UNIVERSITY of OTAGO
Te Whare Wānanga o Otāgo
NEW ZEALAND

BUSINESS SCHOOL
Te Kura Pakihi

ECON 377: Mathematical Economics

Semester 2, 2019

COURSE INFORMATION

Welcome to ECON 377!

Description

Mathematical economics embodies various applications of mathematical techniques to economics, particularly economic theory. This branch of economics dates from the nineteenth century and has developed a rate of natural increase in recent decades. Mathematics is increasingly important in terms of the expression and communication of ideas in economics. A thorough knowledge of mathematics is indispensable for understanding almost all fields of economics, including both applied and theoretical fields.

Advanced economics makes extensive use of formal mathematical models. This course covers the basic mathematical techniques required for rigorous study of economics, and it will provide extensive instruction on applications of these techniques to economic problems. This course provides a comprehensive introduction to the mathematical tools most often used by economists in their research. Wherever possible, familiar micro and macro models will be used to place these tools in economic contexts. Examples and motivation are drawn from important topics in economics.

The class is intended for students with a strong intellectual interest in both mathematics and economics and, in particular, for students who may pursue a graduate degree in economics or related fields.

Learning Outcomes

The principal aims of the course are to introduce students to the basic mathematical methods and to show their application in economic analysis. We will try to give you the intuition to think about economic ideas in mathematical terms, and interpret mathematical concepts in the context of economics. Your understanding of economics and mathematics both will improve after this course.

The principal aim of this course is to extend your facility with those methods of mathematics needed to pursue economic analysis at a more advanced level. By the end of the course you should have extended your knowledge to include the technique of optimisation under inequality constraints, the analysis of dynamic economic models, in particular differential and difference equations and a rudimentary introduction to dynamic optimisation (optimal control theory).

Note that ECON 377 is one of the required papers that students intending to proceed to postgraduate programmes (Honours, PGDip, MEcon, MBus) in economics must include in their Bachelor's degree programme.

After completing the course, you will be able to:

- Use and explain the underlying principles, terminology, methods, techniques and conventions used in the subject;
- Solve economic problems using the mathematical methods described in the subject;
- Develop a set of problem-solving and analytical skills to solve problems in other fields of study and everyday decisions;
- Develop an initial understanding of how to frame economic modelling ideas in mathematical format;
- Possess a solid grasp of essential mathematical tools required for the further studies in economic theory.

Brief Course Outline

ECON 377 is split into two main sections:

1. Ronald Peeters (before the mid-semester break: weeks 1-7)
2. Murat Üngör (after the mid-semester break: weeks 8-13)

Detailed topic outlines and reading lists for each section are provided below.

Teaching Staff

Lecturer

Name: **Ronald Peeters**

Office: 5.07; Otago Business School Office Phone: Ext 8731

Email: ronald.peeters@otago.ac.nz

Website: http://www.otago.ac.nz/economics/staff/otago666118.html

Office Hours: Tuesday / Wednesday 11:30–12:30

Course Coordinator and Lecturer

Name: **Murat Üngör**

Office: 5.11; Otago Business School Office Phone: Ext 8134

Email: murat.ungor@otago.ac.nz

Website: http://www.muratungor.com

Office Hours: Monday / Tuesday / Wednesday 2:00–3:00

Office hours are an excellent resource to get the extra help you may need. Do not hesitate or put off contacting us. We are here to help you and to promote your success. We welcome all feedback from you. You can email us anytime; we usually check our emails throughout the day and evening.

Course Delivery

You will attend three 50-minute lectures and one 50-minute tutorial a week.

Lectures

Three 50-minute lectures will be held each week. Lectures present the key conceptual material through discussion and are supported by readings.

Classes are held on Tuesdays, Wednesdays, and Thursdays between 4:00–4:50pm.

Tutorials

Tutorials are held on Mondays between 4:00–4:50pm.

Tutorials are an integral part of the course, and you should consider attendance at tutorials to be just as important as attendance at lectures. The tutorial programme complements the material covered in lectures and the textbook(s). It contains questions that you should do over the course of the semester. Tutorials take place weekly and begin in the **second week**.

Tutorial questions will be distributed periodically during the semester. You should be prepared to answer (and ask) questions during the tutorial.

Expectations and Workload

As ECON 377 is an 18 point course, using the University's 'rule of thumb' you should therefore plan to devote 12 hours per week to this course throughout the semester (including the mid-semester break and the final examination period). Four of these are spent in lectures and a tutorial – leaving 8 hours per week for your own reading and study.

Course Learning Resources

Blackboard

Blackboard https://blackboard.otago.ac.nz/ provides you with access to course materials, class notices, and resources. The lecture slides and announcements concerning the course will be available on Blackboard. Lecture slides provide an outline of the lectures but are not a substitute for attending lectures and taking your own notes. Problem sets will be posted on Blackboard, and solutions will be posted the day after you turn them in. Lastly, practice questions for the final exam will be posted on Blackboard.

Textbook

Ronald Peeters' section of the course draws on the Baldani-Bradfield-Turner book:

- Baldani, J., Bradfield, J., Turner, R. W. (2005). Mathematical Economics (2nd edition). Thomson/South-Western. [Central Library Main HB135.B4865 2004]
- Baldani, J., Bradfield, J., Turner, R. W. (2013). Mathematical Economics (3rd edition). Linus Publications. [Central Library Main HB135.B4865 2016]

Murat Üngör's section of the course draws on the several editions of different text books:

- Baldani, J., Bradfield, J., Turner, R. W. (2005). Mathematical Economics (2nd edition). Thomson/South-Western. [Central Library Main HB135.B4865 2004]
- Baldani, J., Bradfield, J., Turner, R. W. (2013). Mathematical Economics (3rd edition). Linus Publications. [Central Library Main HB135.B4865 2016]
- Hoy, M., Livernois, J., McKena, C., Rees, R., Stengos, T. (2001). Mathematics for Economics (2nd edition). MIT Press. [Central Library Main HB135.MD563]
- Sydsæter, K., Hammond, P. (2006). Essential Mathematics for Economic Analysis (2nd edition). FT Prentice Hall. [Central Library Main HB135.SZ35 2006]

The textbook readings are meant to clarify or elaborate material presented in class, or to give you an idea of alternative presentations of the same material. Copies of Baldani-Bradfield-Turner (2nd edition, and 3rd edition)[1], Hoy-Livernois-McKena-Rees-Stengos (2nd edition), and Sydsæter and Hammond (2006, 2nd edition) are on close reserve at the Central Library. Additional readings will be posted on Blackboard.

[1] You can use any edition of Baldani-Bradfield-Turner, since there is, basically, no difference between the second and the third edition of this book.

Reading

Several hours of reading will be required each week. You will gain a greater benefit from lectures if you complete the assigned readings (textbook readings) **before** the relevant lecture. Following the lecture, make sure you understand the relevant textbook material, read any hand-out/posted material, and prepare answers to relevant problem set questions.

Blackboard

Blackboard https://blackboard.otago.ac.nz/ provides you with access to course materials, class notices, and resources. Blackboard is used to email the whole class so it is important that you check your student email and *Blackboard* regularly.

All students enrolled in ECON 377 should have access to the ECON 377 Blackboard page.

Study Smart

The library has designed an area, accessible through Blackboard that will help support you with research and referencing. See Study Smart folder in Blackboard.

This Study Smart link contains an interactive study hub of research support resources provided by the Library, Student IT and the Student Learning Centre. It includes:

- Getting Started – what you need to know in your first couple of weeks
- Resources for finding information for your assignments
- Finding information on the web that is appropriate for your studies
- Tools and resources to help you study efficiently
- Where to get help when you need it
- Additional assistance is available from the University Library liaison: http://otago.libguides.com/liaison

Student Webmail

Forward your University email address to an email address that you use regularly as follows:

1. Log into your StudentMail account using your student username and password
2. Click **Cog button (top right corner)** > **Options**
3. Under **Account**, select the **Forward your email** shortcut under the **Short Cuts** menu on the right side of the screen**.**
4. Under the Forwarding heading, type in the email address you want your email to be forwarded to. You can also choose to have a copy of these emails kept on your StudentMail account, so please check the box if you would like this.
5. Click the **Start forwarding** button.

Assessment: Assignments & Final Exam

Your overall grade for this paper is determined as follows (note that plussage will not be applied and detailed descriptions of each assessment will be provided separately):

Internal assessment assignments (four)	40%
Final examination (three hours, closed book)	60%

Internal Assessment Sets (4X 10% = 40%)
Ronald Peeters will set the first two assignments and Murat Üngör will set the other two. The details of each assignment will be provided separately.

Assignments may be submitted in hard copy or by email, providing attachments are in PDF format. They can be submitted by email (as a PDF file) or as a hard copy, which can be placed in the 'ECON 377' pigeonhole (to the left of room CO504) on the due date.

Requests for extensions can only be considered on medical or compassionate grounds and should be made, if at all possible, before the relevant due date. Moreover, it is your responsibility to ensure that your computer files are regularly backed up. Extensions will not be granted due to the accidental loss or deletion of files.

Final Exam (60%)
The final exam will be three hours long (closed book). The final exam will cover the whole course. The University will report the exam timetable later in the semester.

Plussage
There is no plussage on this paper.

Grading System
The grading scheme used at Otago is:

A+	90-100	**C+**	60-64
A	85-89	**C**	55-59
A-	80-84	**C-**	50-54
B+	75-79	**D**	40-49
B	70-74	**E**	<40
B-	65-69		

Special consideration
If you consider your performance in the final exam to be seriously impaired, or if you are too ill to sit an examination, you can apply for Special Consideration. To do this you will need to obtain an application form from the University Information Centre or Student Health. Please note that applications for Special Consideration must be made within five calendar days from the date of the last examination for which the application applies and must be accompanied by supporting documentation, such as a medical certificate.

Academic Integrity

Academic integrity means being honest in your studying and assessments. It is the basis for ethical decision-making and behaviour in an academic context. Academic integrity is informed by the values of honesty, trust, responsibility, fairness, respect and courage. You are expected to be aware of, and act in accordance with, the University's Academic Integrity Policy.

Academic Misconduct, such as plagiarism or cheating, is a breach of Academic Integrity and is taken very seriously by the University. Types of misconduct include plagiarism, copying, unauthorised collaboration, taking unauthorised material into a test or exam, impersonation, and assisting someone else's misconduct. A more extensive list of the types of academic misconduct and associated processes and penalties is available in the University's Student Academic Misconduct Procedures.

To access the information in the Academic Integrity Policy and learn more, please visit the University's Academic Integrity website at www.otago.ac.nz/study/academicintegrity or ask at the Student Learning Centre or Library. If you have any questions, ask us.

Student Learning Support and Information

Student Charter

http://www.otago.ac.nz/about/otago005275.html

Guidelines for Learning at Otago

http://www.otago.ac.nz/hedc/index.html

Student Learning Centre

The Student Learning Centre, which is part of the Higher Education Development Centre, provides learning support, free of charge, to ALL enrolled students. Their services include:

- a workshop programme designed to help students to improve their learning strategies and their generic skills; http://slc.otago.ac.nz/attend-a-workshop/
- free and confidential consultations with a learning adviser for assistance with learning strategies; http://slc.otago.ac.nz/talk-to-a-learning-adviser/
- on-line study skills advice;
- a student leadership programme
- a student-led peer support programme for students of all ages and backgrounds
- conversational English groups for students from a non-English speaking background.

Library Support

The Library website http://www.otago.ac.nz/library provides access to resources and services, including group room bookings, library hours and locations, past exam papers, subject guides, article databases and more.

If you need assistance either check out the self-help guides http://otago.libguides.com/selfhelp, or ask Library staff at the ground floor service desks, or email ask.library@otago.ac.nz

Māori Student Support
Rachel Sizemore (Ngāi Tahu)
Kaiārahi Māori

Rachel provides tautoko to Māori students in the Business School. Offering an ear to listen, help with scholarships, extra tutorials, and to liaise with academic departments and Student Services with regards to those students and their intended course of study. Rachel offers support also to those studying away from their whanau, hapū and iwi, to feel safe and supported.

Tel: +64 3 479 5342 Email: rachel.sizemore@otago.ac.nz

Pacific Islands' Student Academic Advisor
Warm Pacific Greetings

Talofa lava, my name is Esmay Eteuati and my role is to liaise with Academic Departments and Student Services relating to Pacific students' and their course of study. I support both staff and students in the Business School and have a network of Pacific contacts in other Divisions around the University.

Tel: +64 3 479 4756 Email: esmay.eteuati@otago.ac.nz

Disability Information and Support
Students are encouraged to seek support if they are having difficulty with their studies due to disability, temporary or permanent impairment, injury or chronic illness. It is important to seek help early, through one of the contacts below:

Contact either:
The Disabilities Liaison person in the Economics Department is Janet Bryant
(Tel: +64 3 479 8656; Email: janet.bryant@otago.ac.nz; Office: Commerce 5.06)
Or
Disability Information and Support
Tel: +64 3 479 8235; Email: disabilities@otago.ac.nz; Web: http://www.otago.ac.nz/disabilities

International Students
The Otago Business School encourages international students to seek support if they are having difficulties with their studies or meeting other challenges while they are students at the University of Otago. In such instances, international students should feel free to contact International Student Support:

Tel: +64 3 479 8344
Email: international.support@otago.ac.nz
Web: www.otago.ac.nz/international
Location: Archway West Building

Student Feedback

We encourage your feedback. This can be in the form of contacting staff, participating in course evaluation surveys and communicating with class representatives. Continual improvements will be made to this course based in part on student feedback.

Class Representatives

The class (or student) representative system is an avenue for encouraging communication and consultation between staff and students. It provides you with a vehicle for communicating your views on the teaching and delivery of the paper and provides staff with an opportunity to communicate information and gain constructive feedback from students. It contributes to the development of a sense of community within a department and it adds a further dimension to the range of support services offered to students.

Volunteers for the role of class representatives will be called early in the semester. The OUSA invites all class representatives to a training session, conducted by OUSA, about what it means to be a class representative and some of the possible procedures for dealing with issues that arise. They also provide information on the services that OUSA offers and the role OUSA can play in solving problems that may occur. The OUSA provides support to class representatives during the semester. Departmental staff will also meet with class representatives during the semester to discuss general issues or matters they wish to have considered.

Your class representative's name and contact details will be posted on Blackboard early in the semester.

Concerns about the Course

We hope you will feel comfortable coming to talk to us if you have a concern about the course. We will be happy to discuss any concerns you may have. Alternatively, you can report your concerns to the Class Representative who will follow up with departmental staff. If, after making approaches via these channels, you do not feel that your concerns have been addressed, there are University channels that may aid resolution. For further advice or more information on these, contact the departmental administrator or head of department.

Disclaimer

While every effort is made to ensure that the information contained in this document is accurate, it is subject to change. Changes will be notified in class and via Blackboard. You are encouraged to check Blackboard regularly. We hope you enjoy ECON 377. We want every member of the class to succeed, so don't hesitate to ask us for help whenever you need it. The earlier you ask for help with any concepts that are unclear to you, the easier you will find the rest of the course.

We hope you enjoy the course!
Ronald Peeters & Murat Üngör

SCHEDULE OF CLASSES (RONALD PEETERS) Weeks 1-7

WEEK COMMENCING	TUESDAY CLASS (4:00–4:50PM)	WEDNESDAY CLASS (4:00–4:50PM)	THURSDAY CLASS (4:00–4:50PM)	TUTORIALS Monday (4:00–4:50PM)	
8 July	Course outline and goals **TOPIC 1**: An Introduction to Mathematical Economics and Applications	**TOPIC 1**: An Introduction to Mathematical Economics and Applications		No tutorial	
15 July	**TOPIC 2**: Matrix Theory			Tutorial 1	
22 July	**TOPIC 3**: Application of Matrix Theory to Linear Models			Tutorial 2	
29 July	**TOPIC 4**: Multivariate Calculus: Theory and Applications			Tutorial 3	
5 August	**TOPIC 5**: Multivariable Optimisation without Constraints: Theory and Applications			Tutorial 4	Assignment 1 due 4:00pm on Monday (**12 August**)
12 August	**TOPIC 6**: Constrained Optimisation: Theory and Applications			Tutorial 5	
19 August	**TOPIC 6**: Constrained Optimisation: Theory and Applications (continued)			Tutorial 6	Assignment 2 due 4:00pm on Monday (**2 September**)
26 August	**Mid-Semester Break**				

TOPIC 1: AN INTRODUCTION TO MATHEMATICAL ECONOMICS AND APPLICATIONS

Learning Topics and Objectives:

As students with some background in economic analysis, you are probably already accustomed to working with some economic models; these methods, while important and useful, are only part of an economist's mathematical too kit. We start focusing on how mathematical tools are used in economics. The approach is to first introduce mathematical concepts (with a few simple economic examples) and then present detailed economic applications, such as microeconomics applications that draw on economic theory from the fields of industrial organisation, labour economics, and public finance.

Suggested Readings:

1. Lecture Slides
2. Textbook Readings:
   - Baldani-Bradfield-Turner (2nd edition, Chapter 1): "An Introduction to Mathematical Economics" (pp. 1-36)
   - Baldani-Bradfield-Turner (2nd edition, Chapter 2): "An Introduction to Mathematical Economic Applications" (pp. 37-65)

TOPIC 2: MATRIX THEORY

Learning Topics and Objectives:

Most mathematical models used by economists ultimately involve a system of several equations, which usually express how one or more endogenous variables depend on several exogenous parameters. If these equations are all linear, the study of such systems belongs to an area of mathematics called linear algebra. Even if the equations are nonlinear, much may be learned from linear approximations around the solution we are interested in—for example, how the solution changes in response to small shocks to the exogenous parameters. Indeed, such models lie right at the heart of the econometric techniques that form the basis of most modern empirical economic analysis. The analysis and even the comprehension of systems of linear equations become much easier if we use some key mathematical concepts such as matrices, vectors, and determinants. These, as well as their application to economic models, will be introduced.

Suggested Readings:

1. Lecture Slides
2. Textbook Readings:
    - Baldani-Bradfield-Turner (2nd edition, Chapter 3): "Matrix Theory" (pp. 66-95)

TOPIC 3: APPLICATIONS OF MATRIX THEORY TO LINEAR MODELS

Learning Topics and Objectives:

The usefulness of matrix theory extends far beyond its ability to solve systems of linear equations. For instance, in the theory of differential and difference equations, in linear and nonlinear optimisation theory, in statistics and econometrics, the methods of linear algebra are used extensively. We demonstrate several applications of matrix theory, choosing examples from both microeconomics and macroeconomics.

Suggested Readings:

1. Lecture Slides
2. Textbook Readings:
    - Baldani-Bradfield-Turner (2nd edition, Chapter 4): "Applications of Matrix Theory to Linear Models" (pp. 96-121)

TOPIC 4: MULTIVARIATE CALCULUS THEORY AND APPLICATIONS

Learning Topics and Objectives:

We will learn how to use equilibrium conditions that implicitly define equilibrium values of endogenous variables. We will examine the implicit function theorem, which shows under what conditions explicit functions exist that give equilibrium values of endogenous variables as functions of exogenous variables and parameters. We will also see how to derive comparative static results even if the explicit forms of the equilibrium functions are not known. We will study several macro- and microeconomic applications.

Suggested Readings:

1. Lecture Slides
2. Textbook Readings:
    - Baldani-Bradfield-Turner (2nd edition, Chapter 5): "Multivariate Calculus: Theory" (pp. 122-137)
    - Baldani-Bradfield-Turner (2nd edition, Chapter 6): "Multivariate Calculus: Applications" (pp. 138-177)

TOPIC 5: MULTIVARIABLE OPTIMISATION WITHOUT CONSTRAINTS: THEORY AND APPLICATIONS

Learning Topics and Objectives:

Finding the best way to do a specific task involves what is called an optimisation problem. Examples abound in almost all areas of human activity. A farmer might want to know what amount of fertilizer per square yard will maximize profits. An oil company may wish to find the optimal rate of extraction from one of its wells. The first section presents the basic results, illustrated by relatively simple examples and problems. Then we give a more systematic presentation of the theory with two variables. Subsequently we consider how the theory can be generalized to functions of several variables. Much of economic analysis involves seeing how the solution to an optimisation problem responds when the situation changes—for example, if some relevant parameters change.

Suggested Readings:

1. Lecture Slides
2. Textbook Readings:
    - Baldani-Bradfield-Turner (2nd edition, Chapter 7): "Multivariable Optimisation without Constraints: Theory" (pp. 178-195)
    - Baldani-Bradfield-Turner (2nd edition, Chapter 8): "Multivariable Optimisation without Constraints: Applications" (pp. 196-218)

TOPIC 6: CONSTRAINED OPTIMISATION: THEORY AND APPLICATIONS

Learning Topics and Objectives:

Economics is the study of the allocation of scarce resources. Scarcity implies constraints, so much of economic theory has to do with agents who are optimising some objective function subject to a constraint, as is the case, for example, when a consumer maximises utility subject to a budget constraint. Accordingly, this topic considers constrained optimisation problems, and studies the method of Lagrange multipliers in some detail.

Suggested Readings:

1. Lecture Slides
2. Textbook Readings:
    - Baldani-Bradfield-Turner (2nd edition, Chapter 9): "Constrained Optimisation: Theory" (pp. 219-242)
    - Baldani-Bradfield-Turner (2nd edition, Chapter 10): "Constrained Optimisation: Applications" (pp. 243-282)

SCHEDULE OF CLASSES (MURAT ÜNGÖR) Weeks 8-13

WEEK COMMENCING	TUESDAY CLASS (4:00–4:50PM)	WEDNESDAY CLASS (4:00–4:50PM)	THURSDAY CLASS (4:00–4:50PM)	TUTORIALS Monday (4:00–4:50PM)	
26 August	**Mid-Semester Break**				
2 September	Course outline and goals **TOPIC 7**: Optimisation with inequality constraints: Theory and Applications	**TOPIC 7**: Optimisation with inequality constraints: Theory and Applications		Tutorial 7 (Ronald's last tutorial session)	
9 September	**TOPIC 8**: Value Functions and the Envelope Theorem: Theory and Applications		**Introduction to TOPIC 9**: Dynamics and Integration	Tutorial 8	
16 September	**TOPIC 9**: Difference and Differential Equations: Theory and Applications			Tutorial 9	Assignment 3 due 5:00pm on Friday (**20 September**)
23 September	**TOPIC 9**: Difference and Differential Equations: Theory and Applications			Tutorial 10	
30 September	**TOPIC 10**: Optimal Control Theory with Applications			Tutorial 11	
7 October	**TOPIC 10**: Optimal Control Theory with Applications			Tutorial 12	Assignment 4 due 5:00pm on Friday (**11 October**)

TOPIC 7: OPTIMISATION WITH INEQUALITY CONSTRAINTS: THEORY AND APPLICATIONS

Learning Topics and Objectives:

Almost all constraints encountered in economic analysis are inequality constraints, although they are usually treated as if they were equalities. For example, an individual may choose not to spend all her income, so her budget constraint is an inequality. In this topic we introduce the Kuhn-Tucker conditions for optimisation with inequality constraints. First we consider a one-variable maximisation problem where the choice variable is constrained to be nonnegative. Then we impose a functional constraint while ignoring the nonnegativity constraint. Next we formulate the Kuhn-Tucker conditions and generalise them to a many-variable maximisation problem with many constraints. Finally, this topic contains several applications of Kuhn-Tucker analysis.

Suggested Readings:

1. Lecture Slides
2. Textbook Readings:

- Baldani-Bradfield-Turner (2nd edition, Chapter 11): "Optimisation with Inequality Constraints: Theory" (pp. 283-297)
- Baldani-Bradfield-Turner (3rd edition, Chapter 11): "Optimisation with Inequality Constraints: Theory" (pp. 337-353)
- Baldani-Bradfield-Turner (2nd edition, Chapter 12): "Optimisation with Inequality Constraints: Applications" (pp. 298-315)
- Baldani-Bradfield-Turner (3rd edition, Chapter 12): "Optimisation with Inequality Constraints: Applications" (pp. 355-375)
- Hoy-Livernois-McKena-Rees-Stengos (2001, Chapter 15): "Concave Programming and the Kuhn-Tucker Conditions" (pp. 677-697)
- Sydsæter-Hammond (2006, Chapter 14): "Constrained Optimisation" (pp. 503-548)

TOPIC 8: VALUE FUNCTIONS AND THE ENVELOPE THEOREM: THEORY AND APPLICATIONS

Learning Topics and Objectives:

Most economic analysis is comparative statistics. The object is to determine the effects on the equilibrium values of endogenous variable of changes in the values of parameters. The purpose of this topic is to develop value functions and the envelope theorem as tools for comparative static analysis. A *value function* expresses the optimal value of an objective function as a function of the parameters that define an economic agent's environment. The *envelope theorem* is a comparative static relationship between an objective function and its associated value function.

Suggested Readings:

1. Lecture Slides
2. Textbook Readings:
    - Baldani-Bradfield-Turner (2nd edition, Chapter 13): "Value Functions and the Envelope Theorem: Theory" (pp. 316-330)
    - Baldani-Bradfield-Turner (3rd edition, Chapter 13): "Value Functions and the Envelope Theorem: Theory" (pp. 377-394)
    - Baldani-Bradfield-Turner (2nd edition, Chapter 14): "Value Functions and the Envelope Theorem: Duality and Other Applications" (pp. 331-359)
    - Baldani-Bradfield-Turner (3rd edition, Chapter 14): "Value Functions and the Envelope Theorem: Duality and Other Applications" (pp. 395-427)
    - Hoy-Livernois-McKena-Rees-Stengos (2001, Chapter 14): "Comparative Statics" (pp. 631-676)
    - Sydsæter-Hammond (2006, Chapter 14): "Constrained Optimisation" (pp. 503-548)

TOPIC 9: DIFFERENCE AND DIFFERENTIAL EQUATIONS: THEORY AND APPLICATIONS

Learning Topics and Objectives:

This topic introduces difference and differential equations as ways to analyse dynamic processes. Variables must be dated in a dynamic model. We have the option of dating variables at discrete intervals of time (e.g., once per month) or continuously (at every instant of time). A *difference equation* specifies the determinants of the difference between successive values of a variable. Difference equations provide a step by step analysis as it is assumed that variables can change discontinuously from one period to the next. Difference equations are, therefore, appropriate for the analysis of dynamic problems in discrete time.

A *differential equation* is like a difference equation in that it expresses how a variable changes over time except that time is considered to be a very continuous variable. Differential equations can express the rates of change of variables in continuous time. We will focus on different aspects of dynamic processes, such as identification of steady-state (equilibrium) values, analysis of whether those steady states are stable, and description of the time path of adjustment to new steady states.

Suggested Readings:

1. Lecture Slides
2. Textbook Readings:
    - Baldani-Bradfield-Turner (2nd edition, Chapter 15): "Introduction to Dynamics" (pp. 360-396)
    - Baldani-Bradfield-Turner (3rd edition, Chapter 15): "Introduction to Dynamics" (pp. 429-471)
    - Baldani-Bradfield-Turner (2nd edition, Chapter 16): "Difference and Differential Equations: Applications" (pp. 410-448)
    - Baldani-Bradfield-Turner (3rd edition, Chapter 16): "Difference and Differential Equations: Applications" (pp. 487-530)
    - Hoy-Livernois-McKena-Rees-Stengos (2001, Chapter 17-18-19-20-21-22-23-24)

TOPIC 10: OPTIMAL CONTROL THEORY WITH APPLICATIONS

Learning Topics and Objectives:

In this topic we take up the problem of optimisation over time. Such problems are common in economics. For example, in the theory of investment, firms are assumed to choose the time path of investment expenditures to maximise the (discounted) sum of profits over time. In the theory of savings, individuals are assumed to choose the time path of consumption and saving that maximises (discounted) sum of lifetime utility. These are examples of dynamic optimisation problems.

In this topic, we study a new technique, optimal control theory, which is used to solve dynamic optimisation problems. The methodology that classical mathematicians used to solve dynamic problems is known as the calculus of variations. This approach has since been generalized in two ways. First, Richard Bellman, an American mathematician, developed the method of dynamic programming in the 1950s. This method is especially suited to discrete-time problems and is particularly useful for stochastic models. Second, also in the 1950s, a team of Russian mathematicians led by L. Pontryagin developed the maximum principle of optimal control. We demonstrate how to use Pontryagin's technique. The maximum principle is a generalization of the classical calculus of variations in that it provides solutions to problems in which one or more of the constraints involve the derivatives of some of the state variables. This type of constraint is central to the theory of economic growth. Our goal is not to prove the maximum principle but, rather, to provide a heuristic derivation along with a description of the procedure that we follow to use the solutions. This approach will provide us with a set of tools that will allow us to solve the various dynamic models that will be encountered in this topic.

Suggested Readings:

1. Lecture Slides
2. Textbook Readings:
    - Hoy-Livernois-McKena-Rees-Stengos (2001, Chapter 25): "Optimal Control Theory" (pp. 999-1080)