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# Does society benefit from investor overconfidence in the ability of financial market experts?☆

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**Abstract**

This paper develops a securities market model in which participants' beliefs diverge and prices are monotonic in beliefs. Relative to rational expectations (i.e., correct and unanimous beliefs), overconfidence among uninformed traders about the precision of experts' information leads to Pareto-superior equilibria. Efficiency-enhancing departures from rational expectations occur over a dense subset of parameter space, but only for one configuration of beliefs: uninformed traders must be more confident than informed experts. Overconfidence in the form of excessive trust in the predictive ability of experts sets off a virtuous cycle of increased trading that improves liquidity and reduces transaction costs for everyone.



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## 1. Introduction

This article describes a financial market in which systematically errant perceptions about the world can benefit all market participants. Extending Spiegel and Subrahmanyam's (1992) equilibrium model to allow for incorrect beliefs about risk, the paper illustrates the potential for a symbiotic relationship to emerge between risk-averse hedgers and risk-neutral insiders, analogous to the respective roles of the trading public and professional traders on Wall Street. Underlying this phenomenon is a beneficial liquidity externality caused and reinforced by excessive trading.[1] The way this works is as follows.

When non-experts perceive experts to be highly skilled at forecasting the future, they believe that current prices (which reflect the demand of experts) are highly correlated with future prices. This makes securities traded in financial markets seem more effective as an instrument for hedging risk than would otherwise be the case. Therefore, inflated perceptions of the degree to which expert information can predict the future translate into inflated perceptions of the hedging opportunities afforded by securities trading.

Perceiving securities as a more effective risk-reduction mechanism than is truly the case, hedgers who are overconfident in the ability of experts wind up hedging too much. A surprising consequence of overconfidence-induced over-trading is that the market shifts to a new equilibrium that is objectively less risky and where trading is objectively less costly. To understand this, it is helpful to think of the phenomenon of too much hedging as an outward shift in the demand curve for insurance. In response to greater demand, speculators (who are suppliers of insurance) sell a greater quantity and enjoy higher profits. At the new equilibrium generated by overconfident beliefs, the quantity traded (i.e., trading volume, or order-flow) is based more on noise than on information and therefore has a lower signal-to-noise ratio. Relying on this noisier signal (overconfident-demand-driven order-flow), market-makers set price according to a price function that is less sensitive to the order-flow signal. That is, market-makers flatten the function they use to price securities. Flatter, more competitive pricing reduces execution-price uncertainty and lowers the average cost of a trade. Ultimately, overconfidence reduces the objective transaction costs that all traders face without changing the expected losses of market-makers, therefore leading to a Pareto improvement.

It is worth emphasizing that the efficiency gains resulting from misperceived probabilities in this model are not tautological as they would be in a fallacious chain of logic asserting that "Everyone believes they are better off, therefore they *are* better off." In fact, as measured by expected utility evaluated with respect to the true probability distribution, overconfident traders subject themselves to a penalty for being irrational.[2] Irrational types in this model fail

[1] Many studies on the topic of overconfidence have appeared in the last 15 years, establishing its importance in the area of empirical finance (Rabin and Schrag, 1999; Daigler and Wiley, 1999; Barberis et al., 1998; DeBondt and Thaler, 1987), its validity in the experimental laboratory (Bloomfield et al., 2000, 1999; Rabin, 1998; Camerer, 1997), and its relevance to economic theory (Rabin, 2002; Daniel et al., 2001; Gervais and Odean, 2001; Bernardo and Welch, 2001; Hong and Stein, 1999).

[2] In this paper, the term irrationality is a synonym for misperception. This follows the convention of authors such as Richard Thaler, Matthew Rabin and Daniel Kahneman who use the word rational to denote correctly perceived probabilities, similar to John Muth and Robert Lucas' notion of rational expectations. Using terminology in this way has the unfortunate consequence of glossing over the more traditional meaning of rationality as choosing the best feasible alternative. Nonetheless, the remainder of the article uses the words irrationality and misperceptions interchangeably, with the preceding caveat in mind.

to pick the objectively best alternative in their choice sets because they maximize subjective utility instead of objective utility. Offsetting the utility losses associated with misperceiving the probability distribution of the fundamentals, however, are objective benefits of the reduced trading costs described above. This paper examines the conditions under which a heterogeneous-belief equilibrium exists and investigates which belief configurations support the phenomenon of efficiency-enhancing overconfidence.

In a world where departures from perfect rationality are socially beneficial, normative analysis becomes more difficult. For one thing, policies aimed at guaranteeing financial market transparency, insofar as they lead the economy away from a beneficially irrational equilibrium back toward perfect rationality, may generate higher social costs relative to the irrational status quo. According to the model in this paper, greater transparency can reduce liquidity and lead to increased transaction costs.

Although we do not provide an explicit analysis incorporating formal evolutionary dynamics, the static model developed in this paper provides a useful backdrop against which to consider the consequences of mutating beliefs shaped by a selection process in which perfect rationality is not necessarily the fittest psychological profile for individuals to have. In showing that overconfidence can benefit society, the paper implicitly suggests that competitive environments may actually select for overconfidence. That is, the interaction between survival outcomes and individual beliefs, amplified through successive generations of replication, can potentially lead to a process in which the environment tunes the hard-wired beliefs of individuals to a (locally) optimal position that does not coincide with perfect rationality.

Similar results have been reported. Hirshleifer and Luo (2001) present a dynamic model in which overconfident traders survive because they more aggressively exploit information they possess and therefore earn higher profits. De Long et al. (1991) suggest the possibility that overconfidence induces risk-averse individuals to take on extra risk unwittingly, which the market ultimately rewards with increased average return. Another reason why overconfident individuals may outperform rational individuals is given by Kyle and Wang (1997) who model an environment where irrational traders' reputation for over-reaction inhibits rational traders, leaving a larger slice of the profit pie for overconfident types. Dekel (1999) develops a model that generates a similar phenomenon in a bargaining setting. Gintis (2000) argues that *groups* with members whose beliefs lead them to behave reciprocally or coordinate in unusual ways are more adaptive to actual economic environments than groups of *homo economicus* individuals, an idea related to the coordination-gains from non-rational beliefs analyzed in this paper. Gigerenzer and Todd (1999) specify an array of decision-making environments in which making mistakes relative to the Bayesian benchmark can improve fitness. Finally, Bernardo and Welch (2001) model herd-behavior and the socially constructive role overconfident individuals can play in bucking trends; by relying on private information when it would be individually rational to follow the crowd, overconfident types benefit society by making private information public. The ideas in these papers are consonant with our thesis that many important economic environments do not exert pressure on beliefs to converge toward a single version of the truth. This occurs precisely because disagreements about the world can be socially useful.

## 2. A heterogeneous-belief model of financial market equilibrium

Following Spiegel and Subrahmanyam (1992), we consider a securities market comprised of $n$ risk-averse hedgers (non-expert individual investors), $k$ risk-neutral speculators (professional traders), and a single market-maker who observes order-flow $Q$ and chooses the current price of one securities contract $P$. Relegating details to an appendix posted on the JEBO website, we briefly discuss the objective functions of each of these three classes of agents, report the optimal decision rules generated by those objectives, and present conditions that guarantee the existence of a unique equilibrium. These results extend a basic model of financial market equilibrium by weakening the assumption of perfect rationality and allowing for systematically incorrect beliefs about the degree to which insider information is correlated with future spot prices. Thus, each agent maximizes an objective function that is a mathematical expectation taken with respect to a subjective probability distribution that may or may not coincide with the objective frequency distribution of the variables in the market. The equilibrium strategies of each class of agents are therefore a function of the beliefs of all market participants.

The $i$th speculator's demand for securities contracts is denoted $x_i$ and is assumed to be a linear function of the speculator's private information $\delta + \epsilon_i$. This notation indicates that private information has two components: the true deviation of tomorrow's spot price from its unconditional expectation $\delta \equiv S - \mathrm{E}S$, and speculator-$i$-specific noise $\epsilon_i$. This implies

$$x_i = \beta(\delta + \epsilon_i), \tag{1}$$

so that the speculator's choice of $x_i$ is equivalent to choosing $\beta$. Each speculator chooses $x_i$ to maximize expected profit, conditional on private information $(\delta + \epsilon_i)$. That is, the speculator's objective function is

$$E[(S - P)x_i|(\delta + \epsilon_i)]. \tag{2}$$

The expression inside the expectations operator is speculator profit, which equals the value of the securities contracts $x_i$ next period, $Sx_i$, net of initial-period cost $Px_i$. Next-period spot price $S$ and initial-period price (because execution price is uncertain) are both uncertain.

The Appendix A shows that the objective function in (2) is quadratic in $x_i$. As long as the leading coefficient is negative (which it must be, given mild technical assumptions stated explicitly in the Appendix A), there is a unique solution. Assuming joint normality for all the random variables in the model, the optimal choice of $\beta$ is independent of $\epsilon_i$, implying that (because $\epsilon_i$ does not appear in the optimal choice of $\beta$) each speculator chooses the same $\beta$.[3] Each speculator's quantity demanded $x_i$ is unique, however, owing to its dependence on $\epsilon_i$.

[3] Normality assumptions in the finance microfoundations literature go back at least to Kyle (1985). Although times series data from financial markets reveal significant departures from normality, the normality assumption can be justified as a second-order approximation to non-normal distributions.

Taking the decisions of other agents in the model as given, an expression for the optimal choice of $\beta$ can be developed, referred to as the best-response function (for all speculators):

$$\beta^{\mathrm{BR}} = \frac{\psi_{\mathrm{s}}}{\lambda((k+1)\psi_{\mathrm{s}} + 2\phi_{\mathrm{s}})}, \tag{3}$$

where $\psi_{\mathrm{s}} \equiv \mathrm{var}_{\mathrm{s}}(\delta)$, $\phi_{\mathrm{s}} \equiv \mathrm{var}_{\mathrm{s}}(\epsilon_i)$, and $\lambda$ is the slope of the market-maker's price schedule (assumed to be linear in order-flow $Q$), $P = ES + \lambda Q$. The subscript $s$ denotes the belief of a speculator. In other words, $\psi_{\mathrm{s}} \equiv \mathrm{var}_{\mathrm{s}}(\delta)$ is the variance of $\delta$ evaluated with respect to the subjective distribution of speculators. The subscripts h and m introduced below refer, respectively, to the moments of hedgers' and market-makers' subjective probability distributions.

Next, the hedger's best-response function is derived. The $j$th hedger starts off with a random endowment $\omega_j$ and hedges by taking a (partially) offsetting position in securities contracts. That is, given the endowment $\omega_j$, the hedger demands a quantity of securities contracts equal to

$$y_j = \gamma\omega_j, \tag{4}$$

where $\gamma < 0$ indicates the percentage of the initial endowment that is hedged away by trading in the securities market. Next-period wealth is $S(\omega_j + y_j) - Py_j$, consisting of the value of the endowment next-period, $S\omega_j$, summed together with profit from securities trading $(S - P)y_j$. Assuming that risk preferences of hedgers may be represented by the exponential (constant absolute risk aversion) expected utility function $-e^{-A\rho(y_j)}$ with risk-aversion parameter $A$, the standard result allows one to express the objective function solely in terms of mean and variance:

$$E_{\mathrm{h}}[S(\omega_j + y_j) - Py_j|\omega_j] - .5A\ \mathrm{var}_{\mathrm{h}}[S(\omega_j + y_j) - Py_j|\omega_j]. \tag{5}$$

The Appendix A shows that the first-order condition of this quadratic objective in $y_j$ may be expressed as a third-degree polynomial equation in $\gamma$, independent of $\omega_j$. That means all hedgers use an identical best-response strategy $\gamma^{\mathrm{BR}}$. The Appendix A contains the cubic equation that implicitly defines $\gamma^{\mathrm{BR}}$ and proves that its solution is unique on the real line.

Finally turning to the market-maker, its objective is to set the initial-period execution price $P$ by choosing $\lambda$ (in $P = ES + \lambda Q$) to minimize

$$E_m[(S - P)^2|Q], \tag{6}$$

which has the well-known solution

$$P = E_{\mathrm{m}}[S|Q] = \mathrm{E}S + \frac{\mathrm{cov}_{\mathrm{m}}(S, Q)}{\mathrm{var}_{\mathrm{m}}(Q)}Q, \tag{7}$$

where the last equality depends on the joint normality of $S$ and $Q$.[4] Imposing linearity on the market-maker's decision rule, the relationship $P = ES + \lambda Q$ must hold. The slope $\lambda$

[4] The market-maker observes the aggregate of all orders for securities, order flow $Q$, but not any single individual's order. The market-maker's objective function reflects competitive assumptions consistent with zero expected profit (or possibly, an institutional arrangement with transfer payments that compensate market-makers without affecting any trader's marginal cost of making a trade). Of course, competitive market assumptions abstract from a number of strategic opportunities that market-makers in the real world may have to obtain private information and

must equal the coefficient on $Q$ in (7). Therefore, the market-maker's best-response function is

$$\lambda^{\mathrm{BR}} = \frac{\mathrm{cov}_{\mathrm{m}}(S, Q)}{\mathrm{var}_{\mathrm{m}}(Q)} = \frac{\beta k \psi_{\mathrm{m}}}{\beta^2 k(k\psi_{\mathrm{m}} + \phi_{\mathrm{m}}) + \gamma^2 n \sigma_{\mathrm{m}}}, \tag{8}$$

where $\sigma_{\mathrm{m}} \equiv \mathrm{var}_{\mathrm{m}}(\omega_j)$ for all $j$. A necessary step in deriving this expression is to develop the order-flow variable $Q$. By definition,

$$Q = \sum_{i=1}^{k} x_i + \sum_{j=1}^{n} y_j. \tag{9}$$

With the assumption that all random variables in the model are zero-mean and jointly normal, then $\mathrm{E}Q = 0$, and conditional expectations are linear. These results are used repeatedly in deriving all of the best-response functions. See the Appendix A for details.

The best-response function for each class of individuals represents the optimal strategy of each agent given the choices of all other agents in the model. A Nash equilibrium is defined as a profile (i.e., a vector, one for each type of agent) of best-response functions such that each is simultaneously a best-response to the others. Expressing the profile of best-response strategies as the vector function $\begin{pmatrix} \beta^{\mathrm{BR}}(\beta,\gamma,\lambda) \\ \gamma^{\mathrm{BR}}(\beta,\gamma,\lambda) \\ \lambda^{\mathrm{BR}}(\beta,\gamma,\lambda) \end{pmatrix}$ any Nash equilibrium may be characterized as a fixed point of this function.

The first of this paper's main results to emerge from the heterogeneous-expectations extension of the standard model is that disagreement about the world does not usually (although it can) jeopardize the existence of equilibrium.

**Proposition 1.** *For any belief parameters that satisfy*

$$A^2 n\sigma_{\mathrm{m}}(\psi_{\mathrm{s}} + 2\phi_{\mathrm{s}})^2\psi_{\mathrm{h}}^2 > 4k(\psi_{\mathrm{m}}\psi_{\mathrm{s}} + 2\psi_{\mathrm{m}}\phi_{\mathrm{s}} - \psi_{\mathrm{s}}\phi_{\mathrm{m}})\psi_{\mathrm{s}}, \tag{10}$$

and

$$\psi_{\mathrm{m}}\psi_{\mathrm{s}} + 2\psi_{\mathrm{m}}\phi_{\mathrm{s}} - \psi_{\mathrm{s}}\phi_{\mathrm{m}} > 0, \tag{11}$$

there exists a unique Nash equilibrium $[\beta^*, \gamma^*, \lambda^*]'$ with the following form:

$$\gamma^* = \gamma^*(\psi_{\mathrm{h}}, \phi_{\mathrm{h}}, \sigma_{\mathrm{h}}, \alpha, a) = \frac{2\alpha^{\frac{1}{2}} - A\psi_{\mathrm{h}}(1 - ak)}{A\psi_{\mathrm{h}}(1 - ak)^2 + Aa^2k\phi_{\mathrm{h}} + A(n-1)\sigma_{\mathrm{h}}\alpha}, \tag{12}$$

$$\beta^* = \beta^*(a, \alpha, \gamma^*) = -\frac{a}{\alpha^{\frac{1}{2}}}\gamma^*, \tag{13}$$

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profitably trade based upon it. Another caveat that should be pointed out regarding the role of the market-maker is that there are a variety of different institutional structures one finds among actual securities markets around the world. Kyle (1989), for example, contrasts models where traders submit demand functions (limit orders) with those in which traders submit quantities (market orders), analogous to the difference between exchanges that use an order book price setting mechanism and those with market-makers. Our model should be interpreted in the context of floor trading with competitive market makers.

$$\lambda^* = \lambda^*(\alpha, \gamma^*) = -\frac{\alpha^{\frac{1}{2}}}{\gamma^*}, \tag{14}$$

where

$$a = \frac{\psi_{\mathrm{s}}}{(k+1)\psi_{\mathrm{s}} + 2\phi_{\mathrm{s}}}, \tag{15}$$

and

$$\alpha = \frac{ak[\psi_{\mathrm{m}}(1-ak) - a\phi_{\mathrm{m}}]}{n\sigma_{\mathrm{m}}}. \tag{16}$$

Inequality (10) requires that hedgers have a minimal degree of sensitivity to risk (i.e., $A$ must be sufficiently large), and that hedgers believe speculators are moderately well-informed about the true state of the economy (i.e., $\psi_{\mathrm{h}}$ not too small). Inequality (10) also requires that the number of speculators not be too large relative to the number of hedgers (i.e., $k \gg n$ not allowed). Also, existence of an equilibrium requires that the market-maker not hold extreme beliefs about the informativeness of speculators' private information or about the dispersion of hedgers' endowments (i.e., $\frac{\psi_{\mathrm{m}}}{\phi_{\mathrm{m}}}$ should not be too big, and $\sigma_{\mathrm{m}}$ should not be too small). Regarding the market-maker's perception of information versus noise, the inequalities in Proposition 1 point to a favorable normative interpretation of noise itself, akin to the ideas expressed in Fischer Black's article "Noise" (1986). In particular, if there is too much information and too little noise in the market, or if the market-maker believes order-flow is mostly information rather than noise, the market-maker sets price so steeply that no one is willing to trade. A certain level of noise in individual demand behavior is required in order to lubricate the market and make trade possible. At the other extreme, if the market-maker believes order-flow consists mostly of noise and consequently sets the price schedule too flatly, disequilibrium results: the quantity demanded by speculators goes to infinity while the demand of hedgers approaches a finite value.

Comparative statics analysis of the equilibrium strategies expressed in Proposition 1 leads to Proposition 2, stated below. What is unusual about the comparative statics of Proposition 2 is that the changes under consideration reflect changes in beliefs rather than changes in exogenous parameters that are traditionally considered to be under the control of policy makers. Proposition 2 summarizes how the equilibrium strategies of the three classes of decision-makers shift when any individual belief (about the variance of the random variables in the model) changes.

**Proposition 2.** *Equilibrium $\gamma$, $\beta$, $\lambda$ are increasing* $(+)$, *decreasing* $(-)$, *or nonmonotonic* $(\pm)$ *in beliefs, indicated as follows:*

$$\gamma = \gamma(\overset{\pm}{\psi_{\mathrm{s}}}, \overset{\pm}{\phi_{\mathrm{s}}}, \overset{-}{\psi_{\mathrm{h}}}, \overset{+}{\phi_{\mathrm{h}}}, \overset{+}{\sigma_{\mathrm{h}}}, \overset{+}{\psi_{\mathrm{m}}}, \overset{-}{\phi_{\mathrm{m}}}, \overset{-}{\sigma_{\mathrm{m}}}), \quad \beta = \beta(\overset{\pm}{\psi_{\mathrm{s}}}, \overset{\pm}{\phi_{\mathrm{s}}}, \overset{+}{\psi_{\mathrm{h}}}, \overset{-}{\phi_{\mathrm{h}}}, \overset{-}{\sigma_{\mathrm{h}}}, \overset{-}{\psi_{\mathrm{m}}}, \overset{+}{\phi_{\mathrm{m}}}, \overset{+}{\sigma_{\mathrm{m}}}),$$

$$\lambda = \lambda(\overset{\pm}{\psi_{\mathrm{s}}}, \overset{\pm}{\phi_{\mathrm{s}}}, \overset{-}{\psi_{\mathrm{h}}}, \overset{+}{\phi_{\mathrm{h}}}, \overset{+}{\sigma_{\mathrm{h}}}, \overset{+}{\psi_{\mathrm{m}}}, \overset{-}{\phi_{\mathrm{m}}}, \overset{-}{\sigma_{\mathrm{m}}}). \tag{17}$$

The most important aspect of Proposition 2 is that it allows one to understand the connection between overconfident beliefs and equilibrium strategies. In order to state this rigorously, a definition of overconfidence must first be specified. We define overconfidence as an inequality between subjective beliefs and objective expectations (mathematical expectations taken with respect to the true probability distribution) regarding the signal-to-noise ratio of insider information.

Recall that each speculator is privy to an insider signal $\delta + \epsilon_i$. If the speculator were able to observe $\delta$ directly, then next-period's spot price $S$ could be predicted perfectly. At the other extreme in which there is no information about $S$ in the initial period, the best forecast of $S$ is simply the unconditional expectation E$S$. An intermediate case between these two extremes of perfect versus no information occurs when both signal and noise components of insider information have positive variance (i.e., where $\text{var}(\delta) = \psi > 0$ and $\text{var}(\epsilon_i) = \phi > 0$). The quality of the information depends on the relative magnitude of these variance terms. In particular, when $\psi$ is large relative to $\phi$, the information $\delta + \epsilon_i$ is relatively helpful in predicting $S$. This leads to the following definition of overconfidence.

**Definition.** Agents of type $\tau$, $\tau$ = h, s, m, are said to be overconfident *whenever* $\frac{\psi_\tau}{\phi_\tau} > \frac{\psi}{\phi}$. That is, overconfident agents believe insider information is more correlated with the future spot price than it actually is.

It should be acknowledged that different authors define overconfidence in different ways. The most important difference among the various definitions of overconfidence is whether they refer to distorted beliefs about the mean, as opposed to the variance, of random variables. Defined as a first-moment phenomenon, overconfidence refers to decision-makers who believe the mean of a probability distribution is larger than it actually is (e.g., Manove and Padilla, 1999). As a second-moment phenomenon, overconfidence describes agents who make unfounded inferences about the distribution of a random event based on an observable signal (e.g., Odean, 1998). The phenomenon under study in this paper is second-order overconfidence. Overconfident agents in this paper correctly perceive the first moments of all random variables in the model, yet they overestimate the signal-to-noise ratio of insider information.

However, distorted beliefs about second moments also lead to distorted beliefs about conditional means, complicating the second/first-moment distinction described above. The second-moment-overconfident agents in this paper use information available today to revise expectations too far away from the unconditional mean. Thus, second-order overconfidence implies over-conditioned first-order expectations. This concept of overconfidence follows that of Hirshleifer and Luo (2001), Daniel et al. (1998), Benos (1998), with one important difference. Overconfident traders in those papers are, themselves, in possession of the private information whose precision is over-estimated. In contrast, any type of agent in our model can be overconfident about the quality of initial-period information, whether they are in possession of that information or not. In fact, the most interesting welfare consequences of overconfidence arise in the case where uninformed hedgers are the ones who are overconfident in the ability of professional traders (i.e., where those without information place too much faith in the quality of the information possessed by experts). This, in our view, is the more empirically relevant case to consider, judging from extensive anecdotal accounts in the news media of the rise and fall of the U.S. stock market over the last 10 years.

Having defined overconfidence, the consequences of moving from a state of perfect rationality to a state of overconfidence among the uninformed can be described. Proposition 2 illustrates the chain of events following a change in beliefs that starts from perfect rationality and moves in the direction of hedger overconfidence. A shift from correct beliefs to overconfident beliefs among hedgers corresponds to a rise in $\psi_h$ (the numerator of the signal-to-noise ratio) above the correct value $\psi$, holding $\phi$ (the denominator of the signal-to-noise ratio) constant.

Proposition 2 states that a hedger's choice of $\gamma$ will fall as $\phi_h$ increases, meaning that (because $\gamma$ is always negative) the degree of hedging intensifies. According to Proposition 2, speculators speculate more (i.e., $\beta$ increases) as a result of hedgers' overconfidence. What is happening is that hedgers' demand for insurance shifts out because of their systematic misperception, and speculators oblige by supplying an increased quantity. It can be shown that this increased trading activity injects more noise than additional signal into order-flow (i.e., order-flow is less correlated with the future price as a result of overconfidence). Consequently, a rational market-maker conditions on order-flow less than before, lowering the transaction cost $\lambda$ (Proposition 2 indicates that equilibrium $\lambda$ is decreasing in $\psi_h$). Thus, overconfident individual investors set off a self-reinforcing cycle in which speculators and hedgers both trade more, and the market-maker sets $P$ more competitively: as the market-maker reduces $\lambda$ toward zero, $P = \bar{S} + \lambda Q$ becomes closer to $P = \bar{S}$. The equilibrium consequences of hedger overconfidence can be summarized:

$$\psi_h \uparrow \text{ or } \phi_h \downarrow \Rightarrow \gamma \downarrow, \beta \uparrow, \lambda \downarrow . \tag{18}$$

This establishes the positive relationships between beliefs and equilibrium strategies forthcoming from the model, providing a framework for addressing the central question motivating this analysis: how does overconfidence affect the objective well-being of hedgers, speculators and the market-maker?

## 3. Welfare effects of overconfidence

In order to measure objective well-being as a function of possibly errant beliefs, we employ the following device. Plug each agent's decision rule (which depends on possibly errant subjective beliefs) into his or her expected utility function, and compute expectations with respect to the true distribution. Different subjective beliefs lead agents to choose different values of $\beta$, $\gamma$ or $\lambda$. Therefore, distinct values of objective expected utility result, even though the objective probability distribution remains constant. This device permits one to compute a unique functional relationship between beliefs and objective expected utility.

The objective utility functions are derived as follows. Plugging the equilibrium demand of the $i$th speculator, $x_i = \beta(\delta + \epsilon_i)$, into (2), objective utility can be expressed as $(\delta + \epsilon_i)^2 U_*^s$, where

$$U_*^s = \frac{\psi_s\psi + 2\phi_s\psi - \psi_s\phi}{[(k+1)\psi_s + 2\phi_s](\psi + \phi)}\beta^* . \tag{19}$$

Since $(\delta + \epsilon_i)^2$ is positive, $U_*^{\mathrm{s}}$ can be used to compare the speculator's well-being at rational expectations beliefs and at other points in belief-space.

Applying the same procedure as before to derive the objective expected utility for a typical hedger, one inserts equilibrium demand $y_j = \gamma\omega_j$ into the expected utility function (5). From there, the hedger's objective utility may be expressed as $\omega_j^2 U_*^{\mathrm{h}} + \omega_j \bar{S}$, where

$$U_*^{\mathrm{h}} = -0.5A\psi - A\psi(1-ak)\gamma - \{\lambda + 0.5A[\psi(1-ak)^2 + a^2k\phi + \alpha(n-1)\sigma]\}\gamma^2. \tag{20}$$

Since $\omega_j^2$ is positive and $\omega_j\bar{S}$ is a constant and independent of all beliefs, $U_*^{\mathrm{h}}$ can be used for the purpose of making objective welfare comparisons across different beliefs.

Finally, for the market-maker, the expected loss function when evaluated with respect to the true distribution is

$$\begin{aligned} \mathrm{E}[(S-P)^2|Q] = \psi &- \frac{(\beta k\psi)^2}{\beta^2 k(k\psi+\phi)+\gamma^2 n\sigma} - [\beta^2 k(k\psi+\phi)+\gamma^2 n\sigma] \\ &\times \left[\frac{\beta k\psi}{\beta^2 k(k\psi+\phi)+\gamma^2 n\sigma} - \lambda\right]^2, \end{aligned} \tag{21}$$

having replaced $Q^2$ with its expected value, $\mathrm{var}(Q)$. The Appendix A shows that the last bracketed term is zero if the market-maker is rational. Therefore, the irrationality of hedgers and speculators does not affect the market-maker's well-being so long as the market-maker's beliefs are correct.

By plugging in numerical values for all the beliefs in the model and allowing them to vary one at a time, it is easy to demonstrate that the three objective-utility expressions above are nonmonotonic in beliefs. The absence of any global relationships between beliefs and well-being is, at first glance, discouraging and prompts one to question whether anything at all can be said about beliefs and economic efficiency on the basis of the model described above. In fact, a systematic relationship does emerge, although it fails to hold in some regions of the model's parameter space.

One might guess, since speculators and hedgers split the objective expected profit pie in zero-sum fashion, that overconfidence would necessarily hurt at least one type of agent. But because of the diversity of preferences (i.e., because hedgers are risk-averse and speculators are risk-neutral), reallocations of expected profit lead to changes in welfare that are not zero-sum. Numerical analysis of the objective expected utility functions shows that exogenous changes in beliefs can improve economic efficiency.

### *3.1. Which belief configurations serve society best?*

The beliefs of each class of decision-maker (hedgers, speculators, and market-makers) can be categorized into one of three mutually exclusive states: overconfident, rational, or underconfident. Given three classes of decision-makers and three states of beliefs, a grand total of $3 \times 3 \times 3 = 27$ *belief configurations* are possible. By numerically investigating the objective utility of the three agents at representative points in all 27 belief configurations, we seek to identify a pattern relating beliefs to economic efficiency.

To deal with the impossibility of exhaustive grid search over an unbounded parameter space, the following numerical strategy is employed. First, a numerical grid in objective parameter space (not including subjective belief parameters) is defined by choosing a range of five possible values for each of six objective parameters. Those objective parameters are risk-aversion, number of hedgers, number of speculators, the variance of the signal component of insider information, the variance of insider noise, and the dispersion of hedgers' endowment. Combined into a parameter vector, the objective parameters have the symbolic form

$$\theta \equiv (A, n, k, \psi, \phi, \omega). \tag{22}$$

Five values for each of six parameters gives $5^6 = 15,625$ grid points to examine.

At each grid point, objective utilities for different belief configurations are compared to the rational-expectations level of objective utility. Allowing for seven values of each of the three belief parameters, corresponding to zero-deviation and plus or minus 0.1, 1, and 5% deviations, a total of 27 belief configurations are considered. By tallying the number of Pareto improvements that occur as a result of 0.1, 1, and 5% deviations (out of a total of $26\times3 = 78$ deviations from perfect rationality), and doing this at all 15,625 grid points, we seek to characterize the relationship between beliefs and efficiency on a statistical basis.

Any time all three agents' objective utilities are better off with distortions rather than full rationality, a single welfare-improving departure from rational expectations is recorded. it turns out that welfare-improving overconfidence is a fairly frequent phenomenon. At fully 5,896 of the 15,625 points evaluated, at least one of the 26 irrational belief configurations led to a Pareto improvement, meaning that irrationality increased everyone's objective utility function beyond the utility level associated with rational expectations.

**Result 1.** Departures from rational expectations often result in Pareto improvements. This occurs over much, but not all, of parameter space.

Of course, overconfident beliefs represent a strict subset of all possible departures from rational expectations, and the question remains as to which departures are associated with Pareto improvements. The numerical analysis continues by recording which of the 26 non-rational belief configurations produce Pareto improvements. A surprisingly clear-cut pattern emerges.

**Result 2.** Welfare-improving overconfidence occurs only when those who do not possess information are more confident about its precision than those who do.

Result 2 states that a very specific kind of departure from perfect rationality is required in order for economic efficiency to improve. Those without information (hedgers) must be more confident in the precision of the information than those who possess it (speculators). This stands in contrast to most other papers on overconfidence in which overconfidence nearly always applies to the beliefs of those who possess information themselves.

The first panel in Fig. 1 depicts the objective utility of hedgers and speculators, and the equilibrium transaction cost $\lambda$, as a function of speculator beliefs in the neighborhood of the (vertical) perfect-rationality line $\frac{\psi_s}{\psi} = 1$. All other beliefs are set equal to the true values

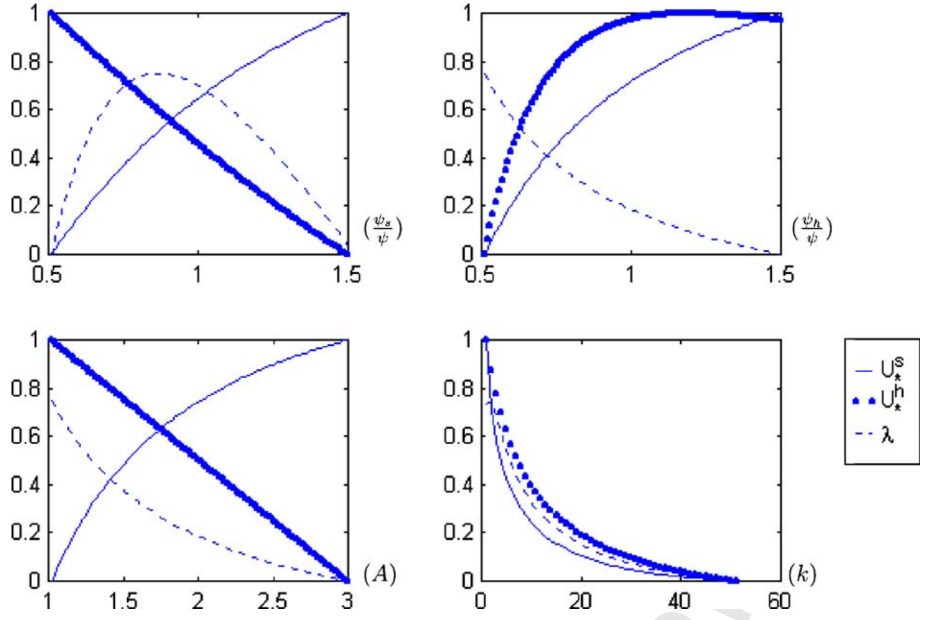


Fig. 1. Objective utility and transaction cost λ. All four panels depict the objective utility of hedgers ($U_*^{\mathrm{h}}$), the objective utility of speculators ($U_*^{\mathrm{s}}$), and transaction cost (inverse liquidity λ). In the first panel, these three quantities are plotted as functions of speculators' confidence. All other beliefs are set equal to the true moments. Speculators are overconfident to the right of the vertical line $\frac{\psi_{\mathrm{s}}}{\psi} = 1$. Overconfident speculators improve their own welfare but hurt hedgers. In the upper right panel, hedgers are overconfident to the right of the vertical line $\frac{\psi_{\mathrm{h}}}{\psi} = 1$. Overconfident hedgers help themselves (for moderate degrees of overconfidence) and benefit speculators. This depicts welfare-improving overconfidence. The lower left panel shows objective utility and λ as functions of hedgers' risk aversion $A$. The last panel plots objective utility and λ as the number of speculators $k$ increases. In the lower two panels, all beliefs are correct. Speculator utility $U_*^{\mathrm{s}}$ and hedger utility $U_*^{\mathrm{h}}$ are normalized to fall in the interval [0,1]. For ease of viewing, the slope of the price schedule λ is normalized to fall in the interval [0,0.75]. The true parameters are set to the following values: $\psi = \phi = \sigma = 10,\ k = n = 20,$ and $A = 2$.

of the corresponding second moments. In the case where speculators are overconfident, the welfare of speculators and hedgers are diametrically opposed. That is, if speculators benefit when they themselves become overconfident, they do so at the expense of hedgers. The slope of the price schedule, λ, is non-monotonic in the beliefs of speculators.

The upper right panel of Fig. 1 depicts the objective utility of hedgers and speculators as a function of hedger beliefs. In contrast to overconfidence in speculators, overconfident hedgers create benefits for both hedgers and speculators. Similar to other panels, the $x$-axis represents the hedgers' belief about the quality of expert information as a percentage of true quality. Perfect rationality occurs when beliefs coincide with the truth, which occurs when the belief-to-truth ratio $\frac{\psi_{\mathrm{h}}}{\psi}$ on the $x$-axis equals 1. To the right of 1 (where $\frac{\psi_{\mathrm{h}}}{\psi} > 1$), hedgers are overconfident. To the left, they are underconfident. The arc-shaped objective utility of hedgers is unmistakably nonmonotonic, increasing in response to moderate overconfidence but decreasing when beliefs become too overconfident. In addition, the overconfidence of hedgers flattens the price schedule (i.e., reduces the transaction cost λ).

The bottom two panels show objective welfare responses as a function of changing degrees of risk aversion and a changing number of speculators respectively. In spite of some sensitivity to the values used for the parameters $A$, $n$, $k$, $\psi$, $\phi$ and $\sigma$, certain regularities emerge. To analyze the strength and qualitative character of those regularities, additional grid-search routines were applied to investigate more effectively the continuous relationship between economic efficiency and the key parameters of interest.

One result that stands out is that the benefits of overconfidence tend to fade away when the number of hedgers becomes large, either in an absolute or relative sense. In contrast, when the number of informed speculators increases, the beneficial effect of overconfidence does not necessarily go away. Another interesting numerical finding concerns the slope of the price schedule $\lambda$. Although the price schedule usually flattens ($\lambda \downarrow$) with $k$, steepening is also possible. This possibility is potentially relevant to policy-making entities such as the Securities and Exchange Commission. By pursuing informational transparency as a policy goal (e.g., enabling more individuals to become experts), the policy maker may unwittingly hurt market liquidity. Hill-Knight (2000) for a description of Regulation Fair Disclosure and some of the issues it raises for experts and the ease with which new entrants may elect to become experts.) Most other patterns match what one would expect. For instance, increased risk aversion ($A \uparrow$) always hurts hedgers and helps speculators.

### 3.2. *Evolution and objective utility*

This section advances an indirect evolutionary argument to try to explain the ubiquity of overconfidence among non-expert traders in the US. Extrapolating from the finding of welfare-improving overconfidence summarized in Result 2, it is possible to articulate a rationale for why the most powerful economy in the world might also be an economy whose trading public is too trusting with regard to the expertise of financial professionals.

Beginning from a state of perfect rationality, consider that by chance, or based on historical events, investor beliefs mutate. What happens next? Does competition or arbitrage activity force beliefs to gravitate back toward full rationality? One often hears the claim that competitive pressure or arbitrage of some kind ensures that markets punish the irrational and reward the rational. It is unclear, however, with heterogeneous risk preferences and heterogeneous beliefs about volatility, what type of arbitrage opportunities arise as the result of overconfidence.

As an example, consider what would happen if the one-period model of this paper were played repeatedly (with no forward looking behavior) and if replication rates were increasing in objective expected utility. It is easy to conclude by Result 1 that portions of the population would grow overconfident through time. Our model actually predicts that the fittest economy, with the highest profits for speculators and the happiest non-experts, is also an economy in which non-experts are overconfident. By this account, the empirical and experimental evidence of widespread overconfidence makes sense. According to the model, overconfidence by non-experts about experts is, evolutionarily speaking, adaptive. Insofar as this simple model corresponds to real-world financial markets, one expects there to be overconfidence. In fact it would be surprising to find the most developed markets in the world exclusively populated by rational traders.

An unusual assertion such as this deserves to be handled with skepticism. In particular, one wonders whether strange or restrictive assumptions are driving the conclusion that mistaken perceptions can be good for the economy. Beyond the technical assumptions used in the derivation of equilibrium strategies (which are stated in the Appendix A), a key ingredient in deriving the main conclusion is the hypothesis that fitness is an increasing function of objective expected utility rather than, say, expected profit. Of course expected utility and expected profit are not the same for those who are risk averse. The issue is whether risk considerations ought to be given any weight in evaluating evolutionary fitness (i.e., whether it makes sense to use expected utility as a measure of adaptiveness). We argue that it may indeed make sense, depending on the particulars of the decision-making environment.

There are many environments in nature with features such as an uncertain water supply where human fitness is clearly improved if agents can take action to reduce volatility, even if that means trading off some amount of expected quantity. Even in modern economies where it is unlikely that basic biological constraints play an important role, there are still reasons to think that risk reduction, and not just high return, is fitness enhancing. For instance, financial stability may enhance society's average quality of health (because of reduced stress levels), increase its rate of innovation (think of leisure-class inventors in the history of science and engineering), and possibly reduce the severity of business cycles (because of less volatile wealth effects on aggregate consumption). Although these extrapolations are admittedly speculative, it seems difficult to defend ruling out risk-reduction (and focusing only on expected return) in analyzing the factors that determine survival probabilities.

Without romanticizing irrationality, the results of this paper invite further speculation about the possibility that modern economic environments may actually select for individuals whose beliefs about probabilities depart from actual long-run frequencies. We interpret the model's results as tentative evidence in favor of the claim that overconfidence observed in the real world may be thought of as an evolved trait, either through (long time-scale) biological or (relatively recent) cultural channels. Although society bears certain costs when beliefs diverge from the truth, there can be important benefits as well.

The next section summarizes additional comparative statics for conditional price volatility and unconditional price volatility with respect to changes in beliefs. This provides an alternative means of considering the welfare implications of overconfidence.

### 3.3. *The effect of overconfidence on alternative measures of efficiency*

Recall that the future value of one securities contract is $S$ and the current price is $P$. Therefore, if the current price is an efficient summary of the available information, conditional variance $\mathrm{var}(S|P)$ should be small. Developing $\mathrm{var}(S|P)$, we have

$$\mathrm{var}(S|P) = \psi - \frac{1}{(1/\psi) + (\phi/k\psi^2) + (n\sigma/k^2\psi^2)(k/n\sigma_\mathrm{m})[\psi_\mathrm{m}(1 + 2(\phi_\mathrm{s}/\psi_\mathrm{s})) - \phi_\mathrm{m}]}, \tag{23}$$

which depends on true moments (with no subscripts) as well as subjective beliefs. The remarkable point about the expression above is that it is independent of the beliefs of hedgers. Although equilibrium $\gamma$, which depends on the hedger's beliefs, enters in several places through $Q$, those beliefs wind up dropping out because of offsetting adjustments made by speculators and the market-maker. Thus, by the conditional volatility efficiency metric, overconfident hedgers do not harm society.

**Proposition 3.** *Equilibrium informational efficiency, as measured by* $\mathrm{var}(S|P)$, *is independent of the beliefs of hedgers and is increasing* $(+)$, *or decreasing* $(-)$, *indicated as follows:*

$$\mathrm{var}(S|P) = \sigma^2_{\delta|P}(\overset{+}{\psi_{\mathrm{m}}}, \overset{-}{\phi_{\mathrm{m}}}, \overset{-}{\sigma_{\mathrm{m}}}, \overset{-}{\psi_{\mathrm{s}}}, \overset{+}{\phi_{\mathrm{s}}}). \tag{24}$$

Turning to unconditional price volatility, the following proposition paints a similar picture regarding the benign nature of overconfidence in non-experts.

**Proposition 4.** *Near any rational expectations parameter, unconditional price volatility is increasing* $(+)$ *or decreasing* $(-)$, *indicated as follows:*

$$\mathrm{var}(P) = \sigma^2_{P}(\overset{+}{\psi_{\mathrm{m}}}, \overset{-}{\phi_{\mathrm{m}}}, \overset{-}{\sigma_{\mathrm{m}}}, \overset{+}{\psi_{\mathrm{s}}}, \overset{-}{\phi_{\mathrm{s}}}). \tag{25}$$

Away from rational expectations belief parameters, the signs on the first three arguments are unchanged, but the signs over the last two arguments may fail to hold. Proposition 4 implies that marginally overconfident speculators (in the neighborhood of rational expectations) make prices more volatile, just as one would expect, but that changes in the beliefs of hedgers are perfectly offset by other adjustments with a null net effect on price volatility.

## 4. Conclusion

Mistaken beliefs can improve the lot of all traders in financial markets. This happens when non-experts, because they are overconfident in the ability of experts, trade too much. Although this departure from perfect rationality comes with a cost, over-trading leads to improved liquidity, lower trading costs, and lower execution-price volatility. When experts who possess insider information are themselves overconfident about their ability to forecast the future, no such efficiency gain occurs.

In light of numerous empirical puzzles from financial markets concerning trading volume and various measures of market liquidity, the link in our model between beliefs and market depth provides a starting point for further analysis. Proposition 2 implies that liquidity, as measured by market depth $(\frac{1}{\lambda^*})$, is an increasing function of overconfidence among non-expert traders. A natural extension of this idea would be a dynamic model with learning in which beliefs are tied to the time path of price volatility and order flow.

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## Appendix A

This section provides the algebraic steps used in the derivation of the three best-response functions and Propositions 1–4.

### *A.1. The speculator's best-response function*

The speculator's objective function is $\mathrm{E}[(S-P)x_i \mid (\delta+\epsilon_i)]$. Substituting in for initial-period price $P = \bar{S} + \lambda Q$, for order-flow $Q = \sum_{l=1}^{k} x_l + \sum_{j=1}^{n} y_j$, and for speculator and hedger demand $x_l = \beta(\delta+\epsilon_l)$ and $y_j = \gamma\omega_j$, the speculator's profit function may be expressed as

$$(S-P)x_i = [1-\lambda\beta(k-1)]x_i\delta - \lambda x_i^2 - \lambda\beta x_i \sum_{l\neq i} \epsilon_l - \lambda\gamma x_i \sum_{j=1}^{n} \omega_j. \tag{26}$$

Taking expectations with respect to the speculator's subjective probability distribution (and assuming that all random variables are zero-mean and jointly normal) yields the objective function

$$\mathrm{E}[(S-P)x_i \mid (\delta+\epsilon_i)] = \frac{\psi_{\mathrm{s}}}{\psi_{\mathrm{s}}+\phi_{\mathrm{s}}}(\delta+\epsilon_i)[1-\lambda\beta(k-1)]x_i - \lambda x_i^2. \tag{27}$$

As a quadratic function with a negative leading coefficient (the market-maker must set $\lambda > 0$ in order to minimize losses), there exists a unique maximizer $x_i$ characterized by the first-order condition

$$\frac{\psi_{\mathrm{s}}}{\psi_{\mathrm{s}}+\phi_{\mathrm{s}}}(\delta+\epsilon_i)[1-\lambda\beta(k-1)] - 2\lambda x_i = 0. \tag{28}$$

Substituting the linear demand rule $x_i = \beta(\delta+\epsilon_i)$, the $i$-specific information $(\delta+\epsilon_i)$ drops out. Solving for $\beta$ leads to the best response function (3).

### *A.2. The Hedger's best-response function*

The hedger's profit function is $S\omega_j + (S-P)y_j$. Substituting in for $P$, and then $Q$, $x_i$, and $y_l,\ l \neq j$, profit can be expressed as

$$\bar{S}\omega_j + [\omega_j + (1-\lambda k\beta)y_j]\delta - \lambda\beta y_j \sum_{i=1}^{k} \epsilon_i - \lambda\gamma y_j \sum_{l\neq j}^{n} \omega_l - \lambda y_j^2. \tag{29}$$

Taking the mean and variance of the above expression with respect to the hedger's subjective probability distribution (and assuming that all random variables are zero-mean and jointly normal) leads to the mean-variance objective function

$$b_0 + b_1 y_j + b_2 y_j^2, \tag{30}$$

with coefficients

$$b_0 \equiv \bar{S}\omega_j - 0.5A\psi_\mathrm{h}\omega_j^2, \tag{31}$$

$$b_1 \equiv -A\psi_\mathrm{h}\omega_j(1-\lambda\beta k), \tag{32}$$

$$b_2 \equiv -\lambda - 0.5A[\psi_\mathrm{h}(1-\lambda\beta k)^2 + \lambda^2\beta^2 k\phi_\mathrm{h} + \lambda^2\gamma^2(n-1)\sigma_\mathrm{h}]. \tag{33}$$

The hedger's first-order condition is $b_1 + 2b_2 y_j = 0$, and the appropriate second-order condition $b_2 < 0$ is easily verified. After substituting $y_j = \gamma\omega_j$ and canceling $\omega_j$ from the first-order condition, it can be written as a third-degree polynomial equation in $\gamma$. Solving the first-order condition necessitates finding the zero(s) of the hedger's *implicit best-response function*, defined as

$$f(\gamma;\beta,\lambda) = c_0 + c_1\gamma + c_3\gamma^3, \text{ with coefficients} \tag{34}$$

$$c_0 \equiv A\psi_\mathrm{h}(1-\lambda\beta k), \tag{35}$$

$$c_1 \equiv 2\lambda + A\psi_\mathrm{h}(1-\lambda\beta k)^2 + A\lambda^2\beta^2 k\phi_\mathrm{h}, \tag{36}$$

$$c_3 \equiv A\lambda^2(n-1)\sigma_\mathrm{h}. \tag{37}$$

Any real root of $f(\gamma;\beta,\lambda) = 0$ in $\gamma$ is a best response, given the actions of others, $\beta$ and $\lambda$. We will argue that $\gamma < 0$ in any finite equilibrium. First note that, by plugging in the speculator's best-response function for $\beta$,

$$1 - \lambda\beta k = \frac{\psi_\mathrm{s} + 2\phi_\mathrm{s}}{(k+1)\psi_\mathrm{s} + 2\phi_\mathrm{s}} > 0, \tag{38}$$

guaranteeing that the coefficient $c_0$ is positive. Inspecting the other two coefficients, it is clear that they too are positive and, therefore, that the implicit best-response function is a strictly increasing function. That is, $f(0;\beta,\lambda) = c_0 > 0$ and $\frac{\partial f(\gamma;\beta,\lambda)}{\partial\gamma} = c_1 + 3c_3\gamma^2$. Every zero of $f$ must be to the left of zero, implying that $\gamma < 0$. And because the auxiliary function is strictly increasing (and its slope never approaches zero), it must have exactly one intersection with the $x$-axis. The best-response function $\gamma^{\mathrm{BR}}$ is therefore unique.

### *A.3. The market-maker's best-response function*

The market-maker picks a linear pricing rule $P = \bar{S} + \lambda Q$ that minimizes expected square loss $(S-P)^2$ conditional on $Q$. It is a standard result that the solution to this problem is $P = \mathrm{E_m}[S|Q]$ which can be developed as follows:

$$\begin{aligned} E_\mathrm{m}[S|Q] &= E_\mathrm{m}S + \frac{\mathrm{cov_m}(S,Q)}{\mathrm{var_m}(Q)}(Q - E_\mathrm{m}Q) \\ &= \bar{S} + \frac{\beta k\psi_\mathrm{m}}{\beta^2 k(k\psi_\mathrm{m} + \phi_\mathrm{m}) + \gamma^2 n\sigma_\mathrm{m}}Q. \end{aligned} \tag{39}$$

Setting the expression above equal to the linear pricing rule assumed earlier forces equality between the coefficients on $Q$ (i.e., $\lambda = \frac{\beta k \psi_{\mathrm{m}}}{\beta^2 k(k\psi_{\mathrm{m}}+\phi_{\mathrm{m}})+\gamma^2 n\sigma_{\mathrm{m}}}$), which is the best-response function given in the text.

### A.4. Proposition 1

The goal is to produce a closed-form Nash equilibrium $\beta,\ \gamma,$ and $\lambda$ in terms of the beliefs $\psi_{\mathrm{s}}, \phi_{\mathrm{s}}, \psi_{\mathrm{h}}, \phi_{\mathrm{h}}, \sigma_{\mathrm{h}}, \psi_{\mathrm{m}}, \phi_{\mathrm{m}}, \sigma_{\mathrm{m}}$ and other parameters $A, k, n$.

The best response functions of the three types form the system:

$$\beta\lambda = \frac{\psi_{\mathrm{s}}}{[(k+1)\psi_{\mathrm{s}} + 2\phi_{\mathrm{s}}]} \equiv a, \tag{40}$$

$$A\psi_{\mathrm{h}}(1-\lambda\beta k) + [2\lambda + A\psi_{\mathrm{h}}(1-\lambda\beta k)^2 + A\lambda^2\beta^2 k\phi_{\mathrm{h}}]\gamma + A\lambda^2(n-1)\sigma_{\mathrm{h}}\gamma^3 = 0 \tag{41}$$

$$\lambda[\beta^2 k(k\psi_{\mathrm{m}} + \phi_{\mathrm{m}}) + n\gamma^2\sigma_{\mathrm{m}}] = k\psi_{\mathrm{m}}\beta. \tag{42}$$

We know, in equilibrium, that $\lambda > 0$. The converse, $\lambda \le 0$, does not make sense and would lead the speculator to demand an infinite quantity. Given that $\lambda$ is strictly positive, we can multiply Eq. (42) by $\lambda$:

$$\lambda^2\beta^2 k(k\psi_{\mathrm{m}} + \phi_{\mathrm{m}}) + n\sigma_{\mathrm{m}}\lambda^2\gamma^2 = k\psi_{\mathrm{m}}\lambda\beta. \tag{43}$$

Using the relation $\lambda\beta = a$ from (40) and rearranging terms produces

$$\lambda^2\gamma^2 = \alpha \equiv \frac{ak\psi_{\mathrm{m}} - a^2 k(k\psi_{\mathrm{m}} + \phi_{\mathrm{m}})}{n\sigma_{\mathrm{m}}}. \tag{44}$$

Because the left-hand side is the square of non-zero terms, $\alpha$ must be greater than zero. Because $\gamma < 0$ and $\lambda > 0$, the square root of the equation above is equivalent to $\lambda\gamma = -\alpha^{\frac{1}{2}}$. Substituting $\alpha$ for $\lambda^2\gamma^2$ in the cubic term of (41) leads to a linear equation in $\gamma$ from which the equilibrium value $\gamma^*$ presented in the text is derived. Substituting $\lambda\beta = a$ and $\gamma = \gamma^*$ in (42) produces $\beta^*$, and the equilibrium value $\lambda^*$ easily follows from (40).

The two inequalities in the hypothesis of Proposition 1 are necessary for $\gamma < 0$ and for $\alpha \ge 0$. Without the first condition, speculators demand an infinite quantity. And without the second condition, no real solution to the hedger's problem exists. The admissible set defined by these inequalities is quite large. Existence requires only that it is non-empty, which it is: simply note that the parameterization in which all parameters equal unity is admissible. Finally, the inequalities are sufficient to produce a negative $\gamma$ and a positive $\alpha$, implying that the necessary conditions are also sufficient for the existence of equilibrium.

### A.5. Proposition 2

The goal is to compute derivatives of the equilibrium strategies with respect to subjective beliefs and determine the sign of those derivatives. To economize on symbolic exposition, the strategy we use is first to compute total derivatives of $\gamma$ with respect to the belief parameters and then use simple algebraic relationships among $\beta$, $\lambda$ and $\gamma$ to find the signs of the other statics results. It is helpful to note that the auxiliary parameter $a$ isolates the effects of the speculator's beliefs and that the market-maker's belief parameters enter $\gamma$ only through the auxiliary parameter $\alpha$.

Writing $\gamma = N/D$, with $N = 2\alpha^{\frac{1}{2}} - A\psi_{\mathrm{h}}(1 - ak)$ and $D = [A\psi_{\mathrm{h}}(1 - ak)^2 + Aa^2k\phi_{\mathrm{h}} + A(n-1)\sigma_{\mathrm{h}}\alpha]$, a useful preliminary fact can be established:

$$\frac{\partial\gamma}{\partial\alpha} = \frac{\alpha^{-\frac{1}{2}}D - NA(n-1)\sigma_{\mathrm{h}}}{D^2} > 0 \tag{45}$$

(since denominator $D$ is positive and, following from $\gamma < 0$, numerator $N = \gamma D$ is negative). Then

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\psi_{\mathrm{m}}} = \frac{1}{n\sigma_{\mathrm{m}}}[ak(1-ak)] > 0 \;\Rightarrow\; \frac{\mathrm{d}\gamma}{\mathrm{d}\psi_{\mathrm{m}}} = \frac{\partial\gamma}{\partial\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\psi_{\mathrm{m}}} = (+)(+) > 0. \tag{46}$$

The last derivative formula as well as the next two are valid because $\psi_{\mathrm{m}}, \phi_{\mathrm{m}}, \sigma_{\mathrm{m}}$ enter $\gamma$ only through $\alpha$. The following computations provide the desired statics for the hedger's (equilibrium) strategy:

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\phi_{\mathrm{m}}} = -\frac{1}{n\sigma_{\mathrm{m}}}[a^2k] < 0 \;\Rightarrow\; \frac{\mathrm{d}\gamma}{\mathrm{d}\phi_{\mathrm{m}}} = \frac{\partial\gamma}{\partial\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\phi_{\mathrm{m}}} = (+)(-) < 0, \tag{47}$$

and

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\sigma_{\mathrm{m}}} = -\frac{1}{\sigma_{\mathrm{m}}}\alpha < 0 \;\Rightarrow\; \frac{\mathrm{d}\gamma}{\mathrm{d}\sigma_{\mathrm{m}}} = \frac{\partial\gamma}{\partial\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\sigma_{\mathrm{m}}} = (+)(-) < 0. \tag{48}$$

Next, the statics for $\beta$ and $\lambda$ in response to a small change in the beliefs of the market-maker are computed. As an intermediate step, partial derivatives are computed using the equilibrium relationships $\beta = -a\alpha^{-\frac{1}{2}}\gamma$ and $\lambda = -\alpha^{\frac{1}{2}}\gamma^{-1}$:

$$\begin{aligned}&\frac{\partial\beta}{\partial\gamma} = -a\alpha^{-(1/2)} < 0, \quad \frac{\partial\beta}{\partial\alpha} = \frac{1}{2}a\alpha^{-(3/2)}\gamma < 0, \quad \frac{\partial\lambda}{\partial\gamma} = \alpha^{(1/2)}\gamma^{-2} > 0, \text{ and}\\ &\frac{\partial\lambda}{\partial\alpha} = -\frac{1}{2}\alpha^{-(1/2)}\gamma^{-1} > 0.\end{aligned} \tag{49}$$

The results above can then be used to find the sign of the total derivatives we are seeking. Because the auxiliary parameter $a$ only depends on the beliefs of speculators, there is no need to consider terms such as $\frac{\mathrm{d}a}{\mathrm{d}\psi_{\mathrm{m}}}$, $\frac{\mathrm{d}a}{\mathrm{d}\phi_{\mathrm{m}}}$, or $\frac{\mathrm{d}a}{\mathrm{d}\sigma_{\mathrm{m}}}$, since they equal zero. Six more monotonic relationships can be computed at this point:

$$\frac{\mathrm{d}\beta}{\mathrm{d}\psi_{\mathrm{m}}} = \frac{\partial\beta}{\partial\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\psi_{\mathrm{m}}} + \frac{\partial\beta}{\partial\gamma}\frac{\mathrm{d}\gamma}{\mathrm{d}\psi_{\mathrm{m}}} = (-)(+) \;+\; (-)(+) < 0, \tag{50}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}\phi_{\mathrm{m}}} = \frac{\partial\beta}{\partial\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\phi_{\mathrm{m}}} + \frac{\partial\beta}{\partial\gamma}\frac{\mathrm{d}\gamma}{\mathrm{d}\phi_{\mathrm{m}}} = (-)(-) \;+\; (-)(-) > 0, \tag{51}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}\sigma_{\mathrm{m}}} = \frac{\partial\beta}{\partial\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\sigma_{\mathrm{m}}} + \frac{\partial\beta}{\partial\gamma}\frac{\mathrm{d}\gamma}{\mathrm{d}\sigma_{\mathrm{m}}} = (-)(-) \;+\; (-)(-) > 0, \tag{52}$$

$$\frac{d\lambda}{d\psi_m} = \frac{\partial\lambda}{\partial\alpha}\frac{d\alpha}{d\psi_m} + \frac{\partial\lambda}{\partial\gamma}\frac{d\gamma}{d\psi_m} = (+)(+) \quad + \quad (+)(+) > 0, \tag{53}$$

$$\frac{d\lambda}{d\phi_m} = \frac{\partial\lambda}{\partial\alpha}\frac{d\alpha}{d\phi_m} + \frac{\partial\lambda}{\partial\gamma}\frac{d\gamma}{d\phi_m} = (+)(-) \quad + \quad (+)(-) < 0, \tag{54}$$

$$\frac{d\lambda}{d\sigma_m} = \frac{\partial\lambda}{\partial\alpha}\frac{d\alpha}{d\sigma_m} + \frac{\partial\lambda}{\partial\gamma}\frac{d\gamma}{d\sigma_m} = (+)(-) \quad + \quad (+)(-) < 0. \tag{55}$$

This establishes that monotonicity in the beliefs of the market-maker prevails in all 9 cases (three agents' strategies with respect to three belief parameters). Next, we turn to the beliefs of hedgers.

Using the fact that $a$ and $\alpha$ do not depend on the beliefs of hedgers, the following calculation is justified:

$$\frac{d\gamma}{d\psi_h} = \frac{-A(1-ak)[Aa^2k\phi_h + A\alpha(n-1)\sigma_h + (1-ak)2\alpha^{\frac{1}{2}}]}{D^2} < 0. \tag{56}$$

Noting that $\phi_h$ enters $\gamma$ only through the denominator, that $\frac{dD}{d\phi_h} = Aa^2k$, and with $N < 0$, we have

$$\frac{d\gamma}{d\phi_h} = \frac{d}{d\phi_h}\left(\frac{N}{D}\right) = -\frac{NAa^2k}{D^2} > 0 \text{ and } \frac{d\gamma}{d\sigma_h} = -\frac{NA(n-1)\alpha}{D^2} > 0. \tag{57}$$

Computing total derivatives with respect to the hedgers' beliefs is easier than it was for the market-maker's beliefs, since the hedgers' beliefs, $\psi_h, \phi_h, \sigma_h$, enter equilibrium $\beta$ and $\lambda$ only through $\gamma$. Thus,

$$\frac{d\beta}{d\psi_h} = \frac{\partial\beta}{\partial\gamma}\frac{d\gamma}{d\psi_h} = (-)(-) > 0, \quad \frac{d\beta}{d\phi_h} = \frac{\partial\beta}{\partial\gamma}\frac{d\gamma}{d\phi_h} = (-)(+) < 0,$$
$$\frac{d\beta}{d\sigma_h} = \frac{\partial\beta}{\partial\gamma}\frac{d\gamma}{d\sigma_h} = (-)(+) < 0, \tag{58}$$

and

$$\frac{d\lambda}{d\psi_h} = \frac{\partial\lambda}{\partial\gamma}\frac{d\gamma}{d\psi_h} = (+)(-) < 0, \frac{d\lambda}{d\phi_h} = \frac{\partial\lambda}{\partial\gamma}\frac{d\gamma}{d\phi_h} = (+)(+) > 0, \frac{d\lambda}{d\sigma_h}$$
$$= \frac{\partial\lambda}{\partial\gamma}\frac{d\gamma}{d\sigma_h} = (+)(+) > 0. \tag{59}$$

As was the case with the beliefs of the market-maker, all nine effects of strategies with respect to the beliefs of hedgers are seen to have a definite sign. Next, we show that monotonicity fails when it comes to the case of speculators' beliefs.

We must show that equilibrium $\beta$, $\gamma$, and $\lambda$ are nonmonotonic in the beliefs of speculators ($\psi_s$, and $\phi_s$). A numerical example suffices for the purpose. Table A.1 contains the values of equilibrium strategies when evaluated at three different values of $\psi_s$, setting all other parameters equal to 1. Table A.1 clearly demonstrates that all the strategies are nonmonotonic in $\psi_s$. Similar numerical demonstrations illustrate nonmonotonicity with respect to $\phi_s$.

Table A.1
Nonmonotonicity of equilibrium strategies as functions of $\psi_s$

| $\psi_s$ | $\beta$ | $\gamma$ | $\lambda$ |
|---|---|---|---|
| 1 | 0.049 | −0.069 | 5.152 |
| 3 | 0.029 | −0.024 | 12.881 |
| 5 | 0.173 | −0.110 | 2.406 |

### *A.6. Proposition 3*

Because all the random variables in the model are normal, we have

$$\mathrm{var}(S|P) = \psi - \frac{1}{\mathrm{d}_0 + \mathrm{d}_1 X}, \tag{60}$$

where $\mathrm{d}_0 \equiv \frac{1}{\psi} + \frac{\phi}{k\psi^2} > 0$, $\mathrm{d}_1 \equiv \frac{n\sigma}{k^2\psi^2} > 0$, and

$$X \equiv \frac{\gamma^2}{\beta^2} = \frac{\gamma^2\lambda^2}{\beta^2\lambda^2} = \frac{\alpha}{a^2}. \tag{61}$$

From the definitions of $\alpha$ and $a$,

$$X = \frac{k}{n\sigma_{\mathrm{m}}}\left[\psi_{\mathrm{m}}\left(1 + 2\frac{\phi_{\mathrm{s}}}{\psi_{\mathrm{s}}}\right) - \phi_{\mathrm{m}}\right]. \tag{62}$$

Thus,

$$X = X(\overset{+}{\psi_{\mathrm{m}}}, \overset{-}{\phi_{\mathrm{m}}}, \overset{-}{\sigma_{\mathrm{m}}}, \overset{-}{\psi_{\mathrm{s}}}, \overset{+}{\phi_{\mathrm{s}}}),\ \text{and}\ \frac{\partial \mathrm{var}(S|P)}{\partial X} = (\mathrm{d}_0 + \mathrm{d}_1 X)^{-2}\mathrm{d}_1 > 0. \tag{63}$$

By the chain rule, the belief parameters affect $\mathrm{var}(S|P)$ in the same qualitative way as they affect $X$, proving Proposition 3.

### *A.7. Proposition 4*

Rearranging the conditional variance formula for two normal variables so that the unconditional variance is on the left-hand side, and using the equilibrium conditions $\lambda\beta = a$ and $(\lambda\gamma)^2 = \alpha$, we have

$$\mathrm{var}(P) = \frac{a^2k^2\psi^2}{\psi - \mathrm{var}(S|P)}. \tag{64}$$

Apart from speculator beliefs, which enter through $a$, the market-maker's beliefs affect both $\mathrm{var}(P)$ and $\mathrm{var}(S|P)$ in the same qualitative manner. The effect of $\psi_s$ on $\mathrm{var}(P)$, which enters through $a$ and $\mathrm{var}(S|P)$), is not obvious at first. Working with Eq. (64), we sign the effect in the neighborhood of a rational expectations equilibrium (i.e., an equilibrium where beliefs are correct). These results are summarized in Proposition 4.

## References

Barberis, N., Shleifer, A., Vishny, R.W., 1998. A model of investor sentiment. Journal of Financial Economics 49, 307–343.

Benos, A.B., 1998. Aggressiveness and survival of overconfident traders. Journal of Financial Markets 1, 353–383.

Bernardo, A.E., Welch, I., 2001. On the evolution of overconfidence and entrepreneurs. Journal of Economics and Management Strategy 10, 301–330.

Black, F., 1986. Noise. Journal of Finance 41, 529–543.

Bloomfield, R., Libby, R., Nelson, M.W., 1999. Confidence and the welfare of less-informed investors. Accounting, Organizations and Society 24, 623–647.

Bloomfield, R., Libby, R., Nelson, M.W., 2000. Underreactions, overreactions and moderated confidence. Journal of Financial Markets 2, 113–137.

Camerer, C.F., 1997. Progress in behavioral game theory. Journal of Economic Perspectives 11 (4), 167–188.

Daigler, R.T., Wiley, M.K., 1999. The impact of trader type on the futures volatility-volume relation. Journal of Finance 54, 2297–2316.

Daniel, K.D., Hirshleifer, D., Subrahmanyam, A., 1998. Investor psychology and security market under- and overreactions. Journal of Finance 53, 1839–1885.

Daniel, K.D., Hirshleifer, D., Subrahmanyam, A., 2001. Overconfidence, arbitrage, and equilibrium asset pricing. Journal of Finance 56, 921–965.

DeBondt, W.F.M., Thaler, R.H., 1987. Further evidence on investor overreaction and stock market seasonability. Journal of Finance 42, 557–581.

Dekel, E., 1999. On the evolution of attitudes toward risk in winner-take-all games. Journal of Economic Theory 87, 125–143.

De Long, J.B., Shleifer, A., Summers, L.H., Waldman, R.J., 1991. The survival of noise traders. Journal of Business 64, 1–19.

Gervais, S., Odean, T., 2001. Learning to be overconfident. Review of Financial Studies 14, 1–27.

Gigerenzer, G., Todd, P., ABC Research Group, 1999. Simple Heuristics That Make Us Smart. Oxford University Press, New York.

Gintis, H., 2000. Game Theory Evolving. Princeton University Press, Princeton.

Hirshleifer, D., Luo, G.Y., 2001. On the survival of overconfident traders in a competitive securities market. Journal of Financial Markets 4, 73–84.

Hong, H., Stein, J.C., 1999. A unified theory of underreaction, momentum trading, and overreaction in asset markets. Journal of Finance 54, 2143–2178.

Kyle, A.S., 1985. Continuous auctions and insider trading. Econometrica 53, 1315–1336.

Kyle, A.S., 1989. Informed speculation with imperfect competition. Review of Economic Studies 56, 317–355.

Kyle, A.S., Wang, A., 1997. Speculation duopoly with agreement to disagree: can overconfidence survive the market test? Journal of Finance 52, 2073–2090.

Manove, M., Padilla, J.A., 1999. Banking (conservatively) with optimists. Rand Journal of Economics 30, 324–350.

Odean, T., 1998. Volume, volatility, price, and profit when all traders are above average. Journal of Finance 53, 1887–1934.

Rabin, M., 1998. Psychology and economics. Journal of Economic Literature 36, 11–46.

Rabin, M., 2002. Inference by believers in the law of small numbers. Quarterly Journal of Economics 117, 775–816.

Rabin, M., Schrag, J.L., 1999. First impressions matter: a model of confirmatory bias. Quarterly Journal of Economics 117, 37–82.

Spiegel, M., Subrahmanyam, A., 1992. Informed speculation and hedging in a noncompetitive securities market. Review of Financial Studies 5, 307–329.