The Puzzle of Uniform Standards and Market Segmentation Among Islamic Financial Institutions

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Abstract

This paper proposes a new answer to a controversial puzzle in Islamic finance described by El-Gamal (2002): “Despite the long development of uniform standards for Islamic finance, the market remains largely segmented.” We explain market segmentation as a separating equilibrium in which Islamic finance premiums serve as a socially beneficial (although costly) signaling mechanism. Market segmentation under a uniform standard of Shariah-compliance occurs when the Shariah Boards of two Islamic Finance Institutions (IFIs) use substantially different degrees of stringency even though they agree on a common set of requirements as determined by Islamic jurisprudence to minimally comply with Shariah Law. Heterogeneous degrees of stringency chosen by different IFIs’ Shariah Boards translate into different premiums paid by different IFIs' customers. One IFI targets the moderately pious consumer segment while the other targets the highly pious segment. The IFI that targets highly pious consumers voluntarily chooses to offer a more limited set of investments and financing products than the IFI targeting moderately pious consumers. By allowing for multiple Muslim communities with distinct group identities and correspondingly variable willingness-to-pay for being able to signal otherwise unobservable piety types, the model provides an explanation for market segmentation in which multiple IFIs choose to offer distinct ranges of products and serve different markets while not disagreeing on the uniform minimum standard.

Key Words: Islamic Finance, norms, piety, devout, loyalty, screening, signaling, lexicographic preference, marketing, segmentation, prestige pricing, legal standard

JEL Classification Code: D72

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Abstract: This paper proposes a new answer to a controversial puzzle in Islamic finance described by El-Gamal (2002): “Despite the long development of uniform standards for Islamic finance, the market remains largely segmented.” We explain market segmentation as a separating equilibrium in which Islamic finance premiums serve as a socially beneficial (although costly) signaling mechanism. Market segmentation under a uniform standard of Shariah-compliance occurs when the Shariah Boards of two Islamic Finance Institutions (IFIs) use substantially different degrees of stringency even though they agree on a common set of requirements as determined by Islamic jurisprudence to minimally comply with Shariah Law. Heterogeneous degrees of stringency chosen by different IFIs’ Shariah Boards translate into different premiums paid by different IFIs’ customers. One IFI targets the moderately pious consumer segment while the other targets the highly pious segment. The IFI that targets highly pious consumers voluntarily chooses to offer a more limited set of investments and financing products than the IFI targeting moderately pious consumers. By allowing for multiple Muslim communities with distinct group identities and correspondingly variable willingness-to-pay for being able to signal otherwise unobservable piety types, the model provides an explanation for market segmentation in which multiple IFIs choose to offer distinct ranges of products and serve different markets while not disagreeing on the uniform minimum standard.
1 Introduction

El-Gamal (2002) describes four related paradoxes in Islamic Finance (IF). First is the substantial gap between economic theory on one hand and the observed product offerings and consumer behaviors observed in the IF industry on the other. Second, the market remains largely segmented. Despite longstanding attempts to develop uniform standards for IF, the Shariah-compliance policies one observes are markedly heterogeneous, implying that there is no universally accepted uniform standard in practice. Third, the number of active jurists is relatively small (and the same jurists frequently serve on multiple firms’ Shariah Boards), leading to substantial overlap among juridical interpretations, despite the demand for juristic expertise to resolve many open questions and heated debates pertaining to Islamic jurisprudence. And fourth, a number of widely used financial contracts that have been approved by Islamic jurists come under sharp criticism (by the same jurists who found them to be permissible under Shariah Law) for being overused. This paper focuses primarily on the second of these apparent paradoxes. In the same article, El-Gamal (2002) provides an answer for the paradox of market segmentation, suggesting that different speeds of innovation between incumbents and new entrants may explain why we observe segmented product offerings based on different judgments by Shariah Boards at Islamic Financial Institutions (IFIs) even though jurists essentially agree on the legal standard that should be applied. We propose an alternative explanation (which is complementary, we think) by showing that market segmentation can also arise as a natural consequence of screening by IFIs aiming to serve different markets consisting of highly pious and moderately pious people.\footnote{We follow El-Gamal (2002) in assuming that there are exogenously given piety types. Our explanation does not require a face-value interpretation of piety types (i.e., the assumption that one individual can be ordered as being more pious than another in the eyes of God). The piety types referred to here (e.g., high, moderate and low) can be interpreted simply as distinct Muslim communities with different views about what it means to be pious, with correspondingly different willingnesses to pay for the privilege of associating with others who share their view.}

This paper describes a game situation in which IFIs set different Shariah-compliance policies that determine the premiums their customers face above the cost of using non-IF products. In this game, a continuum of customers seeking financial services makes decisions about which bank or financial institution to patronize. The customer’s decision is to: (i) bear
the higher cost of the strictest IFI, (ii) bear the moderate cost of a permissive IFI, or (iii) forgo paying any IFI’s premium and instead deal with a conventional financial institution. This decision depends on each agent’s piety type (high, moderate, or low), which is known to him or herself but not observable to others. Given three types of agents, the consumer’s decision (of which financial institution to become a client of) also depends on the heterogeneity of other consumers’ financial profiles that determine each individual’s cost of conventional finance. The heterogeneity of individuals’ outside options using conventional finance generates a continuum of reference points against which the competing Shariah-compliance policies (strict versus moderate) generate a continuum of IF premiums that individual agents face. Agents weigh the benefits of signaling piety to other agents (by bearing the IF premium they choose) against its cost. Although there are only two distinct Shariah-compliance policies set by the two IFIs in the model, these two policies generate a continuum of opportunity costs and consequently variable utility gains from agents’ decisions about how to signal their otherwise private piety type. The decision to become an IF client therefore transmits different information depending on an individual’s outside option, capturing what we think is an important real-world driver of heterogeneity among bank customers’ behavior and IFIs’ strategic marketing decisions when they consider which subpopulations they wish to attract and serve.

Berg and Kim (2014) demonstrate that IF premiums can provide a beneficial signaling technology even in the absence of any direct benefits (intrinsic or otherwise) received by agents who choose to become IF clients. What is new in the model presented in this paper is its focus on two competing IFIs that strategically decide on the stringency of their Shariah-compliance policies. In line with El-Gamal’s (2002) description of “market segmentation,” the choice variables of the competing IFIs in our model are scalar-valued degrees of stringency of each IFI’s Shariah-compliance policy, which emerges as a strategic marketing tool that presupposes no essential disagreement among IFIs’ Shariah Boards regarding the bona fide requirements of Shariah Law.

Although other criteria for evaluating the costs and benefits of the set of permissible portfolios (including debt equivalents achieved with joint ownership of anticipated cash flows together with obligations to make payments), we interpret the stringency of an IFI’s Shariah-compliance policy as a deliberate shrinking of the risk-return bullet (i.e., the menu of choices
facing consumers who demand financial services) whose graph is a strict subset of the unrestricted universe of feasible risk return combinations provided by conventional finance. The IF premium we refer to can be interpreted in units of forgone expected return (which includes premiums paid for debt financing, or as any short position obligating an IF client to make future payments) while holding volatility constant.\footnote{Along every constant-volatility vertical segment that passes through the larger non-IFI bullet (encompassing the unrestricted universe of feasible risk-return pairs using conventional finance) and the smaller bullet offered by the IFI, a vertical distance between these two bullets in units of forgone expected return can be measured. Discussion and figures illustrating possible shapes of IFI and non-IFI sectors of the financial services industry can be found in Berg and Kim, (2014).} All else equal, a more stringent Shariah-compliance policy implies a greater forgone expected return at every fixed level of volatility. This risk-invariant decrease in expected returns that any IF client faces (e.g., on a home purchase, a retirement portfolio, an insurance policy, or a small business investment) is equivalent to paying higher prices for financial services, referred to here as the IF premium. As in Berg and Kim (2014), the modeling technique deliberately abstracts from any intrinsic benefits brought about by following Shariah-compliance, which may be substantial as El-Gamal (2002) and others have pointed out or, alternatively, interpreted as a net social cost in the eyes of critics such as Kahn (2010). This abstraction provides a thought experiment focused on signaling and its role in explaining the emergence of market segmentation in the IF industry.

Section 2 discusses market segmentation in the Islamic finance industry that motivates the theoretical model. The model is described in Section 3. The equilibrium analysis appears in Section 4 followed by a conclusion in Section 5.

2 Market Segmentation and Positioning of IFIs

2.1 Market Segmentation

Market segmentation enables firms to focus their services on targeted homogeneous groups of potential consumers rather than competing for customers over the entire heterogeneous market. This helps the firm to allocate its limited resources efficiently by positioning itself strategically to select the most attractive consumer groups whose preferences match the
range of financial contracts it offers. The ability to segment the market relies on having access to information relevant to both the firm and the targeted consumer group (Van Raaij and Verhallen, 1994). Not only does the firm need to distinguish its products (and sometimes itself) using such identifying variables, it also needs to signal its position to the targeted consumer segments. Market segmentation among IFIs in the real world uses several identifying variables but depends crucially on the range of services and products offered (e.g. securities, insurance, personal banking, and corporate banking). A Priori segmentation uses identifying characteristics that exist in the consumer population, such as demographics, social class, age, or stage in the family life cycle. Post hoc segmentation is often achieved using survey data, from which clusters of consumers can be identified and sorted into market segments (Harrison, 1994).

The IF industry is primarily based on religious preferences. It specializes in financial services that meet certain religious beliefs about economic transactions. Customers of IFIs do not, however, hold the same degree (or even type) of religiosity. Hence, market segmentation of IF services can be viewed as necessary for IFIs (to be competitive in attracting customers) and consumers (to achieve preference satisfaction). Using in-depth interview data collected from financial managers in Singapore and London, Muhamad et al. (2012) describe the Islamic banking market as being comprised of three main groups or segments. One group is motivated primarily by economic incentives. A second group includes those who are cautious about moral values (i.e., accustomed to applying binding moral constraints that limit their own choice sets) but not particularly observant of religion. The third group is primarily motivated by religious principles. Additionally, Muhamad et al. (2012) suggest a fourth group comprised of those who are religious and financial profit-maximizers at the same time.

Religiosity is complex. Representing its multi-dimensionality in any mathematical model as a vector- rather than scalar-valued attribute would be one reasonable approach. But this approach (vector-valued profiles of religiosity) would imply that the “religiosity space” may not be easy to order in any reasonable way of ranking one person’s profile as more religious or more pious than another’s. As a technical convenience, the scalar-valued parameter referred to in our model as piety represents a compressed projection from the potentially vast religiosity space (containing unordered individual profiles of religiosity) into a scalar-valued willingness-to-pay. We assume that more pious consumers are willing to practice
or express their piety by choosing more stringent lifestyles, which includes choosing more stringent—and consequently, more expensive—financial services.

In light of the discussion above and our objective of strategic behavior among two IFIs in choosing different degrees of stringency, a minimally fine taxonomy of types or market segments consists of the following three groups ordered as low, moderate and high piety types:

- **Consumers Whose Preferences are Indifferent Between Conventional Finance and IFIs** are indistinguishable from non-Muslims in their choice of financial institutions, whether they are customers of IFIs or non-IFIs. These consumers have the lowest (or zero) willingness-to-pay for the signaling service of Shariah-compliance credentialing offered by IFIs.

- **Moderately Pious Consumers** choose financial products, at least in part, guided by religious motives, whether or not they have detailed knowledge about the Shariah-compliance of the products they choose. These consumers are willing to sacrifice some range of choice of the financial products they use and profitability for the sake of Shariah-compliance.

- **Highly Pious Consumers** are primarily driven by religious motives, which manifest as higher willingness-to-pay to satisfy their preferences for a specific interpretation of, or high degree of, Shariah-compliance.

Several studies report that many IF customers fall into the first segment, with preferences that are indifferent about the Shariah-compliance credentialing that IFIs offer. For a substantial number of IF customers, preferences over financial products (in terms of quality of services, profitability and convenience as the main criteria for choosing financial institutions) are apparently no different from that of consumers who choose conventional finance. See Rosenblatt et al. (1988) on corporate customers; Erol and El-Bdour (1989); Erol et al. (1990); Hegazy (1995) using data from Egypt; Karim and Afiff (2006) and Rohmah (2006) on Indonesia; Dusuki and Abdullah (2007) on Malaysia; and Naser et al. (1999) on Jordan. There is a substantial number of individual consumers who simultaneously choose and have active accounts with both IFIs and non-IFIs.
Other empirical studies of consumer preferences among IFI customers support religiosity as an important motive. The empirical results vary, however, in terms of the degree to which IFIs’ customers prioritize religiosity with respect to other motives. Insofar as mixed empirical findings about the intensity of religious preference reflect genuine heterogeneity (rather than measurement error), there would seem to be strong evidence for the existence of the latter two consumer segments as enumerated above. For example, studies suggesting that religiosity is the primary motive for choosing an IFI include Metawa and Almossawi (1998) and Omer (1992) using data from the U.K.; Othman and Owen (2001) on Kuwait; and Wakhid and Efrita (2007) and Abduh and Omar (2012) on Malaysia. Contrasting evidence showing that religiosity may not be the primary or sole concern when choosing an IFI includes Awan et al. (2011) using data from Pakistan; and Haron et al. (1994) and Haron and Wan Azmi (2008) on Malaysia.

2.2 Divergence of Opinions among IF Shariah Boards

Shariah-compliance policies among IFIs are, in many cases, heterogeneous. Multiple factors contribute to this heterogeneity, which we emphasize may or may not reflect conflicting religious opinions about IF among jurists who serve on Shariah Boards. One factor is the nature of Islam itself, which does not have a central authority and therefore allows for (or, by many accounts, embraces) diverse interpretations and understandings of religious texts. Insofar as diversity is viewed as an essential religious tenet of Islam, flexibility and freedom among diverse cultural, geographic and social groups of Muslims are to be expected (and indeed celebrated by many practitioners and students of religion). Another factor is the lack of standardization among IFIs. Some countries have specific laws to regulate IFIs; other countries instead have a central authority (not necessarily affiliated with, or appointed by, government) that regulates or more informally manages the IFIs domiciled in a particular country or geographic region; and some countries have no authority responsible for regulating IFIs apart from an individual IFI’s owners and Shariah Board (El-Hawary et al., 2007).

Malaysia provides an interesting example in which distinct Shariah-compliance policies are readily observed. Some Malaysian IFIs permit contracts such as Bay'-al-'inah,\textsuperscript{3} which

\textsuperscript{3}Bay'-al-'inah is a contract that involves sale and buyback of an asset at a markup over the original price.
would not pass the Shariah-compliance policies adopted by IFIs in the Gulf countries. The consequences of permitting a medieval contract such as Bay’-al-‘inah are crucial for researchers of, and active practitioners in, IF. A line of Islamic financial products (e.g., Islamic Credit Cards and Negotiable Islamic Debt Certificates (NIDC)) may not be approved in jurisdictions where Shariah standards do not allow the underlying Bay’-al-‘inah contract (Shaharuddin, 2012). Bay’ al-dayn (sale of debt) is another contract that is permissible in Malaysia (used in the design of Malaysian Islamic Bonds; see Rosly and Sanusi, 1999) but not in the Middle-East. Our model presented in the next section is motivated by the hypothesis that, in the example of Malaysia as well as in others, greater heterogeneity of Shariah-compliance policies is a predictable outcome of the population’s religious and ethnic heterogeneity (e.g., Malaysia’s large non-Muslim minority subpopulations living alongside the Muslim majority).

Heterogeneity in the stringency of Shariah Boards’ decisions is not restricted to comparisons between jurisdictions, regions and countries. Heterogeneous decisions by Shariah Boards can be found among IFIs within the same region or city. One account from a senior manager at a Dubai-based asset management firm, for example, describes how his firm’s Shariah Board approved certain products for his own firm but then faced delays in completing sales because the Shariah Boards of other IFIs in Dubai did not permit those same financial products (Torchia, 2012).

2.3 Shariah Scholars’ Reputations

How should we interpret observed heterogeneity in stringency among Shariah Boards? One problem of interpretation is that some IF customers (many perhaps?) may not know very much about Shariah-compliant financial services. Regardless of the extent to which consumers place weight on religious motives (signaling that they belong to the moderate- or high-piety groups as described above), we must acknowledge the empirical evidence showing that many IF customers have little understanding of Shariah-compliance (even for the IF products they choose). For example, Al-Ajmi et al. (2009) report data showing weak familiarity with Shariah-compliance in Bahrain. They find very low familiarity with most

This contract is permissible under the Shafi’ Islamic (madhab) school of thought but prohibited by others.
IF products except for *murabaha*, which is the dominant contract used in Islamic banking. Rammal and Zurbrugg (2007) report similar results among Muslims in Australia.

Given this evidence of consumers’ very limited familiarity with Shariah-compliance (despite their expressed and acted-upon desire to comply), these consumers must somehow infer the degree of Shariah-compliance (i.e., stringency) among different IFIs and IF products. One important source of information used to make such inferences is the composition of an IFI’s Shariah Scholars Board (SBB) that approves the operations and products of the IFI. Shariah scholars who serve on SBBs (referred to in this paper as Shariah Boards) are very few, with twenty well-known scholars holding 621 different SBB positions at multiple IFIs (Ünal, 2011; Farook and Farooq, 2011). Alman (2012) reports evidence associating multiple memberships among the top-20 Shariah scholars (ranked by the number of SBB positions that a single scholar holds) with higher levels of risk in the loan portfolios of Islamic Banks. A very small number of elite Shariah scholars’ judgments guide the Shariah-compliance decisions of IFIs. The reputations of top-ranked Shariah scholars (who are household names, widely known to consumers in the highly pious segment of the market) appear to also play a significant role in attracting highly pious customers.

Shariah scholars are paid by IFIs to provide *fatwas* (i.e., religions decisions) on specific financial products. Conflicts of interest are a serious problem according to many practitioners and academics. Responding to this criticism, Shariah scholars defend these potentially conflicting religious and financial motives built into their remunerated relationships with IFIs on the ground that they are accountable before God and, in addition, have a reputation to protect. The reputational component of this argument is observable and, we argue, plays an important role in IFIs’ marketing strategies in regard to different choices over stringency of the Shariah Board.

It is interesting to consider what economic theory might tell us to expect under the usual economic assumptions of rational choice theory. What if the Shariah scholars serving on Shariah Boards were modeled as if they were money-maximizers subject to a uniform minimum standard of Shariah-compliance? Does it follow from rational choice theory that the Shariah-compliance policies of IFIs across the world would tend to converge to this hypothetical uniform minimum standard? We argue that the answer to this question is “No.” Our model shows that Shariah Boards could still have a strong incentive to choose hetero-
geneous Shariah-compliance standards, helping to achieve informational efficiency and, with it, improve social welfare. Considering the stringency of Shariah-compliance standards as a choice variable reveals how it can be used as a screening device, enabling IFIs to differentiate their brands and product lines to better match different segments of the consumer side of the market.

Two interlinked problems arise. The IF industry’s raison d’être is to provide financial services to the two pious segments of the consumer market (i.e., those with a strictly positive willingness-to-pay for Shariah-compliance). If IFIs were to offer excessively lenient products that resemble or are indistinguishable from conventional finance, then IFIs could lose their primary market insofar as they fail to deliver Shariah-compliance credentialing that consumers use to signal their otherwise private piety type. Therefore, it likely serves the interest of IFIs to differentiate themselves from conventional finance by offering more stringent products (as in El-Gamal’s, 2002, model). Shariah Boards need to offer sufficiently stringent fatwas so as to establish and maintain the IFI’s reputation for stringency, which is the key mechanism that provides consumers with signaling technology which (at least in the context of our model) drives consumer demand. Analysis in later sections shows what our model can say about whether these reputational and signaling incentives will lead to escalation of stringency (i.e., an arms race of ever-increasing stringency among IFIs) as IFIs compete for the highly pious segment; a spiraling toward permissiveness, as IFIs compete for the moderately pious segment; or a stable outcome with heterogeneous Shariah-compliance decisions among different IFIs that serve different segments of the market, despite the scholars all sharing a single understanding (i.e., a uniform minimum standard) of just-sufficient Shariah-compliance.

2.4 Questions

The model in the next section addresses two main questions: (i) Under what conditions will there exist a separating equilibrium in which the heterogeneous choices of stringency in Shariah-compliance can be rationalized (i.e., where one IFI chooses moderate stringency and another chooses high stringency, while neither has any incentive to deviate)?; and (ii) Can our theoretical mechanism based on signaling explain heterogeneous stringencies of Shariah-
compliance decisions by IFIs Shariah Boards as a marketing strategy (while neutralizing the variation in fatwas that stems from genuine disagreements in the interpretation of Shariah Law)?

Thus, we set out to make progress toward better explaining market segmentation and observed tension between the heterogeneity of Shariah Boards’ decisions versus the small number of scholars who comprise them with significantly overlapping Board memberships. We undertake to develop testable implications from theory regarding the role that signaling plays in rationalizing the premiums paid by IFI consumers (i.e., consumers who voluntarily restrict the risk-return choice set from which their portfolios and wealth management services are chosen). To analyze segmentation with heterogeneous decisions by Shariah Boards, we consider what we think is probably the simplest model in which to pursue this objective, by assuming there are three types of consumers (two of which have strictly positive willingness-to-pay for signaling services by choosing an IFI over conventional finance) and two IFIs.

3 Model

The model adapts Hotelling’s linear city model, where physical distance is interpreted as proportional to the additional financing costs of becoming an IFI consumer rather than using more cheaply available non-Islamic financing, which is available to all agents at an exogenously given cost normalized to zero. Agents’ locations are assumed to be distributed uniformly on the unit interval representing heterogeneity in the publically observable costs borne by individuals when they choose to restrict choice over financial products or, equivalently, paying a premium for Shariah-compliance provided by the IFI’s Shariah Board. Physical movement in Hotelling’s linear city model is interpreted as the decision to move away from the outside option of conventional finance and become an IF consumer at one of two IFIs whose distances (i.e., premiums, or degrees of stringency) from each agent’s initial position are, in general, different.

The model assumes that there are two IFIs, A and B. Each potential IFI consumer faces a cost of becoming an IFI client that is proportional to the distance travelled from his or her initial position to the “location” of the IFI. If an IFI is located at $z_i \in [0, \infty)$, then the cost borne by an agent at location $x$ when he or she becomes a client of that IFI (by choosing
to use its financial products) is $t|z_i - x|$, where $t > 0$ is the unit transportation cost and the distance norm is given by absolute value of the difference between the two locations. The assumption that $z_i$ can take on any value in the non-negative real line while agents’ initial locations are restricted to the unit interval implies that the IFIs may be located either inside or outside the continuum where agents and their non-Islamic financing options are located.

The model assumes that agents receive no direct utility from IFIs (beyond that of the non-Islamic outside option which, as mentioned before, is normalized to zero). By abstracting from the multiplicity of motives among pious IFI clients in the real world, this assumption that zero intrinsic utility is derived from IFI-client status simply focuses attention on the signaling mechanism and shows the economically interesting case of voluntary payment by different agents of two different premiums. Different premiums are required for an agent to enjoy client status at different IFIs. In turn, the IFIs voluntarily choose their own premiums even though the Shariah scholars who serve on the Shariah Boards agree on the minimum requirements of Shariah Law. Agents whose initial positions are farther away from an IFI are endowed with the possibility (or curse) of publically incurring a greater cost, which reveals and makes public their otherwise private piety types.

An agent’s true but unobservable piety type $\omega$ is assumed to take on one of three types: low ($\omega = L$), medium ($\omega = M$), or high ($\omega = H$), $L < M < H$. We denote $\Delta M \equiv M - L$ and $\Delta H \equiv H - M$, which represent exogenously given preference parameters that determine different types’ willingness to pay for the social benefits of signaling their piety types as represented by the payoff functions introduced below. An agent’s piety type is private information that other agents, including IFIs and their Shariah Boards, do not know and can never observe directly. What is common knowledge, however, are the proportions of each type within the whole population, denoted $\mu_L, \mu_M, \mu_H \in (0, 1)$, which satisfy the adding-up condition $\mu_L + \mu_M + \mu_H = 1$. For notational convenience, the symbol $\mu \equiv \mu_L + \mu_H$ is introduced, which represents the proportion of the population that is pious to some extent (either M- or H-type). It is also assumed that each agent’s initial location ($x$) is known to all agents in the model. Possible interpretations of $x$ would include the agent’s credit score or neighborhood income, which influence the costs of financial services available to different

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4If intrinsic utility from being an IFI client is included in the model, then the qualitative results in Theorems 1 and 2 continue to hold as explained in the final section of Berg and Kim (2014).
Agents are assumed to make inferences that generate beliefs about the piety of others by observing whether that person is a non-IF client, a client of A, or a client of B. The observability of piety is a key component of the signaling mechanism analyzed in this paper and has two main components. The two publically observable variables, which are assumed to be common knowledge, are an agent’s initial location $x$ and that agent’s observed movement away from the initial location, coded as the choice variable $m$: $m = 0$ codes the decision to be a non-IF client and instead use the outside financing option from a non-IF institution; $m = A$ codes the agent’s choice of IFI $A$; and $m = B$ codes the agent’s choice of IFI $B$. Observing both $x$ and $m$ is required to fully reveal piety types in the separating equilibrium that follows. Agents are assumed to update their beliefs about others’ piety types according to Bayes Rule. Based on another agent’s initial location and observed IFI status, other agents form posterior beliefs about that agent’s piety type, which (in the context of the stylized model) completely characterize each agent’s reputation.

The next step is to specify the social production function that maps an agent’s true piety type $\omega$ and perceived type $\hat{\omega}$ into a utility flow. The assumption motivating the specification of the utility function that follows is that agents receive extrinsic utility directly from their reputations as a function of their true piety type. Together, the exogenous data that generate unobservable piety types combine with endogenous beliefs, based on observable initial locations and IFI client-status decisions, to produce social interactions and financial outcomes. These combined social and financial outcomes are then valued differently as a function of each agent’s true piety type. The model assumes that more pious types value their reputations in terms of being willing to bear a higher cost to publically express piety than less pious types. Interactions with fellow religious group members who have reputations as high-piety types generate positive utility, which would include provision of aid to fellow high-piety types in need, benefits resulting from coordination, communication of religiously informed insights among the pious, lower transaction costs contracting with fellow pious group members, and no doubt other channels as well.\footnote{In Muslim societies (and communities within countries that are not predominantly Muslim), many institutions and important decisions are headed by individual leaders whose reputation for piety is a necessary qualification. For example, it may be necessary that political leaders such as the President or the Prime}
The benefit an individual gets from interacting with another agent depends on his or her true piety type and the perceived type as well. We denote this benefit of social interaction as $V(\omega, \hat{\omega})$, where $\partial V/\partial \omega > 0$ and $\partial V/\partial \hat{\omega} > 0$. The assumption that the benefit of social interaction is an increasing function of one’s own piety type (holding the reputation of the other agent constant) represents the idea that piety, as interpreted by a long tradition of Islamic jurisprudence, is of greater importance to those who study and abide by the teachings of the Koran. El-Gamal (2001) emphasizes that Koranic principles and the interpretations of Islamic jurists reflect value judgments that are highly consistent with principles of social justice, which are easily recognizable to students of welfare economics. The assumption that $V(\omega, \hat{\omega})$ is an increasing function of the other’s reputation (holding one’s unobservable piety type constant) represents the value of having a reputation for piety in a Muslim community.

For simplicity, we assume that $V(\omega, \hat{\omega}) = \omega \hat{\omega}$. This functional form implies that the marginal productivity of one’s reputation in the eyes of other agents depends on those other agents’ piety types, capturing the idea that, all else equal, pious agents value other agents’ piety more highly than non-pious agents do.\(^6\)

Given the definitions above, the model is a three-stage game described as follows. Given any unified standard that may have developed among Islamic jurists (i.e., minimum requirements) for IFIs to comply with Shariah, the two IFIs’ Shariah Boards choose their location $z_i \in [0, \infty)$, interpreted as a choice of stringency or restrictiveness of the financial services offered by the IFI (i.e., the IFI’s Shariah-compliance policy). Together with the client’s outside financing option as determined by his or her location, the locations that Shariah Boards $A$ and $B$ choose, in turn, determine the IF premiums that IFI client faces. Second, individual agents decide which of the three elements in their choice set they will choose: non-IF client, client of IFI $A$, or client of IFI $B$.\(^7\) And finally in the third stage of the game, all agents make

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\(^6\)Minister are sufficiently pious. In decentralized social hierarchies such as those that govern Friday prayers at Mosques, the Imam who leads the prayers should have a reputation of sufficiently high piety.

\(^7\)If interacting with others involves the activity of producing a public good, for example, the functional form $V(\omega, \hat{\omega})$ means that a more pious agent (who has larger $\omega$) will value any provision of the public good more than a less pious agent does (assuming that both exert the same level of effort).

\(^7\)In reality, many banks sell both IF and non-IF products. Thus, an individual’s choice can alternatively be interpreted as whether he or she buys an IF product or a non-IF product from a single financial institution offering both.
inferences about the piety types of other agents based on the locations of others (i.e., their outside financing option) and their decisions about becoming an IFI clients. The decision over the choice set of non-IFI client (choosing the outside option of conventional financing), client of A, or client of B, is represented by the choice variable \( m, m \in 0, A, B \).

When considering the three available actions represented by \( m \), the agent of type \( \omega \) with initial location \( x \) (i.e., facing the distances from IFI \( i \), which represent the IF premium) sees the following payoff function:

\[
\begin{aligned}
    u(m, x; \omega) &= \begin{cases} 
        V(\omega, \hat{\omega}(x, 0)) & \text{if } m = 0, \\
        -t|x - z_A| + V(\omega, \hat{\omega}(x, A)) & \text{if } m = A, \\
        -t|x - z_B| + V(\omega, \hat{\omega}(x, B)) & \text{if } m = B.
    \end{cases}
\end{aligned}
\]

(1)

The payoff of an IFI is determined by two variables: the number of its customers (i.e., the mass or measure of customers, because there is a continuum of agents in the model) and their average piety, denoted by \( \pi_i(P_i, N_i) \), where \( P_i \) is the average piety and \( N_i \) is the number of customers. We assume that \( \partial\pi_i/\partial P_i, \partial\pi_i/\partial N_i > 0 \). We also assume that the primary concern of IFI \( A \) is the measure of its customers, \( N_A \), and that the primary concern of IFI \( B \) is the mean piety of its customers, \( P_B \). For simplicity, this difference in the marketing objectives can be formulated as lexicographic preferences. Let the preference relation of IFI \( i \) be denoted \( \succ_i \). Then, the IFIs’ preferences are described by the following orderings:

\[
(P_1^A, N_1^A) \succ_A (P_2^A, N_2^A) \iff N_1^A > N_2^A \text{ or } P_1^A > P_2^A \text{ if } N_1^A = N_2^A;
\]

(2)

\[
(P_1^B, N_1^B) \succ_B (P_2^B, N_2^B) \iff P_1^B > P_2^B \text{ or } N_1^B > N_2^B \text{ if } P_1^B = P_2^B.
\]

(3)

4 Analysis

As the main solution concept, we will employ weak Perfect Bayesian equilibrium (wPBE).\(^8\)

Let the equilibrium location of IFI \( i \) be \( z_i^* \). Then, the next lemma will turn out to be conveniently used for our analysis.

---

\(^8\)The wPBE concept is defined as a profile of strategies and beliefs such that (i) each type of agent makes a payoff-maximizing choice based on beliefs at each information set and (ii) the posterior beliefs must be updated according to Bayes’ Rule whenever possible. Further details on the definition of wPBE are found in Mas-Colell et al. (1995).
Lemma 1 \( z^*_A, z^*_B > 1 \).

Proof. This follows directly from Theorem 1 of Berg and Kim (2014).

The intuition is as follows. If \( z_i \in [0, 1] \) for some \( i = A, B \), then there exists an agent at \( x \in [0, 1] \) who is very near the IFI and consequently faces an arbitrarily small IF premium (i.e., \( |z_i - x| \) is very small). Because the social benefit is strictly positive and there exists an agent for whom the cost is arbitrarily small, such an agent will choose to become a client of IFI \( i \) regardless of piety type. If the IFI sets its premium too low, then it is impossible to exclude the impious type (\( L \)).

Lemma 1 implies that, in any equilibrium, the locations chosen by the two IFIs are strictly to the right of the unit interval: \( z^*_A, z^*_B > 1 \). This means that the IFIs must impose a strictly positive premium on all clients in equilibrium. Since our goal is to explain market segmentation as an equilibrium outcome, we focus on the case of \( z^*_B > z^*_A > 1 \). Later, we will also consider the possibility of a pooling equilibrium in which \( z^*_A = z^*_B > 1 \).

The first issue is whether a fully separating equilibrium exists. If it does exist, then we will show that the separating equilibrium is described as follows: (i) the IFIs choose \( z^*_A = 1 + \frac{L(M-L)}{t}, \quad z^*_B = 1 + \frac{L(H-L)}{t} \); (ii) for any \( x \in [0, 1] \), a type-\( L \) agent chooses non-IFI client status, a type-\( M \) agent chooses \( A \), and a type-\( H \) agent chooses \( B \); and (iii) true piety types are perfectly revealed by the publically observable choices of IF client status, according to which \( \hat{\omega}(x, 0) = L, \quad \hat{\omega}(x, A) = M, \quad \hat{\omega}(x, B) = H \), for any \( x \in [0, 1] \). If this could be an equilibrium, then IFI-client status would be shown to provide a signaling service that fully reveals otherwise unobservable piety status: the impious choose non-IFI client status and save themselves the cost of IF premiums; the moderately pious choose the IFI with modest IF premiums (or restrictiveness as decided by the IFI’s Shariah Board), \( A \); and the highly pious choose to incur the highest degree of restrictiveness, selecting \( B \).

The uniformity of the minimum Shariah-compliance criteria becomes important at this point. Both \( A \) and \( B \) want to maximize the number of clients who satisfy this uniform standard (i.e., both IFIs want to avoid low-piety types and are happy to have as many medium- and high-type clients as they can attract). We interpret this uniform standard to be a common or widely shared understanding of authoritative Islamic jurists and their writings explicating Shariah Law. Given this presumed uniformity of the piety standard (granted at
least for the sake of the thought experiment that the model entails), the goal is to investigate how segmentation into different degrees of stringency could emerge as strategic choices by different Shariah Boards working, by design, to serve the objectives of their associated IFIs (since they are directly paid by the IFIs that employ them). Both IFIs are constrained by the uniform minimum standard to avoid transacting with non-pious L types. But they are both happy to transact with either M or H types (consistent with them keeping safely within the strict interior of the constraint set imposed by the uniform minimum standard required for Sharia-compliance). Both IFIs would like to maximize the number (quantity or mass) of clients, subject to the condition that clients meet the uniform minimum standard. Therefore, the IFIs’ choices of stringency that just meet this condition of excluding L types is:

\[
z^*_A = 1 + \frac{L(M - L)}{t}, \quad \text{and} \quad z^*_B = 1 + \frac{L(H - L)}{t}.
\]  

(4)

At the locations given by the equations above, each IFI satisfies the incentive compatibility constraint which requires that no L type chooses to be their client, while achieving perfect revelation of all three piety types. Were such an equilibrium to exist, it would be welfare improving in the sense of achieving perfect informational efficiency (i.e., the signaling technology would perfectly reveal all three customers’ piety types) and payoff-enhancing for both IFIs and their customers as well. The property of informational efficiency would explain why the more stringent B has an incentive to persist in applying more stringent Shariah criteria than A instead of becoming more permissive (moving toward A). We will show that this action profile describing an IF market that is perfectly segmented by varying degrees of stringency with respect to different IFIs’ Shariah-compliance policies cannot be an equilibrium. But we proceed to show that a nearly perfect separating equilibrium does exist, which can be chosen arbitrarily close to the perfectly segmented market profile.

Beliefs about individual agents’ types are generated (as posterior Bayesian beliefs) in the separating equilibrium proposed above according to the belief function:

\[
\hat{\omega}(x, m) = \begin{cases} 
L & \text{if } x \text{ chooses } m = 0 \\
M & \text{if } x \text{ chooses } m = A \\
H & \text{if } x \text{ chooses } m = B.
\end{cases}
\]  

(5)

Next, we consider each type of agent’s decision. Given \(z^*_A\) and \(z^*_B\), an individual of type L...
located at \( x \in [0, 1] \) faces payoffs:

\[
u(m, x; L) = \begin{cases} 
V(L, \hat{\omega}(x, 0)) = L^2 & \text{if } x \text{ chooses } m = 0, \\
-t(z_A - x) + V(L, \hat{\omega}(x, A)) = -t(z_A - x) + LM & \text{if } x \text{ chooses } m = A, \\
-t(z_B - x) + V(L, \hat{\omega}(x, B)) = -t(z_B - x) + LH & \text{if } x \text{ chooses } m = B. 
\end{cases}
\]  

(6)

Because both IF premiums are larger, the farther away an agent’s location \( x \) is, it follows that all \( L \) types will prefer \( m = 0 \) if the \( L \) type located at \( x = 1 \) does. All agents face at least as large an IF premium as the agent at \( x = 1 \), since the IFIs are to the right of 1. An \( L \) type located at \( x = 1 \) will choose \( m = 0 \) if \( L^2 \geq \max\{-t(z_A^* - 1) + LM, -t(z_B^* - 1) + LH\} \). Since \( \max\{-t(z_A^* - 1) + LM, -t(z_B^* - 1) + LH\} = 0 \) at \( z_A^* = 1 + \frac{L(M - L)}{t} \) and \( z_B^* = 1 + \frac{L(H - L)}{t} \), the condition guaranteeing that all \( L \) types prefer \( m = 0 \) is trivially satisfied.

Similarly, the payoff of an \( M \) type located at \( x \in [0, 1] \) is:

\[
u(m, x; M) = \begin{cases} 
ML & \text{if } x \text{ chooses } m = 0, \\
-t(z_A - x) + M^2 & \text{if } x \text{ chooses } m = A, \\
-t(z_B - x) + MH & \text{if } x \text{ chooses } m = B. 
\end{cases}
\]  

(7)

The \( M \) type at \( x = 1 \) will prefer \( A \) over \( B \) if 

\[-t(z_A - x) + M^2 \geq -t(z_B - x) + MH, \]

which is equivalent to the condition:

\[z_B - z_A \geq \frac{M(H - M)}{t}.\]  

(8)

Because \( z_B^* - z_A^* = \frac{L(H-M)}{t} < \frac{M(H-M)}{t} \), this condition is not satisfied, implying that there exists a set of \( M \) types near \( x = 1 \) who prefer \( B \) over \( A \). The existence of \( M \) types who want to defect from the fully separating outcome described above means that it cannot be supported as an equilibrium.

The intuition for this negative result is that \( B \)’s Shariah-compliance policy (in the perfectly segmented action profile), \( z_B^* \), is set to barely prevent \( L \) types from mimicking \( H \) types and therefore is not stringent enough to prevent some \( M \) types from mimicking \( H \) types. Thus, it is a natural to conjecture that a fully separating equilibrium may be possible if \( z_B^* \) were chosen farther away from \( z_A^* \) (i.e., \( B \) chooses a more stringent Shariah-compliance policy) to make it just costly enough for \( M \) types to mimic \( H \) types that no \( M \) type will choose \( B \). Below, we show that this turns out to be the case. The following theorem is our main result addressing the first of the two questions posed in Section 2.4.
Theorem 1 There exists a separating equilibrium if \( t \leq (\Delta M)^2 \). In the separating equilibrium, (i) \( z^*_A = 1 + \frac{L \Delta M}{t} \), \( z^*_B = 1 + \frac{L \Delta M + M \Delta H}{t} \); (ii) for any \( x \in [0, 1] \), all \( L \) types choose \( m = 0 \) (i.e., non-IF client status), all \( M \) types choose \( m = M \), and all \( H \) types choose \( m = B \); and (iii) \( \hat{\omega}(x, A) = M, \hat{\omega}(x, B) = H, \hat{\omega}(x, 0) = L \) for any \( x \in [0, 1] \).\(^9\)

Proof. See the appendix.

If \( B \) deviates from \( z^*_B \) to \( z^*_B - \epsilon, \epsilon > 0 \), (moving slightly toward \( A \)), then some \( M \) type at \( x \in (1 - \epsilon, 1] \) could mimic \( H \) types by choosing \( B \). Then \( B \) could attract slightly more clients at a slight sacrifice of its clients’ mean piety. Because the primary concern of \( B \)’s Shariah Board is the average piety of its clients, it will not choose to deviate.\(^10\) It is clear that \( B \) will not choose to deviate by moving to \( A \)’s location. It is also clear that \( A \) will not choose to deviate. If \( A \) were to choose \( z^*_A - \epsilon \), then \( A \) would attract some unwanted \( L \)-type clients. If \( A \) were to choose \( z^*_A + \epsilon \), then it would lose some of its \( M \)-type clients, which contradicts the goal described by its preferences.

The next possibility we consider is a partially separating equilibrium. In the separating equilibrium characterized in Theorem 1, each agent of a particular type has a uniform best response (within its subpopulation of agents of the same type), regardless of location. All \( H \) types choose \( B \); all \( M \) types choose \( A \); and all \( L \) types choose non-IF client status. This action profile holds uniformly even when \( x_H < x_M \leq 0 \), i.e., \( t \leq (\Delta M)^2 \). One may wonder

\(^9\)To complete the off-the-equilibrium path strategy specification, we can interpret the strategy \( m = A \) as going to a closer and therefore less stringent IFI if and only if \( z_A < z_B \); and we can interpret the strategy \( m = B \) as going to a farther-away and therefore more stringent IFI if and only if \( z_A < z_B \). Then, the beliefs about types can be pinned down off the equilibrium path (i.e., when \( z_i \neq z^*_i \)). If \( z_A = z_B \), both \( M \) and \( H \) randomly choose between \( m = A \) and \( m = B \), and the perceived type is just the weighted average \( \frac{\mu_M M + \mu_H H}{\mu} \).

\(^10\)Some may suspect that this result relies critically on asymmetric preferences of two IFIs. But this is not the case. Because \( z_B < z^*_B \) is off the equilibrium path, other agents may assign arbitrary beliefs to such action profiles. For example, if the most pessimistic belief, \( \hat{w} = L \), is assigned to off-equilibrium action profiles, then this belief mechanism, which is outside the IFI’s control, could support the choices by IFIs given in Theorem 1 as an equilibrium, without requiring asymmetric preferences among IFIs. IFI \( B \) would not have an incentive to choose \( z_B < z^*_B \), because the IFI’s Shariah Board (and possibly its clients) would believe that all agents choosing to be clients of \( B \) are \( L \) types, which results in a substantial loss according to its objective function.
if there exists an interior solution \( x_M > 0 \) when the transportation cost is very low. In other words, might there be a partially separating equilibrium in which \( M \) types located at \( x \in [x_H, x_M) \) choose \( m = 0 \) over \( m = A \)? This would imply that the choice of \( m = 0 \) by an agent at \( x \in [x_H, x_M) \) does not fully separate types, because others might believe that the agent is an \( L \) or \( M \) type. The next theorem shows that this is not possible.

**Theorem 2** There does not exist a partially separating equilibrium in which (i) \( z_A^{**} < z_B^{**} \); (ii) \( H \) types choose \( m = B \) if \( x \geq x_H \) (possibly 0), and otherwise \( m = 0 \); \( M \) types choose \( m = A \) if \( x \geq x'_M \), and otherwise \( m = 0 \); and type-\( L \) agents choose \( m = 0 \) for any \( x \in [0, 1] \); (iii) with the belief profile given by:

\[
\hat{\omega}(x, m) = \begin{cases} L & \text{if } m = 0, \\ M & \text{if } m = A \text{ for } x \geq x_M, \\ H & \text{if } m = B. \end{cases}
\]

\[
\hat{\omega}(x, m) = \begin{cases} \frac{\mu_M}{\mu} M + \frac{\mu_L}{\mu} L & \text{if } m = 0, \\ \text{any belief} & \text{if } m = A \text{ for } x \in [x_H, x_M), \\ H & \text{if } m = B. \end{cases}
\]

\[
\hat{\omega}(x, m) = \begin{cases} \mu_L L + \mu_M M + \mu_H H & \text{if } m = 0, \\ \text{any belief} & \text{if } m = A \text{ for } x \in [0, x_H), \\ \text{any belief} & \text{if } m = B. \end{cases}
\]

In other words, there exists no cut-off value \( x'_M \in (0, 1) \) that supports a partially separating outcome as an equilibrium.

**Proof.** See the appendix.

One important feature that distinguishes the partially separating equilibrium from the fully separating equilibrium is that an \( M \)-type agent located at \( x = x'_M - \epsilon \) chooses non-IF-client status in the partially separating equilibrium. In this case, when an agent at any location in the half \( \epsilon \)-ball to the left of \( x_M \) is observed to choose conventional finance, this choice does not wrongly reveal the \( M \) type as an \( L \) type. Rather, the observed choice of non-IF conventional finance sends a noisy and therefore less-than-perfectly-revealing signal: other agents perceive that an \( M \) type choosing conventional finance is either type \( L \) or type
The fact that the reputational loss is not a total loss (as a consequence of choosing \( m = 0 \)) makes this choice more attractive relative to choosing \( m = B \). Conditional on \( m = 0 \), the agent’s reputation does not fall all the way to \( L \) but rather to an intermediate reputation somewhere between \( L \) and \( M \). Staying at one’s initial location and choosing the outside option of non-IF status is always cheaper than being an IF client and, in the partially separating action profile described in Theorem 2, incurs less of a reputational loss than in the fully separating equilibrium. The fact that the reputational losses among higher-piety agents choosing conventional finance goes down when the most stringent IFI becomes more stringent, because posterior beliefs about agents’ types conditional on \( m = 0 \) become more diluted (i.e., less informative), also shows why a smaller equilibrium value of \( z_B^{**} \) is expected. In that case, however, an \( L \) type located at \( x = 1 \) would deviate to \( m = B \), because the cost of pretending to be a high-piety type declines to zero as \( z_B^{**} \) approaches 1 from above.

5 Conclusion

In this paper, we extended the model of Berg and Kim (2014) to include three types of consumers (high-, medium-, and low-piety types) and two IFIs whose Shariah Boards choose intensities of stringency for Shariah-compliance. The model provided an explanation for market segmentation in the IF industry, a puzzle first raised by El-Gamal (2002), based on a screening mechanism. We characterized a separating equilibrium in which one IFI targets more highly pious customers than the other, choosing to voluntarily increase the stringency of its Shariah-compliance policy more than is minimally required according to jurists’ unified understanding of the legal standard. In the future, we think it would be worthwhile to pursue further empirical implications linking the features of this simple model of heterogeneous Shariah-compliance policies and market segmentation to field data and testing in the experimental lab.

Appendix

Proof of Theorem 1
If $z_B^* = 1 + \frac{L\Delta M + M\Delta H}{t}$, the condition for $M$ types to not mimic type $H$, which is given by inequality (8), is satisfied, and all $M$ types prefer $m = A$ over $m = B$. If $-t(z_A^* - x) + M^2 \geq ML$ (i.e., $x \geq 1 - \frac{(\Delta M)^2}{t} \equiv x_M$), then all $M$ types also prefer $m = A$ over $m = 0$. Therefore, if $t \leq (\Delta M)^2$, it is optimal for $M$ types at any $x \in [0, 1]$ to choose $m = A$.

The payoff for an individual of type $H$ located at $x \in [0, 1]$ is:

$$u(m, x; H) = \begin{cases} HL & \text{if } x \text{ chooses } m = 0, \\
-t(z_A - x) + HM & \text{if } x \text{ chooses } m = A, \\
-t(z_B - x) + H^2 & \text{if } x \text{ chooses } m = B. \end{cases}$$ (9)

Any $H$ type will prefer $m = B$ over $m = A$ if $-t(z_B - x) + H^2 \geq -t(z_A - x) + HM$ (i.e., $z_B - z_A \leq \frac{H(H-M)}{t}$). Because $z_B^* - z_A^* = \frac{M(H-M)}{t}$, the previous inequality is satisfied. Therefore, all $H$ types prefer $m = B$ over $m = A$. And if $-t(z_B^* - x) + H^2 \geq HL$, i.e., $x \geq z_B^* - \frac{H(H-L)}{t} = 1 + \frac{\psi}{t} \equiv x_H$, where $\psi = L\Delta M + M\Delta H - H(\Delta H + \Delta M)$, then it is also true that all $H$ types prefer $m = B$ over $m = 0$. Because $-\psi = H(\Delta H + \Delta M) - (L\Delta M + M\Delta H) = (H - M)\Delta H + (H - L)\Delta M = (\Delta H)^2 + (\Delta M)^2 + (\Delta H)(\Delta M) \geq (\Delta M)^2 \geq t$, we observe that $H$ types located at any $x \in [0, 1]$ will choose $m = B$. This completes the proof. ||

Proof of Theorem 2

Define $x'_M(z_B)$ as the value of $x$ that satisfies $-t(z_B - x) + MH = M\left(\frac{\mu M}{\mu} M + \frac{\mu L}{\mu} L\right)$. This value of $x$ makes an $M$ type at that location indifferent between $m = 0$ and $m = B$.

Lemma 2 In equilibrium, it must be the case that $x'_M = x_M$.

Proof. Suppose $x'_M < x_M$. Consider $x \in (x'_M, x_M)$. Because all $M$ types at $x \geq x_M$ are supposed to choose $m = A$, it must be the case that $x_M = x_M$. Since $M$ types at $x \in (x'_M, x_M)$ prefer $m = B$ to $m = 0$, this is a contradiction. If $x_M = x'_M$, then $M$ types at $x \in (x'_M, x_M)$ are supposed to choose $m = A$; but they prefer $m = 0$ over $m = A$, because $x < x_M$. This is also a contradiction. The equality in the statement of the Lemma can be similarly proved in the case where $x'_M > x_M$.

Lemma 3 In a partially separating equilibrium, it must be the case that $z_B'^* < z_B^*$.

Proof. By definition of $x'_M(z_B)$, we have:

$$-t(z_B - x'_M) + MH = M\left(\frac{H_M}{\mu} M + \frac{H_L}{\mu} L\right).$$
At \( z_B = z_B^* \), it follows that:

\[
x_M'(z_B^*) = 1 + \frac{L(M - L) - M(M - \frac{\nu M}{\mu}L)}{t} > 1 - \frac{(M - L)^2}{t} = x_M.
\]

Because \( x_M' \) is increasing in \( z_B \), the equality \( x_M'(z_B) = x_M \) implies that \( z_B^{**} < z_B^* \).

We will show that Lemma 3 implies that \( M \) types’ incentive compatibility condition is violated. The condition guaranteeing that the \( M \) type located at \( x \) \( \bar{x}_M \) will prefer \( m = A \) over \( m = B \) is \( z_B - z_A \geq \frac{M(H-M)}{t} \). Given that \( z_A = 1 + \frac{L(M-L)}{t} \), it must be the case that \( z_B \geq 1 + \frac{L(M-L)}{t} + \frac{M(H-M)}{t} = z_B^* \). This is a contradiction. ||

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