corresponding VAR of (23) as

\[
\begin{bmatrix}
s_t \\
b_t
\end{bmatrix} = A_0^{-1} A_1^* (L) \begin{bmatrix}
s_{t-1} \\
b_{t-1}
\end{bmatrix} + A_0^{-1} \begin{bmatrix}
\varepsilon_{1,t}^* \\
\varepsilon_{2,t}^*
\end{bmatrix}
\]

where \(A^*(L) = P(L)^{-1}\), implying that the coefficients of the VAR will not yield the correct cross-equation restrictions implied by (23) when there is foresight. Campbell and Shiller (1987) show that the restrictions on the VAR coefficients implied by the present-value constraint are given by

\[
a_{11} + a_{21} = 0, \quad a_{22} + a_{12} = \beta^{-1}
\]

With foresight, however, the restrictions given by (25) will not hold even though the present-value constraint is satisfied. The VAR estimates give

\[
a_{11} + a_{21} = \frac{\eta \rho_1 \rho_2 \beta A(\beta) C(\beta)}{\rho_2 C(\beta) - \rho_1 A(\beta)}, \quad a_{22} + a_{12} = \frac{A(\beta) \eta \rho_2 \rho_1 (C(\beta) - A(\beta))}{\beta (\rho_2 C(\beta) - \rho_1 A(\beta))}
\]

where \(\eta = (1 + [A(\beta)C(\beta)]^2)^{-1/2}\). Therefore, the econometrician will incorrectly reject the null hypothesis that the present-value constraint holds.

Sargent (1981) calls for Granger (1969)-Sims (1972) causality tests to play a key role in helping the econometrician determine which variables properly belong in agents’ information

---

\(^{16}\)Given the structure of the non-invertibility, the invertible representation is obtained as in (17).
sets. For example, causality tests are commonly used to justify treating variables as exogenous for purposes of inference.\textsuperscript{17} Causality tests, however, are misspecified if agents have fiscal foresight.\textsuperscript{18} To see this more clearly, return to the analytical model of section 3 with one quarter of foresight and an \textit{i.i.d.} tax rule. The (true) moving-average representation, on the left, and the (econometrician’s) fundamental representation, on the right, in the variables $(\tilde{\tau}_t, k_t)'$ are given by

$$
\begin{pmatrix}
\tilde{\tau}_t \\
k_t
\end{pmatrix} = 
\begin{bmatrix}
L & 0 \\
-\frac{\kappa}{1-\alpha L} & \frac{1}{1-\alpha L}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_{\tau,t} \\
\varepsilon_{A,t}
\end{pmatrix} = 
\begin{bmatrix}
\delta & -\kappa \delta L \\
0 & [\delta (1-\alpha L)]^{-1}
\end{bmatrix}
\begin{pmatrix}
\delta (\varepsilon_{\tau,t-1} + \kappa \varepsilon_{A,t-1}) \\
\delta (-\varepsilon_{\tau,t-1} + \kappa \varepsilon_{A,t-1})
\end{pmatrix}
$$

$$
\begin{pmatrix}
\varepsilon_{\tau,t} \\
\varepsilon_{A,t}
\end{pmatrix} = \mathcal{D}(L) \epsilon_t
$$

where $\delta = (1 + \kappa^2)^{-1/2}$. Note that the zero appearing in the true MA will appear in the opposite off-diagonal in the econometrician’s representation. By theorem 1 of Sims (1972), the econometrician’s representation implies that $\tilde{\tau}$ fails to Granger-cause $k$; in fact, $\tilde{\tau}$ lies in a proper subspace of $k$, and hence $k$ fails to Granger-cause $\tilde{\tau}$. By ignoring foresight, the econometrician effectively reverses the Granger-causal ordering of the true dynamics.

4.5. \textbf{Practical Implications.} The nature of the econometric problem is such that important aspects of empirical research regarding fiscal policy can be misinformed. One such example is the short-run forecasts of macroeconomic aggregates in response to legislative changes. This is an important empirical issue if, as is often the case, the tax change is designed to operate at business cycle frequencies. Econometricians who do not adequately handle foresight may get the forecast of aggregates wrong, especially in the short run. Figure 2 demonstrates that with two-quarter foresight most of the discrepancy between the impulse response functions of the agent and econometrician dissipates only after six quarters. This is due to the moving-average component of the equilibrium and the degree of foresight. As the degree of foresight increases, this discrepancy will become more prominent at longer horizons and the econometrician’s short-run forecast will deviate further from the truth. Hence, it is essential that foresight be confronted when examining the response of macroeconomic aggregates to fiscal stimulus.

Short-run dynamics are also crucial for forecasting the revenue consequences of proposed legislation. Pay-as-you-go (PAYGO) budget rules, which were in effect in the U.S. Congress for fiscal years 1991-2002, then allowed to lapse before being reestablished for one year in 2007 and then abandoned in favor to Alternative Minimum Tax relief, require that tax cuts be offset by spending reductions of other tax increases. This system places great reliance on forecasting tax revenues over short horizons—the horizons over which failing to account

\textsuperscript{17}Rom and Rom (2007b) argue that the failure of output to Granger-cause their measure of tax news reinforces their specification of news as exogenous in an output regression.

\textsuperscript{18}Leep (1990) shows that fiscal foresight can imply that money growth Granger-causes deficits in an equilibrium in which deficits are systematically monetized.
for fiscal foresight can have dramatic consequences. Revenue forecasts play a central role in determining U.S. state government budgets because most states are required to balance budgets every fiscal year or over a two-year budget cycle. Spending decisions then rely heavily on accurate forecasts of revenues.

5. Assessing the Ex-Post Approach

In this section and the next, we evaluate existing empirical efforts to quantify the impacts of fiscal foresight in light of the theoretical framework developed above. We examine both ex-post approaches, which estimate conventional fiscal VARs and attempt to align the agents’ and the econometrician’s information sets through clever identification schemes, and ex-ante approaches, which reject conventional VARs in favor of narrative evidence of fiscal news. Although superficially very different in the way they tackle fiscal foresight, the ex-post and ex-ante approaches can be understood as sharing a common technique: avoid estimating the non-invertible moving average part of macroeconomic time series by obtaining “data” on the exogenous component of anticipated tax changes. This data augmentation can be interpreted as adding information to the standard VAR through instrumental variables. The ex-post approach achieves this “internally” by imposing identification schemes that aim to isolate the exogenous component of anticipated tax changes, while the ex-ante approach achieves this “externally” by bringing into the analysis forecasts generated by other models.

As section 4 shows, fiscal foresight implies the equilibrium moving-average representation will be non-invertible, and the VAR process will not capture the true dynamics of the economy. Suppose the equilibrium moving-average representation is given by

\[ x_t = \sum_{j=0}^{\infty} H_j \epsilon_{t-j} - H(L) \epsilon_t \]

where \( \epsilon_t \) is an \((n \times 1)\) vector of white noise and \( \sum_{j=0}^{\infty} \text{trace} H_j H_j^\top < \infty \). If the representation is invertible in non-negative powers of \( L \), then

\[ A(L)x_t = (A_0 - A_1 L - A_2 L^2 - \cdots) x_t = \epsilon_t \]

where \( A(L) = H(L)^{-1} \). The true equilibrium dynamics may be captured by estimating

\[ x_t = A_0^{-1}[A_1 x_{t-1} + A_2 x_{t-2} + \cdots + \epsilon_t] \]


With fiscal foresight, the representation is non-invertible and the invertible Wold representation obtained by the econometrician will not yield the true dynamics. Writing the invertible
representation as $x_t = \mathcal{H}^\ast(L)e_t^\ast$, the econometrician seeks to identify the parameters in

$$x_t = A^\ast_{0}^{-1} [A^\ast_1 x_{t-1} + A^\ast_2 x_{t-2} + \cdots + e_t^\ast]$$

where $A^\ast(L) = \mathcal{H}^\ast(L)^{-1}$, not the parameters in (27).

We now examine two prominent identification strategies that have acknowledged foresight in the fiscal VAR literature—Blanchard and Perotti (2002) and Mountford and Uhlig (2008). We document the conditions under which these identification schemes will correctly align the information sets and discuss the assumptions that both approaches impose.

Lags in legislative responses to economic shocks are critical to the identification strategies in both Blanchard-Perotti and Mountford-Uhlig. Blanchard-Perotti identify two types of fiscal lags: legislative lags and implementation lags. Legislative lags are the time between the arrival of an economic shock and the response of fiscal policy; implementation lags are the delay between the policy decision and actual implementation of the tax change. Together, the two types of lags lead to fiscal foresight.

Blanchard-Perotti rely heavily on legislative lags to achieve identification. They construct a quarterly VAR in output, $x_t$, government revenues net of transfers, $t_t$, and government spending, $g_t$, and write the reduced-form residuals as

$$t_t = a_1 x_t + a_2 e_t^g + e_t^t$$

$$g_t = b_1 x_t + b_2 e_t^g + e_t^g$$

$$x_t = c_1 t_t + c_2 g_t + e_t^x,$$

where $e_t^t$, $e_t^g$, and $e_t^x$ denote the uncorrelated structural shocks. Blanchard-Perotti achieve identification by arguing that legislative lags ensure that there can be no within-quarter adjustment of fiscal policy to unexpected changes in GDP, other than “automatic effects of activity on taxes and spending under existing fiscal policy rules.” Automatic effects operate through parameters $a_1$ and $b_1$, which are elasticities of tax revenues and government purchases with respect to output.\(^{19}\)

Blanchard-Perotti themselves note that this timing assumption becomes tenuous if agents have long periods of foresight. With fiscal foresight, it is likely that the reduced-form residuals for tax revenues and government expenditures will respond to past innovations in output. More specifically, $a_{11} x_{t-1}$ and $b_{11} x_{t-1}$ would be added to the above equations. Now the identification requirements become more stringent. For example, if there is one quarter of foresight, the Blanchard-Perotti identification strategy requires no discretionary response of fiscal policy to output realizations both this quarter and last quarter. As the total period of foresight increases, this assumption becomes increasingly more untenable. Yang (2007a)

\(^{19}\)Blanchard-Perotti then show that once $a_1$ and $b_1$ are estimated, $t_t - a_1 x_t$ and $g_t - b_1 x_t$ can be used as instruments in estimating $c_1$ and $c_2$. The final two variables are set according to either $a_2 = 0$ and $b_2 \neq 0$ or vice versa to achieve identification of the structural shocks by triangularizing the fiscal sector.
finds that the average time elapsed between a proposal announcement by a president and enactment for major postwar U.S. income tax legislation is about seven months. Mertens and Ravn (2008) report that the median implementation lag is six quarters. If we add these estimates to obtain a measure of total foresight length, we have to impose the unacceptable identifying assumption that fiscal authorities cannot respond to economic conditions for two years or more.

Once this stringent assumption is placed on the model, Blanchard-Perotti then argue that the identified structural shocks of the VAR can be used as instruments for the future tax rates that enter the agent’s information set. To understand why this may not be the case, return to the model of sections 3 and 4.1 with one quarter of foresight and an \textit{i.i.d.} tax rule. The (true) moving-average representation and the (econometrician’s) fundamental representation are given by (26). The VAR representation is found by inverting $D^*(L)$:

$$
\begin{bmatrix}
\delta^{-1} & \delta(L\kappa(1-\alpha L)) \\
0 & \delta(1-\alpha L)
\end{bmatrix}
\begin{bmatrix}
\hat{\tau}_t \\
k_t
\end{bmatrix}
= \begin{bmatrix}
\varepsilon^*_{\tau,t} \\
\varepsilon^*_{A,t}
\end{bmatrix}
$$

$$D^*(L)^{-1}x_t = \epsilon^*_t$$

The fundamental representation obtained by the econometrician implies there is no response of capital to the tax shock, which is the correct outcome in the no-foresight equilibrium [see equation (6) when $q = 0$]. Writing out the VAR representation yields

$$k_t = \alpha k_{t-1} + \eta^k_t,$$  \hfill (28)

$$\hat{\tau}_t = -\kappa \delta^2 k_{t-1} + \kappa \alpha \delta^2 k_{t-2} + \eta^\tau_t.$$  \hfill (29)

where $\eta^k_t = \delta^{-1} \varepsilon^*_{A,t}$ and $\eta^\tau_t = \delta \varepsilon^*_{\tau,t}$

Blanchard-Perotti point out that (28) and (29) are misspecified if foresight is taken seriously. They augment the right-hand side of (28) with $E_t \hat{\tau}_{t+1}$ to align the dynamics associated with the agent’s information set (i.e., capital today should respond to taxes tomorrow with one-quarter foresight). They then argue that the tax shock recovered from (29), $\eta^\tau_t$, pushed forward one period, can be used as an instrument for $E_t \hat{\tau}_{t+1}$. Even assuming that Blanchard-Perotti’s strong identifying assumption holds, the approach still identifies $\eta^\tau_{t+1} = \delta \varepsilon^*_{\tau,t+1}$ as the true tax shock. This is because Blanchard-Perotti begin with a VAR representation and attempt to attack the problem \textit{ex-post}. But even with one-quarter foresight, the true dynamics do not admit a VAR representation. From (26), $\varepsilon^*_{\tau,t+1} = \delta \varepsilon_{\tau,t} + \kappa \varepsilon_{A,t}$. Using the tax rule with one period of foresight, $\hat{\tau}_{t+1} = \varepsilon_{\tau,t}$, the instrument is

$$\eta^\tau_{t+1} = \delta^2 E_t \hat{\tau}_{t+1} + \delta \kappa \varepsilon_{A,t}$$  \hfill (30)

Evidently, $\eta^\tau_{t+1}$ is not a strong instrument as it is only partially correlated with $\varepsilon_{\tau,t}$; indeed, shocks to technology are likely to dominate fluctuations in the instrument. The dynamics
estimated using the Blanchard-Perotti strategy are then
\[ k_t = \alpha k_{t-1} + c \eta_{t+1} + \xi^k_t \]  
(31)
and the tax rule in (29), where the coefficient \( c \) in (31) is a function of the estimates from the instrumental variables regression and \( \xi^k_t \) is a statistical shock that does not correspond to the true innovation in capital. Estimates of the impact of tax foresight, as summarized by \( c \), will reflect anticipated tax rates, but are quite likely to be badly polluted by the influence of current technology shocks.

Moving beyond the theoretical model in this paper, it is not clear to what extent the Blanchard-Perotti approach alleviates the econometric issues associated with fiscal foresight. It is clear that the VAR fails to capture the true dynamics: both the tax rule and the evolution of capital are misspecified. The upshot is that by estimating a VAR first without acknowledging foresight and then attempting to deal with foresight \textit{ex-post}, the Blanchard-Perotti methodology is not able to align the information sets exactly. Given these concerns about the Blanchard-Perotti identification strategy, it is not surprising that they find anticipated tax changes have no significant effect on output and the majority of their results are estimated under the assumption of unanticipated tax policy changes.

Mountford and Uhlig (2008) propose a different solution, which combines the sign restriction method with zero restrictions to identify a VAR. Similar to Blanchard-Perotti, they argue that anticipated fiscal policy changes can be identified by imposing zero restrictions on the responses of fiscal variables over the period of fiscal foresight, reflecting the idea that the isolated policy shock is news about a change in future, but not present, policy variables. Mountford-Uhlig take the ambitious approach of identifying multiple exogenous disturbances: a government revenue shock, a government spending shock, a “business cycle” shock, and a monetary policy shock.

To assess the identification scheme, we consider two aspects of the scheme. First, fiscal foresight does not imply a zero response of all fiscal variables over the foresight period. The various fiscal rules considered in the previous section suggest that this is an exceptional situation. In the special case where the tax rate is exogenous and follows the simple rule
\[ \hat{\tau}_t = e^{u}_{\tau,t} + \varepsilon_{\tau,t-q} \]  
(32)
when news arrives in period \( t \), the tax rate does not change until period \( t + q \). Mountford-Uhlig’s zero restriction, if it were applied to the tax rate, would work in this case.

But Mountford-Uhlig impose the zero restriction on tax revenues. They find that higher anticipated revenues reduce output—and, therefore, the tax base—over the period of foresight. Lower output, coupled with the restriction that revenues are fixed, delivers the eccentric implication that a particular sequence of unanticipated tax-rate increases, \( \{ e^{u}_{\tau,t} \} \), is imposed to identify an anticipated tax hike. Considering that in most countries automatic
stabilizers in the tax code would lower rates when output falls, it is difficult to believe that the identification scheme has isolated the effects of fiscal foresight.

Second, it is not clear that the sign restriction methodology will be able to correctly identify non-fiscal shocks. Mountford and Uhlig claim to identify a business cycle shock and a monetary policy shock, but when a non-invertibility exists, these shocks are typically complicated convolutions of all shocks entering the agent’s information set. Fiscal foresight is a pervasive problem and forcing the impulse response function of fiscal variables to react in a certain way does not impose enough structure on the system to correctly identify the other shocks [see figure 3].

6. Assessing the Ex-Ante Approach

We share the view of the ex-ante approach that, in the presence of fiscal foresight, conventional fiscal VARs misalign the information sets of economic agents and the econometrician. In the context of the model in section 3, conventional VARs estimate systems in current and past values of capital (or output) and revenues. Fiscal foresight implies that those systems are not invertible and do not adequately capture the fiscal news to which agents respond. When tax rates are exogenous, as in the simple example, information sets are correctly aligned by including future tax rates in the VAR. This results in a VAR system in \( \{k_t, \tau_{t+q}\} \) and now the fundamental representation is invertible. The ex-ante approach essentially applies this principle by seeking instruments for expected future tax obligations.

The discussion in section 4 and expression (11) make this interpretation more precise. Although non-invertibility of the moving average representation implies there is no autoregressive representation in which the true fiscal news is a function of current and past endogenous variables, there is an autoregressive representation in the fiscal news and future endogenous variables. The ex-ante approach uses forecasts of revenue changes associated with tax legislation to instrument for the information agents possess about future taxes. To infer the effects of anticipated taxes on output, the ex-ante approach regresses output against forecasted revenue changes, among other variables, and interprets the estimated coefficients causally. To assess the ex-ante approach, we examine the quality of instruments employed.

Of course, tax rates are not exogenous. They are the outgrowth of a complex set of economic and political decisions. Recognizing the intrinsic endogeneity of tax policy decisions, Romer and Romer (2007a,b) use a narrative method to compile data series that decompose the forecasted revenue consequences of federal tax changes into “endogenous” and “exogenous” components. Mertens and Ravn (2008) use the Romers’ compiled data series. They generalize the Romers’ empirical work and lay out an intricate DSGE model to interpret their estimates of the impacts of anticipated and unanticipated changes in taxes. Whereas the Romers find only weak evidence that private agents react to anticipated tax changes,
Mertens and Ravn obtain provocative and striking results reminiscent of Branson, Fraga, and Johnson’s (1986) argument about the Reagan tax cuts: anticipated tax cuts induce sharp economic slowdowns during the period of fiscal foresight, and may even produce recessions.

In this section we use a standard real business cycle model with proportional capital and labor tax rates to simulate equilibrium data, including forecasted revenue changes induced by anticipated and unanticipated tax disturbances. We then run regressions using simulated data and compare the estimated effects of foreseen changes in tax rates to the true effects of fiscal foresight. Because the simulated data and revenue forecasts are generated by a single coherent model, if the ex-ante approach is efficacious, the regressions should recover the true effects almost exactly.

Before we can proceed with this test of the ex-ante method, we first must embed the narrative identification scheme in a formal theoretical model.

6.1. **Formalizing the Narrative Identification.** The Romers distinguish between “endogenous” changes in taxes—ones induced by short-run countercyclical concerns and those undertaken because government spending was changing—and “exogenous” changes in taxes—those that are responses to the state of government debt or to concerns about long-run economic growth. To avoid confusion with other definitions, we shall refer to these as “RR endogenous” and “RR exogenous” components of tax policy behavior.

We specify a tax rule that includes the various motivations for tax changes that the Romers consider and embeds both anticipated and unanticipated shocks to taxes. Alternative parametric specifications of policy coincide with different formalizations of the narrative identification scheme. Our message is that the performance of the ex-ante approach hinges critically on the precise formalization attributed to the narrative identification.

To reflect the distinction the Romers draw between “endogenous” countercyclical concerns and “exogenous” long-run concerns, it is convenient to decompose output into business cycle, $y^C_t$, and trend, $y^T_t$, components. A rule for tax rates that embeds this multiplicity of motivations for tax changes is given by

$$
\tau_t = \rho(L)\tau_{t-1} + \sum_{j=-P}^{P} \mu_j^C E_t y^C_{t+j} + \sum_{j=-M}^{M} \beta_j E_t g_{t+j} \\
+ \sum_{j=-P}^{P} \mu_j^T E_t y^T_{t+j} + \sum_{j=-N}^{N} \gamma_j E_t s^B_{t+j} + \varepsilon_{\tau,t-q} + \varepsilon_{\tau,t} 
$$

(33)
The fiscal authority’s choice of the current tax rate is permitted to respond systematically to current, past, and expected fluctuations in output at both business cycle and trend frequencies and to current, past, and expected changes in government spending \((g_{t+j})\) and government indebtedness as measured by the debt-to-output ratio \((s_{t+j-1}^{B})\). The rule also embeds an unanticipated shock, \(e_{\tau, t}^{u}\), and “news” about the tax rate that arrived \(q\) periods in the past, \(\varepsilon_{\tau, t-q}\). Both of these shocks are assumed to be unrelated to economic conditions.\(^{20}\)

To study the Romers’ identification, we simplify (33) by restricting the “RR endogenous” component and the feedback from trend output movements in the “RR exogenous” component. We also specialize the timing of the response to the state of government debt to coincide with the period of foresight, \(q\), and allow only one lag of the tax rate to enter, \(\rho(L) = \rho\). This simplifies (33), written in terms of its anticipated and unanticipated parts, to

\[
\tau_t = \rho \tau_{t-1} + \mu^C y_t + \xi_{t-q} + e_{\tau, t}^{u}
\]

where

\[
\xi_{t-q} = \mu^T y_{t-q-1} + \gamma s_{t-q-1}^{B} + \varepsilon_{\tau, t-q}
\]

is the fiscal foresight, which stems from both systematic responses of taxes to past economic and fiscal conditions and exogenous news about tax legislation. By simplifying the tax rule to restrict the sources of feedback from the economy to expected future tax rates, we are likely to bias our results in favor of the *ex-ante* narrative approach.

It might seem like a stretch to model the response of tax policy to concerns about long-run economic growth as we do in the definition of \(\xi_{t-q}\). But several large tax bills that Romer and Romer (2007b) label as long-run, “exogenous” tax changes could easily be categorized as “endogenous” responses to short-term economic conditions. Stein (1996) documents that President Kennedy was prompted to change his position on a tax cut by the stalled recovery in 1962 and 1963 from the 1960-1961 recession.\(^{21}\) The Economic Recovery Act of 1981 signed by President Reagan is widely regarded as driven by philosophical considerations. But the supply-side promise to stimulate growth without triggering inflation, is arguably an endogenous reaction to the stagflation of the 1970s and early 1980s. The Romers classify two recent tax cut bills signed by President Bush—part of the Economic Growth and Tax Relief Reconciliation Act of 2001 and the Working Families Tax Relief Act of 2003—as long-run “exogenous” events. The Economic Report of the President (2002, p. 44) argues, “The President laid a strong foundation for growth in 2001 with the Economic Growth

\(^{20}\)Although in their papers the Romers do not explicitly interpret tax legislation as containing shock components, in private communication David Romer confirmed that this interpretation is not inconsistent with their views.

\(^{21}\)The unemployment rate fell from 6.7 percent in October, 1961 to 5.5 percent in March, 1962 and then leveled off for the remainder of 1962. Output growth was slower than in the previous year. Stein (1996) writes that the proposal to cut taxes “was a delayed response to a chronic condition after hopes of a spontaneous recovery were dimmed” [p. 408].
and Tax Relief Reconciliation Act. This package provides a powerful stimulus for future growth....” But the tax cut bills enacted in 2001, 2002, and 2003 were also clearly linked to the recession in 2001 and its subsequent “jobless” recovery. Congressional Quarterly Press (2006) documents that in the case of the 2003 tax cut, President “Bush continued to insist that tax cuts were the best way to deal with both the budget deficit and the slow pace of job creation” [p. 42]. Evidently there is no sharp distinction between tax cuts motivated by countercyclical considerations and those driven by a desire to boost economic growth in the long run.

We have specified a rule for future tax rates, but the Romers and Mertens and Ravn employ forecasts of tax revenues. We simulate the model to generate data and model-generated forecasts of revenue changes due to both the unanticipated, $e_{u,t}$, and anticipated, $\varepsilon_{r,t-q}$, exogenous disturbances to capital and labor tax rates. Although the Romers estimate single-equation regressions, we reproduce the slightly more general estimated VARs that Mertens and Ravn use to report the dynamic impacts of the two kinds of tax shock. Specifically, we estimate

$$X_t = A + CX_{t-1} + \sum_{i=0}^{24} D_i T_{t-i}^u + \sum_{i=0}^{24} F_i T_{t-i}^a + \sum_{i=1}^{6} G_i T_{t+i}^a + u_t,$$

(36)

where $X_t$ is a data vector that includes output, consumption, investment, and hours worked, $X_t = [\ln y_t, \ln c_t, \ln i_t, \ln l_t]'$. $T_t^u$ is revenue changes divided by output due to the unanticipated tax shock and $T_{t+i}^a$ is the out-of-sample forecast of revenue changes for anticipated tax policy divided by output.22 Forecasts are conditional on information at $t$, for each date in the simulated data. Since the DSGE model we use to generate data has separate exogenous shocks for capital and labor tax rates, estimation of (36) is done separately for the two taxes; therefore, to estimate the effects of an anticipated capital tax cut, $T_{t+i}^a$ and $T_t^u$ in (36) are associated with capital tax changes, and vice versa for labor taxes.23 Details of the neoclassical growth model and the forecast procedure appear in Appendix B.

Romer and Romer and Mertens and Ravn share the critical maintained assumption that forecasted revenue changes are the exogenous news about taxes. This assumption explains why the system in (36) does not include an equation that describes the evolution of revenues or government debt over time. Implicitly in Romer and Romer and explicitly in Mertens

---

22The Romers' data set scales revenues by actual future output, which treats a function of future shock realizations as a regressor in (36). We follow their procedure in the simulations.
23The Romers and Mertens and Ravn do not distinguish between capital and labor tax changes in their empirical work. Sorting revenue forecasts into those due to capital and labor tax policy changes is a difficult task, as a single provision in a tax bill often affects both capital and labor income taxes simultaneously. For example, an across-the-board individual income tax rate reduction would change both types of taxes. In addition, Yang (2005) shows that anticipated capital and labor taxes can have very different effects and that assuming a single tax rate on both sources of income can mask the impacts of fiscal foresight.
and Ravn’s theoretical model, lump-sum transfers are assumed to adjust to keep the government solvent. But this Ricardian assumption conflicts with the way that “RR exogenous” changes in taxes are constructed: as the rule in (33) makes clear, that constructed measure includes legislative actions that are a response to budget deficits or the state of government indebtedness.

6.2. Simulation Results. Revenue forecasts provide an important input to fiscal decisions by policy makers at both the federal and the state levels. Large fluctuations in tax bases make revenues notoriously difficult to forecast accurately. One way to mimic the difficulties inherent in forecasting revenues is to add measurement error that is unrelated to economic fundamentals. An alternative, more economically grounded method, is simply to build into the theory multiple sources of uncertainty. In addition to unanticipated and anticipated shocks to capital and labor tax rates, the DSGE model used to simulate data includes several other sources of random variation—shocks to technology, preferences over leisure, government spending, and government transfers. Multiple sources of uncertainty imply that forecasted tax rates, \( E_t \tau_{t+q} \), are a function of many different structural disturbances whose effects on taxes operate through the endogenous variables.

Figures 4-7 depict the paths of consumption and output in response to six-period foresight about cuts in labor and capital tax rates. Shocks to tax rates are assumed to be correlated, though not perfectly, as they are in data. Panels (a)-(d) reflect alternative parametric formalizations of the narrative identification. These impulse response functions are derived from estimates of (36) using 1000 sample paths generated by the growth model.\(^{24}\) Heavy solid lines are the true theoretical impacts; thin solid lines are the means of the estimated impacts; dashed lines are 68 percent probability bands for the estimated responses.

Panel (a) is the best-case scenario for the narrative approach. It shuts down all responses of tax rates to economic conditions and has lump-sum transfers adjust to stabilize debt. The policy rule becomes \( \tau_t = \rho \tau_{t-1} + \varepsilon_{\tau,t-q} + \varepsilon_{\tau,t}^B; \) transfers evolve according to \( T_t = \gamma_T s_{t-1}^B \), with \( \gamma_T > 0 \). Across all four figures, estimates of (36) do a very good job of recovering the theoretically correct responses.\(^{25}\)

Once tax rates respond to debt, estimates based on the VAR in (36) can go badly astray over both the period of foresight and longer horizons. Panels (b)-(d) each impose that labor and capital tax rates adjust to stabilize debt \((\gamma_T > 0)\); they differ in the degree to which tax policy choices react to output and in the relative variability of anticipated and unanticipated exogenous disturbances to taxes. Panel (b) comes from a model that allows for

\(^{24}\)See Appendix B.4 for details on how the impulse response functions are computed.

\(^{25}\)Discrepancies between the thin and the thick solid lines arise from the fact that the Romers and Mertens and Ravn scale forecasted revenue changes by actual future GDP, a procedure that we mimic, whereas the true theoretical responses do not include this scaling.
Figure 4. Responses of Consumption to 6-Period Foresight of Labor Taxes.

Panel (a) $\mu_C = 0, \mu_T = 0, \gamma_T = -0.1, \sigma_K = 0.025, \sigma_L = 0.02$

Panel (b) $\mu_C = 1, \mu_T = 0, \gamma_T = 0.05, \sigma_K = 0.025, \sigma_L = 0.02$

Panel (c) $\mu_C = 1, \mu_T = 0.5, \gamma_T = 0.05, \sigma_K = 0.025, \sigma_L = 0.02$

Panel (d) $\mu_C = 1, \mu_T = 0.5, \gamma_T = 0.05, \sigma_Ka = 0.0375, \sigma_Ku = 0.0125, \sigma_La = 0.03, \sigma_Lu = 0.01$

automatic stabilizers in the tax code ($\mu_C > 0$); panel (c) includes both automatic stabilizers and fiscal foresight that includes a systematic response to past output ($\mu_T > 0$); panel (d) includes both of these components but raises the variance of anticipated tax shocks relative to unanticipated tax shocks, reflecting the fact that because most tax changes are implemented with a lag, anticipated changes are more prevalent and more important. Modeling “RR exogenous” tax changes as including a systematic response of tax rates—as opposed to lump-sum transfers—to the state of government debt is fully consistent with the Romer’s narrative, so panels (b)-(d) of the figures provide more appropriate assessments of the ex-ante approach.

The ex-ante approach may perform quite well over the period of foresight, as it does in estimating the response of consumption to foresight about a capital tax rate cut in figure 5 (see also figure 6). But it can also perform very poorly. Figure 4 shows that an anticipated cut in labor taxes creates a boom in consumption in the foresight period, while estimates of (36)
Figure 5. Responses of Consumption to 6-Period Foresight of Capital Taxes.

Panel (a) $\mu_C = 0, \mu_T = 0, \gamma_T = -0.1, \sigma_K = 0.025, \sigma_L = 0.02$

Panel (b) $\mu_C = 1, \mu_T = 0, \gamma_T = 0.05, \sigma_K = 0.025, \sigma_L = 0.02$

Panel (c) $\mu_C = 1, \mu_T = 0.5, \gamma_T = 0.05, \sigma_K = 0.025, \sigma_L = 0.02$

Panel (d) $\mu_C = 1, \mu_T = 0.5, \gamma_T = 0.05, \sigma_{K_a} = 0.0375, \sigma_{K_u} = 0.0125, \sigma_{L_a} = 0.03, \sigma_{L_u} = 0.01$

find that a substantial recession is quite likely. A less pronounced slump in consumption is estimated for the response of output to a foreseen capital tax cut, when the correct theoretical response is a mild expansion [figure 7]. The inference that a recession occurs before an anticipated cut in taxes coincides closely with Merten and Ravn’s results from estimating (36) based on the Romer’s data on changes in tax liabilities.

Difficulties with the ex-ante approach are not limited to inferences about the effects of foresight over the short run. Figure 4 shows that over horizons of five or more years, it is very unlikely that estimates of (36), which die out rather quickly, will recover the medium-run decline in consumption following a reduction in labor tax rates. The source of the mispredictions is that the VAR system in (36) treats the changes in revenues forecasts, the $T_{t+i}$ terms, as exogenous “shocks” that are not systematically related to the state of the economy. This treatment fails to provide agents with the structural information that debt-financed tax cuts will ultimately bring forth higher tax rates still farther in the future. In
Figure 6. Responses of Output to 6-Period Foresight of Labor Taxes.
Panel (a) $\mu^C = 0, \mu^T = 0, \gamma_T = -.1, \sigma_K = .025, \sigma_L = .02$
Panel (b) $\mu^C = 1, \mu^T = 0, \gamma_T = .05, \sigma_K = .025, \sigma_L = .02$
Panel (c) $\mu^C = 1, \mu^T = .5, \gamma_T = .05, \sigma_K = .025, \sigma_L = .02$
Panel (d) $\mu^C = 1, \mu^T = .5, \gamma_T = .05, \sigma_{Ka} = .0375, \sigma_{Ku} = .0125, \sigma_{La} = .03, \sigma_{Lu} = .01$

other words, given how the revenue forecasts are constructed, treating them as evolving autonomously amounts to misspecifying the tax rule. Panels (b)-(d) of figure 4 make clear that misspecification of the tax rule is the source of the medium-run mispredictions: when lump-sum transfers adjust to stabilize debt, as in panel (a), the estimated system in (36) nails the responses at longer horizons.

Our simulation exercise dramatically understates the uncertainty inherent in revenue forecasts because our model forecaster knows the true structure of the economy. If the *ex-ante* approach cannot consistently work in our idealized laboratory, the noise associated with actual revenue forecasts is likely to hinder severely the method’s ability to recover anticipated tax effects.

This assessment of the *ex-ante* approach employs a barebones real business cycle model and a relatively crude specification of tax policy behavior. A model with many more parameters
and internal propagation mechanisms or a more sophisticated characterization of policy can generate far more exotic dynamics. But greater complexity does not alter the basic message: success of the *ex-ante* approach hinges on how the narrative method of identifying tax news is formalized. Even simple theory can produce a wide range of conclusions about the efficacy of the approach. Two factors emerge as critical to the success of the *ex-ante* approach: the degree to which forecasted revenue changes reflect exogenous changes in taxes and the relative volatility of the random components of tax decisions.

7. **Concluding Remarks**

We have explicitly shown how fiscal foresight introduces econometric difficulties that complicate the interpretation of VARs. The crux of the issue is that foresight introduces non-invertible moving average representations into the linear equilibrium process. Non-invertible
moving average representations imply that the mapping between the true tax news that agents observe and the “tax shocks” that the econometrician identifies can be vastly different. The models illustrate the kinds of mistaken inferences an econometrician is likely to draw. Many of the econometric techniques in macroeconomists’ toolboxes—impulse response functions, variance decompositions, Granger-causality tests, tests of cross-equation restrictions—can be badly distorted by empirical methods that do not adequately estimate the non-invertible moving average components of equilibrium time series.

Existing time series work on fiscal foresight—what we have termed the ex-post and ex-ante approaches—seeks instrumental variables that contain the agents’ information about future tax changes. Because foresight implies that conventional VARs recover “tax shocks” that are convolutions of all the exogenous disturbances, the VAR innovations employed in the ex-post analyses are likely to be poor instruments. We have shown that the success of the ex-ante approach depends critically on how the narrative identification scheme is formalized. We modeled a class of parametric formalizations and found that a wide variety of conclusions about the approach’s efficacy are possible.

Both approaches also fail to model the information flow associated with fiscal news. Ex-post analysis misspecifies the tax policy rule, while ex-ante analysis leaves the policy rule unspecified. In the presence of fiscal foresight, neither approach correctly identifies the behavior underlying tax policy choices.

Moving beyond the narrow issue of identifying the impacts of fiscal foresight, the ex-ante approach is likely to be of limited utility even if it successfully identifies the impacts of exogenous news about future tax changes. Imagine that some non-tax shock raises the market value of government debt; for example, an open-market sale of bonds. In our formalization of the narrative method, this would raise expected future tax rates. Because the initial shock carries with it the expectation of higher taxes, the shock’s impacts will be different than if, say, lump-sum transfers adjust to stabilize debt. But the ex-ante approach, by treating forecasted revenue changes as exogenous regressors, does not model the linkages between non-tax shocks and subsequent changes in revenues. Even if the approach correctly identifies fiscal foresight, because it does not identify fiscal policy behavior, it does not lead to a way to integrate fiscal policy into the broader research programs being pursued at central banks and policy institutions that are estimating DSGE models.

Future research will focus on developing the econometric techniques necessary to handle the issues implied by the analytical framework established here. In so doing, the research will inform how to integrate fiscal analysis into models designed to address a broad range of economic questions.
More on the Econometrics of Fiscal Foresight

This appendix provides a more detailed discussion of the econometric issues of fiscal foresight. While there are several quality references on this issue, a few of the derivations in the paper have been done without full rigor. For example, deriving the VAR representation for a non-invertible MA representation.

Suppose we have a multivariate MA(1) process where $y_t$ is a vector of length $k$ and $\epsilon_t$ is a white noise process

$$y_t = \epsilon_t + \Omega_1 \epsilon_{t-1} \quad (37)$$

then recursive substitution implies

$$\epsilon_t = y_t - \Omega_1 \epsilon_{t-1} = y_t - \Omega_1 (y_{t-1} - \Omega_1 \epsilon_{t-2}) = \cdots$$

$$= y_t - \Omega_1 y_{t-1} + \cdots + (-\Omega_1)^n y_{t-n} + (-\Omega_1)^{n+1} \epsilon_{t-n-1}$$

$$= y_t + \sum_{i=1}^{\infty} (-\Omega_1)^i y_{t-i}$$

While this relationship holds for all $t = 0, \pm 1, \pm 2, \ldots$, it is only valid if $\Omega_1^i$ goes to zero as $i$ goes to infinity. Hence, $\{y_t\}$ and $\{\epsilon_t\}$ span the same space if and only if $\lim_{i \to \infty} (-\Omega_1)^i \to 0$. In order for this condition to hold, the eigenvalues of $\Omega_1$ must be less than unity in modulus. That is, $\det(\Omega_1 - \lambda I_k) \neq 0$ for $|\lambda| \geq 1$. Note that a completely analogous condition is given by rewriting (37) as $y_t = (I_k + \Omega_1 L) \epsilon_t$ and requiring $\det(I_k + \Omega_1 z) \neq 0$ for $|z| \leq 1$. If either of these conditions hold, the corresponding VAR representation is given by

$$y_t = -\sum_{i=1}^{\infty} (-\Omega_1)^i y_{t-i} + \epsilon_t. \quad (38)$$

This representation implies that even though an economic equilibrium may contain moving average representations, a VAR with sufficient lags will provide an arbitrarily good approximation to the underlying equilibrium process. However when foresight is present, this approximation is no longer valid because the VAR representation given by (38) does not exist. These conditions can be generalized to an economic equilibrium with generic moving average representation, $y_t = M(L) \epsilon_t$. The requirement for invertibility is the determinant of $M(z)$ contain no zeros inside the unit circle.

While this condition is easy to check in theory, in practice it may be easier to follow Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007) and derive these conditions using a state-space representation. The time invariant state space system is given
by
\[ x_{t+1} = Ax_t + B\epsilon_t \]
\[ y_t = Cx_t + D\epsilon_t \]
where \( \epsilon_t \) are the true economic shocks. Using the lag operator \( L \) and solving for \( y_t \) gives
\[ y_t = [C(LI - A)^{-1}B + D]\epsilon_t = H(L)\epsilon_t, \tag{39} \]
which states the relationship between the observable vector and underlying shocks of the model. If the determinant of \( H(z) \) has no zeros inside the unit circle (or equivalently if the eigenvalues of \( A - BD^{-1}C \) are inside the unit circle), then the representation is invertible.

Fiscal foresight and other types of non-invertibility can be thought of as a reduction in the state variables in the state-space system available to the econometrician. The assumption made throughout the paper is that agents are able to see the realizations of shocks and that future taxes are a function of current shocks. The econometrician who only observes taxes, therefore, is conditioning on a smaller set of state variables—denote this set of variables by \( \hat{x}_t \). Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007) show that for every state space representation there exists an innovations representation (one pertaining to the econometrician’s information set) given by
\[ \hat{x}_{t+1} = A\hat{x}_t + B\xi_t \]
\[ y_t = C\hat{x}_t + D\xi_t \]
where \( \xi_t \) is the one-step-ahead forecast error of predicting \( y_t \) conditional on its past (i.e., \( \epsilon_t \equiv y_t - P[y_t|y_{t-1}, y_{t-2}, \ldots] \)). Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007) then show that the VAR innovations of the econometrician are a combination of the true economic shocks \( D\epsilon_t \) and the error associated from estimating the state \( C(x_t - \hat{x}_t) \).26

**APPENDIX B. GROWTH MODEL AND FORECASTS OF REVENUE CHANGES**

The model is completely conventional and is a stripped down version of the model employed by Mertens and Ravn (2008).

**B.1. The Model.** The section describes the data generating process for the simulated data used in the simulation exercises in section 6. The household chooses consumption, \( c_t \), capital, \( k_t \), hours worked, \( l_t \), and one-period government bonds, \( b_t \), to maximize expected utility, given by
\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\mu} - 1}{1 - \mu} + \chi_t \frac{(1 - l_t)^{1-\theta} - 1}{1 - \theta} \right], \]

26 An additional benefit to using the state space representation is that the Kalman filter will automatically solve the correspondence between the true economic shocks and the statistical shocks.
subject to the budget constraint
\[ c_t + k_t + b_t = \left(1 - \tau^K_t\right) r_t k_{t-1} + \left(1 - \tau^L_t\right) w_t l_t + (1 - \delta) k_{t-1} + b_{t-1} R_{t-1} + T_t, \]
taking all prices and policies as given. \( E_t \) is the mathematical expectation conditional on the household’s information set at \( t \). \( \beta \) is the discount factor \((0 < \beta < 1)\), \( \mu \) and \( \theta \) are the inverses of elasticities of intertemporal substitution of consumption and leisure \((\mu > 0\) and \( \theta \geq 0)\), \( \tau^K_t \) and \( \tau^L_t \) are proportional tax rates levied on capital and labor income, \( T_t \) is lump-sum transfers if positive (taxes if negative), \( \delta \) is the capital depreciation rate \((0 \leq \delta \leq 1)\), and \( R_{t-1} \) is the gross real interest rate on government bonds. \( \chi_t \) is the preference weight on leisure, which follows the process
\[ \chi_t = \chi e^{\chi_t}, \]
and \( \chi_t \sim iid N(0, \sigma^2_\chi) \).

A representative firm rents capital and labor from the household to maximize profit
\[ y_t - w_t l_t - r_t k_{t-1}, \]
where \( w_t \) is the real wage, \( r_t \) is the capital rental rate, and, in log-linearized form, technology obeys
\[ \ln A_t = \rho_A \ln A_{t-1} + \varepsilon^A_t. \]
Total goods produced each period are \( y_t = A_t k_{t-1}^{1-\alpha}. \)

Tax policy rules allow tax rates to respond to endogenous variables and to anticipated (superscripted “a”) and unanticipated (superscripted “u”) random components, which are uncorrelated. We allow for \( q \) periods of foresight. The tax rates themselves are permitted to be correlated, reflecting the fact that typical tax legislation usually changes both labor and capital tax rates. In log-linearized forms the rules are
\begin{align*}
\hat{\tau}^K_t &= \rho_{\tau} \hat{\tau}^K_{t-1} + \mu C y_t + \varepsilon^K_t + \xi^K_t, \quad (40) \\
\hat{\tau}^L_t &= \rho_{\tau} \hat{\tau}^L_{t-1} + \mu C y_t + \varepsilon^K_t + \xi^K_t, \quad (41) \\
\xi^K_t &= \mu T y_{t-1} + \gamma_{\tau} s^B_{t-1} + \varepsilon^K_t, \quad (42) \\
\xi^L_t &= \mu T y_{t-1} + \gamma_{\tau} s^B_{t-1} + \varepsilon^K_t, \quad (43)
\end{align*}
and
\begin{align*}
\xi^K_t &= \mu T y_{t-1} + \gamma_{\tau} s^B_{t-1} + \varepsilon^K_t, \quad (42) \\
\xi^L_t &= \mu T y_{t-1} + \gamma_{\tau} s^B_{t-1} + \varepsilon^K_t, \quad (43)
\end{align*}
where \( s^B_{t-1} = b_{t-1}/y_{t-1} \). We allow correlation between the two tax rates, so
\begin{align*}
\varepsilon^K_t &= \rho_{K L} \varepsilon^L_t + \varepsilon^K_t \quad (44) \\
\varepsilon^K_t &= \rho_{K L} \varepsilon^L_t + \varepsilon^K_t \quad (45) \\
\varepsilon^K_t &= \rho_{K L} \varepsilon^L_t + \varepsilon^K_t \quad (46) \\
\varepsilon^K_t &= \rho_{K L} \varepsilon^L_t + \varepsilon^K_t \quad (47)
\end{align*}
where $\rho_{KL}$ characterizes the correlation between two tax shocks, $\varepsilon_{t}^{Ku} \sim iid N(0, \sigma_{Ku}^2)$, $\varepsilon_{t}^{Lu} \sim iid N(0, \sigma_{Lu}^2)$, $\varepsilon_{t}^{Ka} \sim iid N(0, \sigma_{Ka}^2)$, and $\varepsilon_{t}^{La} \sim iid N(0, \sigma_{La}^2)$.

Tax rules (40) and (41) imply that, in addition to the AR(1) component, the tax rates allow for an automatic stabilizer ($\mu_{C} \geq 0$), and the two tax rates at $t$ depend on unexpected exogenous shocks, $\varepsilon_{t}^{Ku}$ and $\varepsilon_{t}^{Lu}$, and the announced changes, $\xi_{t-q}^{K}$ and $\xi_{t-q}^{L}$. Fiscal foresight, as characterized by $\xi_{t-q}^{K}$ and $\xi_{t-q}^{L}$ in (42) and (43) model two motivations explicitly: controlling debt ($\gamma_{T} > 0$) and counteracting output declines ($\mu_{T} > 0$), plus the random disturbances ($\varepsilon_{t}^{Ka}$ and $\varepsilon_{t}^{La}$). We allow one-quarter legislative lags to reflect the delay between government’s recognition of the need to act and the actual passage of tax law changes. Actual lags are typically longer than one quarter.

To complete the model specification, transfers and government spending follow AR(1) processes:

$$\hat{s}_{t}^{G} = \rho_{G}\hat{s}_{t-1}^{G} + \varepsilon_{t}^{G}$$
and

$$\hat{s}_{t}^{T} = \rho_{T}\hat{s}_{t-1}^{T} + \gamma_{T}\hat{s}_{t-1}^{B} + \varepsilon_{t}^{T},$$

where $s_{t}^{G} = g_{t}/y_{t}$, $s_{t}^{T} = T_{t}/y_{t}$, $\varepsilon_{t}^{G} \sim iid N(0, \sigma_{G}^2)$ and $\varepsilon_{t}^{T} \sim iid N(0, \sigma_{T}^2)$. Some exercises will allow lump-sum transfers to adjust to stabilize debt ($\gamma_{T} < 0$, $\gamma_{T} = 0$) while others will have tax rates adjust to debt ($\gamma_{T} > 0$, $\gamma_{T} = 0$).

The government’s budget constraint at each period is

$$b_{t} = g_{t} + R_{t-1}b_{t-1} - \tau_{t}^{L}w_{t}l_{t} - \tau_{t}^{K}r_{t}k_{t-1} + T_{t},$$

B.2. Revenue Forecasts. The Romers and Mertens and Rvan use government produced revenue forecasts of tax bills as proxies to capture agents’ knowledge about future tax policy changes. In our theoretical environment, government forecasts are not available. Instead, we compute forecasts in a way that captures the information set upon which government revenue forecasts are based.

In practice, when a tax bill is enacted at $t$, revenue forecasts are produced conditional on the available information at $t$. In the model, tax changes are enacted each period, which signal tax policy to be implemented $q$ quarters later. The size of this tax rate change is characterized by $\xi_{t}^{K}$ and $\xi_{t}^{L}$. To generate the revenue forecast of $t+q$ for the enacted tax change at $t$, we multiply the size of the tax rate change of $t+q$ known at $t$ with the forecasted tax base of $t+q$, conditional on information at $t$.

To be specific, the solution to the DSGE model can be written as

$$X_{t} = AX_{t-1} + M\varepsilon_{t}$$
where $X_t$ is a vector including all endogenous and exogenous variables in the model and $\varepsilon_t$ is a vector including all the exogenous shocks in the model.\(^{27}\) Let $\varepsilon^t = \{\varepsilon_t, \varepsilon_{t-1}, \ldots, \varepsilon_1\}$ denote the history of exogenous shocks up to date $t$ and $\varepsilon^{t+} = \{\varepsilon_{t+1}, \varepsilon_{t+2}, \ldots, \varepsilon_{T+K}\}$ denote future exogenous shocks. For each date $t = 1, 2, \ldots, T$ through the sample of length $T$, we compute the forecast conditional on $\varepsilon^t$, with $\varepsilon^{t+} = 0$. This yields the projection of tax rates changes of $t + q$ based on $\xi^K_t$ and $\xi^L_t$ and the tax base $y_{t+q|t}$. Let $d\tau_{t+q|t}^\text{for}$ represent the projection of capital or labor tax rate changes to be implemented at $t + q$ conditional on information at $t$. Revenues projected in period $t + q$, conditional on information at $t$, are then $d\tau_{t+q|t}^\text{for} \times y_{t+q|t}$.

Following the Romers and Mertens and Ryan, we normalize these revenue projections by the actual value of output in period $t + q$, so the $T_{t+q}^\text{a}$ series used in the regression in equation (36) is given by

$$T_{t+q}^\text{a} = \frac{d\tau_{t+q|t}^\text{for} \times y_{t+q|t}}{Y_{t+q}}. \quad (52)$$

When forecasting revenues of enacted tax laws, government budgeting agencies shut down the feedback effects on tax revenues through the tax base.\(^{28}\) Therefore, the tax base used to compute revenue forecasts is the one forecasted under the existing law. In a general equilibrium framework, tax policy changes can arise from endogenous reasons. It is infeasible in our theoretical framework to forecast the tax base assuming the absence of tax changes (i.e. setting $\xi^K_t = \xi^L_t = 0$), except when $\mu_T = \gamma_T = 0$. Instead, we compute the forecasts of the tax base conditional on information at $t$. But to be consistent with actual government revenue forecasting procedure, we hold the tax base constant when computing revenue differences as shown in (52).

### B.3. Calibration.

In the simulations, we set $T = 240$ quarters to mimic the length of post-war macro time series and the simulation starts from the model’s steady state. Most of the parameters are set to the values comparable to those in the RBC literature

$$\beta = .99, \alpha = .36, \mu = \theta = 1, \delta = .025, \rho_A = .95, s^G = .2$$

We set $\chi = 3$ so that the steady state time share devoted to work is about 0.2. Based on Braun’s (1994) estimate of the standard deviation of technology shocks in a model with technology and fiscal shocks, we set $\sigma_A = 0.009$.

The steady state capital and labor tax rates are set to $\tau^K = .39$ and $\tau^L = .21$, based on the average capital and labor tax rate series (1947:1Q-2007:2Q) constructed by Jones’s (2002) definition. Blanchard and Perotti (2002) estimate the quarterly elasticity of tax revenues with respect to output and obtain an average over the post-war period of 2.08, which implies

\(^{27}\)The original form of the solution has a VARMA(1,6) representation. By including exogenous variables in $X_t$ and introducing a series of dummy variables, we are able to put the solution in a seemingly VAR (1) format, as presented in (51).

\(^{28}\)For the practice of revenue forecasts, see Joint Committee on Taxation (2005) and Auerbach (2005).
that the elasticity of tax rates to output is approximately 1; we set $\mu^C = 1$ in (40) and (41).

To calibrate $\rho_T, \rho_{KL}, \sigma_{Ka}, \sigma_{Ku}, \sigma_{La},$ and $\sigma_{Lu}$, we estimate two reduced-form tax equations based on federal data only:

\begin{align*}
\tau^K_t &= \alpha_0 + \alpha_1 \tau^K_{t-1} + \alpha_2 y_t + \alpha_3 s^B_{t-1} + \epsilon^K_t \\
\tau^L_t &= \beta_0 + \beta_1 \tau^L_{t-1} + \beta_2 y_t + \beta_3 s^B_t + \epsilon^L_t,
\end{align*}

where variables are in logarithms. Estimation of (53) results in $\hat{\sigma}_{e^K} = .036, \hat{\sigma}_{e^L} = .028,$ and the correlation between $e^K_t$ and $e^L_t$ is about .5. Assume the variance of anticipated and unanticipated exogenous tax shocks are of the same size. Setting $\sigma_{Ka} = \sigma_{Ku} = .025, \sigma_{La} = \sigma_{Lu} = .02,$ and $\rho_{KL} = .25$ makes the standard errors of exogenous capital and labor tax shocks and their correlation in the model roughly match the reduced-form estimation results.

In addition, $\hat{\alpha}_1 = .91$ and $\hat{\beta}_1 = .89$ so we set $\rho_T = .9$ in (40) and (41). For tax legislation considered in Mertens and Ravn, the maximum implementation lag is six quarters. For modeling simplicity, we assume all anticipated tax policy changes have an implementation lag of six quarters, so $q = 6$.

When capital and labor tax rates are assumed to adjust to clear the government’s budget, set $\gamma_T = .05$ and $\gamma_T = 0$. When lump-sum transfers clear the budget, set $\gamma_T = 0$ and $\gamma_T = -.1$. The steady state transfer to output ratio is set to $s^T = .07$. Along with other fiscal parameters, this implies the debt to output ratio is 0.475 in the steady state, roughly equal to the postwar historical average share of federal government debt held by the private sector. We set $\mu^T$ in (42) and (43) to .5.

Finally, the model assumes an exogenous component in the process of government spending, transfers, and the preference weight on leisure, the standard errors for these three shocks are set to .02.

**B.4. Computing Impulse Response Functions.** The simulation exercises compare the impulse responses estimated from simulated data with theoretical responses to an exogenous anticipated tax shock. Assume the economy is at the steady state before time 1. To find the theoretical responses to an anticipated capital tax cut, following the usual procedure, we set $\varepsilon^K_{1} = -1, \{\varepsilon^K_i\}_{i=2,3,...} = 0,$ and all other shocks to zero. The moving average representation of the DSGE solution (51) is then used to compute the theoretical impulse responses. To make units comparable to the empirical responses, the size of the exogenous capital tax cut is converted to one percent of output at time 1. The theoretical responses are plotted as the thick solid lines in figures 4-7. The impulse responses to an anticipated labor tax shock are computed analogously.

To compute the empirical responses, we follow the procedure adopted by Mertens and Ravn (2008). For each draw of the simulated data, we estimate the following empirical
model\textsuperscript{29}

\begin{equation}
X_t = A + CX_{t-1} + \sum_{i=0}^{24} D_i T_{a_{t-i}}^a + \sum_{i=0}^{24} F_i T_{t-i}^u + \sum_{i=1}^{6} G_i T_{a_{t+i}}^a + u_t. \tag{54}
\end{equation}

We begin by setting $T_a^a = -1$, $X_0 = 0$, and all other $T_{a_i}^a$ and $T_{u_i}^a$ to zero. The estimated VAR system of (54) is then iterated to trace out the responses of the system ($X_t$) to an anticipated cut in capital taxes equal to one percent of output. Specifically, the system’s responses for 30 quarters from time 1 to an anticipated capital or labor tax cut to be implemented at time 7 can be computed by

\[
\hat{X}_1 = -\hat{G}_6, \\
\hat{X}_i = \hat{C}\hat{X}_{i-1} - \hat{G}_{6-i+1}, \text{ for } i = 2, \ldots, 5, \\
\hat{X}_i = \hat{C}\hat{X}_{i-1} - \hat{F}_{i-6}, \text{ for } i = 6, \ldots, 30.
\]

After 1000 draws of sequences of simulated data, the exercise produces 1000 empirical impulse responses. Their mean values are the thin-solid lines in figures 4-7. The error bands of the impulse responses are 16th and 84th percentile of the responses sorted quarter by quarter, and are represented by the dotted-dashed lines in the figures.

\textsuperscript{29}The only difference from Mertens and Ravn’s model is that we include more lags of anticipated and unanticipated tax regressors.
References


