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Comment on Relative Price Variability and Inflation in Reinganum's Consumer Search Model

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David Fielding* and Christopher Hajzler†

Abstract

There is now a large empirical literature on the effect of the aggregate inflation rate on (i) the dispersion of prices across goods or locations (relative price variability, or RPV) and (ii) the dispersion of inflation rates across goods or locations (relative inflation variability, or RIV). In the early part of this literature, empirical modelling is explicitly based on theoretical macroeconomic models incorporating signal extraction problems. However, more recent empirical research is less directly connected to theory, and several authors report results that are inconsistent with signal extraction models. In particular, while RIV is increasing in the absolute value of inflation shocks, RPV is a negative monotonic function of inflation shocks. In this paper, we show that such a result is predicted by consumer search models in the style of Reinganum (1979). A proper understanding of the dynamics of price dispersion in 21st century economies will require a renewed interest in the theoretical foundations of empirical models.

Keywords: Relative Price Variability; Inflation; Search models

1 Introduction

There is a substantial body of evidence on the effect of aggregate inflation on (i) the dispersion of prices across goods or locations (relative price variability, or RPV) and (ii) the dispersion of inflation rates across goods or locations (relative inflation variability, or RIV). The earliest papers in this literature, such as Parks (1978), Hesselman (1983) and Glezakos and Nugent (1986), are explicitly based on macroeconomic theory. Using a model incorporating a signal extraction problem, as in Lucas Jr (1972), it is possible to show that both RPV and RIV will be a non-decreasing function of the the absolute value of inflation shocks. Early empirical

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papers are designed to estimate the slope of this function using data on aggregate inflation and RIV (but not RPV). Most papers report a statistically significant relationship.

These early papers have led to a voluminous empirical literature, which we review in Fielding *et al.* (2011). This literature estimates RPV-inflation and RIV-inflation functions using data from a wide range of countries and allowing for a wide range of different functional forms. Most papers find that the RIV-inflation function is non-monotonic, and although there is some disagreement about its precise shape - for example, whether it is U-shaped or V-shaped - the broad patterns reported in the early papers are reproduced by a majority of later studies. However, this functional form is not replicated for the RPV-inflation relationship, with several papers finding that RPV is strictly decreasing in aggregate inflation shocks; see for example Reinsdorf (1994). Such a result is inconsistent with a signal extraction model, and lacks a theoretical explanation. In most recent papers the empirical regression equations are not derived explicitly from theory, so the discrepancy between the RIV and RPV results is a puzzle.

In addition to the signal extraction model, there are several theories that could be used to interpret a relationship between RPV/RIV and aggregate inflation. However, some of these, such as the menu cost model of Rotemberg (1983) and Danziger (1987), or the monetary search model of Head and Kumar (2005), relate mainly to anticipated rather than unanticipated inflation. However, one type of model - models of consumer search introduced by Reinganum (1979) and extended by Bénabou (1993) and Bénabou and Gertner (1993) - does relate directly to inflation shocks. This model was originally developed to demonstrate how search costs give rise to price dispersion in a market with many producers of a homogenous product, and Bénabou and Gertner (1993) examine the effects of inflation volatility on consumer search and price dispersion in a similar environment. However, these models also provide a simple and tractable framework for illustrating the impact of consumer search on the relationships between unanticipated inflation and RPV/RIV.

In this paper we show that Reinganum's (1979) model can explain the discrepancy between the empirical results for RPV and those for RIV, as long as search costs are moderate. (That is, as long as search costs are not so high that there is never an incentive to search.) We show that the model predicts a negative monotonic relationship between RPV and unanticipated inflation, and, when relative marginal costs and prices across sellers are sufficiently persistent, a non-monotonic (U-shaped) relationship between RIV and unanticipated inflation. The negative monotonic relationship between RPV and unanticipated inflation is also predicted to be stronger for low or negative inflation shocks compared to large positive shocks.

2 Model Overview

Relationships between unanticipated inflation and both RPV and RIV are derived in a model of price setting under imperfect competition and consumer search similar to Reinganum (1979), Bénabou (1993) and Bénabou and Gertner (1993). Following Reinganum (1979) we consider a continuum of identical buyers that each observe the a single price and decide whether to incur a fixed cost of obtaining an additional price quote from a known distribution of prices of heterogenous sellers. In equilibrium, buyers optimally search to obtain additional price quotes as long as previously quoted prices exceed a reservation price and purchase the product at any price below while sellers, under certain conditions on consumer demand, never charge a price that exceeds the reservation price, and therefore no search takes place. This equilibrium without active search contrasts Bénabou (1993), who extend Reinganum (1979) to include buyers with heterogenous search costs in a dynamic setting. Buyers are assumed to be identical in Bénabou and Gertner (1993), but the authors show that active search takes place under duopolistic price competition when search costs are sufficiently low. Their focus is on the the relationship between price dispersion and the the variability of sellers' marginal costs. The environment we consider here closely follows Reinganum (1979), but our main relationships we examine are qualitatively similar to those implied by the duopoly model of Bénabou and Gertner (1993). (We show this in the Appendix.)

2.1 Product Market and Production Costs

A continuum of sellers of measure J produce a homogenous product but are assumed to differ in their productivity. Specifically, we assume that sellers' marginal production costs, expressed in natural logarithms, are continuously distributed over the interval $[c^-, c^+] \subset \mathbb{R}$ according to the cumulative distribution function $G(c)$. The distribution is common knowledge to all market participants. Each seller observes only its own cost $c_i - \log(C_i)$. Following Bénabou and Gertner (1993), it will be convenient to decompose producer costs into average cost, $\bar{c}$, a common shock, θ, and an idiosyncratic component γ_j resulting in the following log-linear cost function:

$$c_i = \bar{c} + \theta + \gamma_j.$$

where θ is drawn from CDF $T(\theta)$ with bounded support $[\theta^-, \theta^+]$, γ_j is drawn from CDF $H(\gamma)$ with bounded support $[\gamma^-, \gamma^+]$, and $E[\theta] = E[\gamma_j] = 0$ ($c^- = \bar{c} + \theta^- + \gamma^-$ and $c^+ = \bar{c} +$

$\theta^+ + \gamma^+$). We assume that $G(\theta)$ and $H(\theta)$ are continuously differentiable. These assumptions imply a continuously differentiable distribution over marginal costs, and it can be shown that the equilibrium pricing strategies of sellers result in a non-degenerate distribution over price quotes that continuously differentiable almost everywhere which is known to both buyers and sellers (see Reinganum, 1979).

2.2 Buyers' Strategy

A continuum of buyers of measure N each observe their first price quote free, and decide whether to incur cost $\sigma > 0$ for each additional price quote, which are sampled with recall. With a fixed cost of searching, buyers will optimally search sequentially and actively search whenever the expected gain from an additional price quote exceeds this cost. This decision is based on knowledge of sellers' cost distribution and pricing strategies summarized by CDF $F(p)$ (with support $[p^-, p^+]$). Following Reinganum (1979) and Bénabou and Gertner (1993), we adopt the additional assumption that buyer utility is quasi-linear, implying that indirect utility is separable in prices and income. We can therefore express the buyer's indirect utility from purchasing the product at the lowest price quote received across all searches, p_l, as $U(\overrightarrow{\mathbf{p}}, p_l, M) = S(\overrightarrow{\mathbf{p}}, p_l) + M$, where M is income, $\overrightarrow{\mathbf{p}}$ is a vector of prices corresponding to all other consumption goods, and $S'(\overrightarrow{\mathbf{p}}, p) = \partial S/\partial p < 0$. (The function $S(\overrightarrow{\mathbf{p}}, p)$ can be interpreted as the consumer surplus from purchasing the product at price p, and $D(p) = -S'(\overrightarrow{\mathbf{p}}, p)$ is consumer demand in the absence of search. See also Bénabou and Gertner 1993.) The buyer optimally continues to search provided

$$V(p_l) = \int_{p^-}^{p_l} \left[S(\overrightarrow{\mathbf{p}}, p) - S(\overrightarrow{\mathbf{p}}, p_l)\right] dF(p) > \sigma.$$

As outlined in Lippman and McCall (1976), the structure of the optimal stopping rule is characterized by a single reservation price, p_r, such that searching stops once a price at least as small as p_r is quoted. This reservation price is implicitly defined by

$$V(p_r) = \int_{p^-}^{p_r} \left[S(\overrightarrow{\mathbf{p}}, p) - S(\overrightarrow{\mathbf{p}}, p_r)\right] dF(p) = \int_{p^-}^{p_r} D(p) dF(p) = \sigma.$$

For sufficiently high search costs and bounded price support, $p_r = p^+$ and the buyer is never tempted to search. In this case sellers set prices independently and there is no relationship between inflation shocks and relative prices. We therefore restrict our attention to the interesting case where the incentive to search influences seller strategies.

2.3 Sellers' Strategy

With a continuum of sellers, the law of large numbers implies that each buyer's reservation price search strategy will yield an eventual price quote below their reservation with probability one. Because buyers are assumed to be identical the reservation price is the same for all buyers and demand faced by each seller charging a price below the reservation price is strictly positive and zero otherwise. (Sellers with marginal costs above the reservation price receive no customers and exit the market.) Expected profits for each seller $j \in J$ is therefore

$$E[\Pi_j(p_j)] = \begin{cases} (p_j - c_j)D(p_j)E[\lambda_j] & \text{if} \quad p_j \leq p_r \; \& \; c_j \leq p_r \\ 0 & \text{otherwise} \end{cases}$$

where λ_j is the measure of buyers obtaining a price quote from seller j. Taking p_r as given, it is optimal for a seller with costs below some threshold c^* to charge the monopoly profit-maximizing price because all buyers who sample this price will abstain from searching and buy from this seller. For $c_i \in (c^*, p_r]$, a seller prefers to quote price p_r rather than the monopoly price (since this yields positive expected profits).

2.4 Equilibrium

Reinganum (1979) considers the special case of iso-elastic demand, $D(P) = P^{-\eta}$, where $\eta > 1$ and $c^+ \leq c^- + \mu$, where $\mu = \log(\eta/(\eta - 1))$. These assumptions imply that the optimal price charged by the lowest price seller will always exceed the marginal cost of the highest cost seller, and given $p_r > p^-$, there is no incentive for any seller to exit the market. The equilibrium price distribution over prices features bunching at $p = p_r$:

$$F(p) = \begin{cases} G\big(p - \mu\big) & \text{for} \quad p \leq p_r \; (c \leq c^*) \\ 1 & \text{for} \quad p > p_r \; (c > c^*) \end{cases}$$

Therefore no buyer observes a first price quote in excess of the reservation price, and no buyer searches in equilibrium. As sellers are assumed to be identical apart from their marginal cost draw, it is natural to also assume that each seller serves the same measure of buyers, such that for all j, $E[\lambda_j] = \lambda = N/J$.

This also describes equilibrium price setting and search in Bénabou and Gertner's (1993) duopolistic price setting model when search costs are not too low. When search costs are low, they show that for some $c' > c^*$ the seller will optimally set a price above p_r and some buyers actively search. The reason is that, with few price quotes and buyer recall, there is a non-trivial probability that search unveils relatively high prices only and makes their purchases from the producer with the initially quoted price, and expected profits are positive for $p \geq p_r$. Apart from this special case, however, the equilibrium solution is as described in Reinganum (1979).

We derive relationships between unanticipated inflation shocks and both RPV and RIV in the no-search equilibrium. Following Reinganum (1979) and Bénabou and Gertner (1993), we assume an iso-elastic demand curve, $D(P) = P^{-\eta}$, and the monopoly price below p_r is a constant markup over marginal costs, given by:

$$p(c_j) = \bar{c} + \theta + \gamma_i + \mu.$$

Taking the equilibrium reservation price p_r and price distribution $F(p)$ as given, we evaluate the expected effect of a common cost shock θ on standard measures of both RPV and RIV.

3 Relative Price Variability

The standard RPV measure for a market with J price quotes is (expressed in terms of log prices) the coefficient of variation:

$$v = \sqrt{\frac{1}{J} \int_{j \in J} \left(p(c_j) - \bar{p}\right)^2 dj}$$

where $\bar{p} = \int_{p^-}^{p_r} p dF(p)$ is the average price across sellers. In order to evaluate the effect of average inflation shocks on this measure, we assume that the econometrician observes (*ex post*) the common cost shock θ, and we examine the expected value of v given θ. (The anticipated change in inflation is captured by $\bar{c}$, which has no impact on relative prices in this model.) That is, we focus on the expected value of squared price deviations given θ: $E[(p(c_j) - \bar{p}(\theta))^2 \,|\theta]$.

It will be convenient to begin by deriving an expression for the average price level as a function of θ. Actual price deviations depend on seller-specific costs, γ_i. When a seller's marginal cost rises above the threshold c^* it posts a price p_r. Therefore, conditional on θ, there is a threshold value of γ_i above which the seller's price is $p(c^*) = p_r$ and below which the monopoly price is charged, which we define as

$$\gamma^*(\theta) = p_r - \bar{c} - \mu - \theta. \tag{1}$$

Given $\gamma^*(\theta)$, the average price level is

$$\begin{aligned}\bar{p}(\theta) &= \int_{\gamma^-}^{\gamma^*(\theta)} \big(\bar{c} + \theta + \mu + \gamma\big) dH(\gamma) + \big(1 - H(\gamma^*(\theta))\big) p_r \\ &= p_r - H\big(\gamma^*(\theta)\big)\gamma^*(\theta) + \int_{\gamma^-}^{\gamma^*(\theta)} \gamma dH(\gamma).\end{aligned} \tag{2}$$

Using Leibniz rule, the derivative of $\bar{p}(\theta)$ with respect to θ is $\bar{p}'_\theta(\theta) = H\big(\gamma^*(\theta)\big) \in (0, 1)$. Given $\bar{p}(\theta)$, the relationship between θ and v is summarized by

$$E[(p(c_j) - \bar{p}(\theta))^2 \,|\theta] = \int_{\gamma^-}^{\gamma^*(\theta)} \big(\bar{c} + \theta + \mu + \gamma - \bar{p}(\theta)\big)^2 dH(\gamma) + \big(1 - H(\gamma^*(\theta))\big)(p_r - \bar{p}(\theta))^2.$$

The derivative of this expression with respect to θ is

$$\begin{aligned}\frac{\partial E[(p_i - \bar{p})^2 \,|\theta]}{\partial \theta} =& 2\big(1 - H(\gamma^*(\theta))\big) \int_{\gamma^-}^{\gamma^*(\theta)} \big(\gamma - \gamma^*(\theta)\big) dH(\gamma) \\ =& 2\big(1 - H(\gamma^*(\theta))\big)\big(\bar{p}(\theta) - p_r\big) < 0.\end{aligned} \tag{3}$$

Therefore there is a negative, monotonic relationship between θ and RPV in this model.

This result has a simple and intuitive interpretation. The integration term is the expected reduction in price dispersion resulting from a rise in the average price of monopoly sellers relative to producers charging the no-search price p_r (that is, relative to those producers having an idiosyncratic cost shock of $\gamma^*(\theta)$ or above). The multiplicative term in front is simply the measure of sellers drawing an idiosyncratic shock $\gamma^*(\theta)$ or above. (For instance, if the only values of θ considered imply $\gamma < \gamma^*(\theta)$ for all sellers, then all charge the monopoly price and there is no effect of the common cost shock on relative price dispersion.)

Note also that if idiosyncratic marginal costs γ_i are sufficiently dispersed over the support, such that any decrease in $H(\gamma^*(\theta))$ is not too large relative to changes in θ, the negative relationship between θ and RPV will tend to be stronger for negative shocks compared with positive shocks. The significance of this condition is interpreted as follows: as θ increases and $\gamma^*(\theta)$ decreases, a proportion of relatively high-cost sellers charging the monopoly price (which are disperse), $h(\gamma^*(\theta))$, transition to the group of sellers charing the reservation price, which reduces overall price dispersion. If this mass of sellers charging close to the reservation price is particularly large some value of $\gamma^*(\theta)$, an increase in θ may reduce price dispersion at an increasing rate. When $h(\gamma^*(\theta))$ is not too large, however, the negative slope is smaller in absolute value for higher values of θ because, as the average price charged by monopoly-price sellers approaches the reservation price, θ has a smaller overall impact on price dispersion. Therefore the effect of a positive inflation shock will tend to be dampened compared to the effect of a negative inflation shock. These competing factors can be summarised by considering the second derivative of (3) with respect to θ:

$$\frac{\partial E[(p_i - \bar{p})^2 \,|\theta]^2}{\partial \theta^2} = 2h(\gamma(\theta)) \int_{\gamma^-}^{\gamma^*(\theta)} \big(\gamma_i - \gamma^*(\theta)\big) h(\gamma_i) d\gamma_i + 2\big(1 - H(\gamma^*(\theta))\big) H(\gamma^*(\theta)).$$

The first term in this expression is negative and represents the increase in proportion of sellers transiting from charging a monopoly price to price p_r (scaled by the extent of overall price dispersion). The second term in this expression is positive, and represents the decrease in average price gap reductions at higher levels of θ.

4 Relative Inflation Variability

We now consider the impact of common cost shocks on relative inflation variability. Inflation is inherently a dynamic concept, and our analysis of price dispersion thus far has been within a static framework. Extending the analysis of the previous sections to multiple periods is straightforward if we assume that the buyers' reservation price equates the discounted expected benefit of searching for an additional price quote, given the price(s) already observed, with the cost of search.[1] Nevertheless, to analyze the impact of inflation shocks on inflation variability, one must make additional assumptions concerning the evolution of idiosyncratic shocks γ_j. Perhaps the natural assumption in the context of this static model is that relative productivity across sellers is constant across time: $\gamma_{j,t} = \gamma_{j,t+1} = \gamma_j$. It turns out, however, that the implied relationship between inflation shocks and RIV is very different when relative cost differences are persistent compared to the case where they are intertemporally independent. In what follows, therefore, we allow for varying degrees of intertemporal dependence in

[1] With a small finite number of price quotes, the reservation price would evolve according to the updated beliefs about the actual price distribution. In our model with a continuum of sellers, however, the posterior price distribution would be invariant to observed price changes for a single seller.

relative costs between two periods $t = 0$ and $t = 1$, summarized by joint probability density function $h(\gamma_0, \gamma_1)$, and contrast the cases where correlation between the variables is weak and when it is strong.

The standard RIV measure for a duopoly market, expressed in terms of percent inflation rates, is:

$$w = \sqrt{\frac{1}{J}\int_{j\in J}(\pi_j - \bar{\pi}(\theta))^2 dj} \tag{4}$$

where $\pi_j = \Delta p(c_j)$ and $\bar{\pi}(\theta) = \int_{\gamma^-}^{\gamma^+}\int_{\gamma^-}^{\gamma^+}\big(p(\gamma_1,\theta) - p(\gamma_0)\big)h(\gamma_0,\gamma_1)d\gamma_0 d\gamma_1$ are the change in log firm prices and the average change in log price, respectively. For simplicity we consider the effect of a change in θ from the unconditional zero mean, so that the marginal impact of θ on RIV can be thought of as a single-period impulse response.

We begin by deriving an expression for $\bar{\pi}(\theta)$. In the initial period, $\theta = 0$ and the threshold level of γ_j above which sellers charge the reservation price is

$$\gamma_0^* = p_r - \bar{c} - \mu.$$

The corresponding threshold in period 1 is given by equation (1).

The change in price for a seller that draws $\gamma = \gamma_0$ in the initial period and $\gamma = \gamma_1$ in the next is

$$p(\gamma_1,\theta) - p(\gamma_0) = \begin{cases} \theta + \gamma_1 - \gamma_0 & \text{if} \quad \gamma_0 \le \gamma_0^*, \gamma_1 \le \gamma^*(\theta) \\ \gamma_0^* - \gamma_0 & \text{if} \quad \gamma_0 \le \gamma_0^*, \gamma_1 > \gamma^*(\theta) \\ \gamma_1 - \gamma^*(\theta) & \text{if} \quad \gamma_0 > \gamma_0^*, \gamma_1 \le \gamma^*(\theta) \\ 0 & \text{if} \quad \gamma_0 > \gamma_0^*, \gamma_1 > \gamma^*(\theta). \end{cases}$$

where we use the fact that $\gamma^*(\theta) + \theta = \gamma_0^*$. The average price change across all sellers is

$$\bar{\pi}(\theta) = \int_{\gamma^-}^{\gamma_0^*} \int_{\gamma^-}^{\gamma^*(\theta)} \big(\theta + \gamma_1 - \gamma_0\big) h(\gamma_0, \gamma_1) d\gamma_0 d\gamma_1 + \int_{\gamma^-}^{\gamma_0^*} \int_{\gamma^*(\theta)}^{\gamma^+} \big(\gamma_0^* - \gamma_0\big) h(\gamma_0, \gamma_1) d\gamma_0 d\gamma_1$$
$$\int_{\gamma_0^*}^{\gamma^+} \int_{\gamma^-}^{\gamma^*(\theta)} \big(\gamma_1 - \gamma^*(\theta)\big) h(\gamma_0, \gamma_1) d\gamma_0 d\gamma_1$$
$$= \int_{\gamma^-}^{\gamma^*(\theta)} \gamma_1 dH(\gamma_1) - \int_{\gamma^-}^{\gamma_0^*} \gamma_0 dH(\gamma_0) + H(\gamma^*(\theta))\theta + \big[H(\gamma_0^*) - H(\gamma^*(\theta))\big]\gamma_0^*$$

where $H(z) = \int_{-\infty}^{z} \int_{-\infty}^{\infty} h(x, y) dy dx$. Given $\bar{\pi}(\theta)$, the relationship between RIV and θ is summarized by

$$\begin{aligned} E\big[\,(\pi_j - \bar{\pi}(\theta))^2\,|\theta\big] = &\int_{\gamma^-}^{\gamma_0^*} \int_{\gamma^-}^{\gamma^*(\theta)} \big(\theta + \gamma_1 - \gamma_0 - \bar{\pi}(\theta)\big)^2 h(\gamma_0, \gamma_1) d\gamma_1 d\gamma_0 \\ &+ \int_{\gamma^-}^{\gamma_0^*} \int_{\gamma^*(\theta)}^{\gamma^+} \big(\gamma_0^* - \gamma_0 - \bar{\pi}(\theta)\big)^2 h(\gamma_0, \gamma_1) d\gamma_1 d\gamma_0 \\ &+ \int_{\gamma_0^*}^{\gamma^+} \int_{\gamma^-}^{\gamma^*(\theta)} \big(\gamma_1 - \gamma^*(\theta) - \bar{\pi}(\theta)\big)^2 h(\gamma_0, \gamma_1) d\gamma_1 d\gamma_0 \\ &+ \int_{\gamma^*(\theta)}^{\gamma^+} \int_{\gamma_0^*}^{\gamma^+} \bar{\pi}(\theta)^2 h(\gamma_0, \gamma_1) d\gamma_1 d\gamma_0. \end{aligned}$$

We are interested in the impact of θ on this measure. Differentiating with respect to θ yields

$$\begin{aligned} \frac{\partial}{\partial\theta} = &\int_{\gamma^-}^{\gamma_0^*} \Bigg\{ \frac{\partial\gamma^*(\theta)}{\partial\theta} (\theta + \gamma^*(\theta) - \gamma_0 - \bar{\pi}(\theta))^2 h(\gamma_0, \gamma^*(\theta)) \\ &\quad + 2\big(1 - \bar{\pi}'_\theta(\theta)\big) \int_{\gamma^-}^{\gamma^*(\theta)} \big(\theta + \gamma_1 - \gamma_0 - \bar{\pi}(\theta)\big) h(\gamma_0, \gamma_1) d\gamma_1 \\ &\quad - \frac{\partial\gamma^*(\theta)}{\partial\theta} \big(\gamma_0^* - \gamma_0 - \bar{\pi}(\theta)\big)^2 h(\gamma_0, \gamma^*(\theta)) \\ &\quad -2\bar{\pi}'_\theta(\theta) \int_{\gamma^*(\theta)}^{\gamma^+} \big(\gamma_0^* - \gamma_0 - \bar{\pi}(\theta)\big) h(\gamma_0, \gamma_1) d\gamma_1 \Bigg\} d\gamma_0 \\ &+ \int_{\gamma_0^*}^{\gamma^+} \Bigg\{ 2\left(-\frac{\partial\gamma^*(\theta)}{\partial\theta} - \bar{\pi}'_\theta(\theta)\right) \int_{\gamma^-}^{\gamma^*(\theta)} \big(\gamma_1 - \gamma^*(\theta) - \bar{\pi}(\theta)\big) h(\gamma_0, \gamma_1) d\gamma_1 \\ &\quad +2\bar{\pi}'_\theta(\theta) \int_{\gamma^*(\theta)}^{\gamma^+} \bar{\pi}(\theta) h(\gamma_0, \gamma_1) d\gamma_1 \Bigg\} d\gamma_0. \end{aligned}$$

where $\bar{\pi}'_\theta(\theta) = \partial\bar{\pi}/\partial\theta = H(\gamma^*(\theta)) \in (0,1)$ and $\partial\gamma^*(\theta)/\partial\theta = -1$. Substituting these terms, it is possible to show that this expression reduces to

$$\begin{aligned}\frac{\partial E\left[\,(\pi_j - \bar{\pi}(\theta))^2\,|\theta\right]}{\partial\theta} =& 2\big(1 - H(\gamma^*(\theta))\big)\left\{\int_{\gamma^-}^{\gamma_0^*}\int_{\gamma^-}^{\gamma^*(\theta)}\big(\gamma_1 - \gamma_0\big)h(\gamma_0,\gamma_1)d\gamma_1 d\gamma_0\right.\\ &\left.+\int_{\gamma_0^*}^{\gamma^+}\int_{\gamma^-}^{\gamma^*(\theta)}\gamma_1 h(\gamma_0,\gamma_1)d\gamma_1 d\gamma_0\right\}\\ &+2H(\gamma^*(\theta))\left\{\int_{\gamma^-}^{\gamma_0^*}\int_{\gamma^*(\theta)}^{\gamma^+}\gamma_0 h(\gamma_0,\gamma_1)d\gamma_1 d\gamma_0\right\}\\ &-2\gamma_0^*\left\{\int_{\gamma^-}^{\gamma_0^*}h(\gamma_0)\left(\int_{\gamma^*(\theta)}^{\gamma^+}h(\gamma_1|\gamma_0)d\gamma_1\right)d\gamma_0\right\}\\ &+2\big(1 - H(\gamma^*(\theta))\big)\Big(H(\gamma^*(\theta))\theta + \big[H(\gamma_0^*) - H(\gamma^*(\theta))\big]\gamma_0^*\Big) \qquad (5)\end{aligned}$$

where $h(x) = \int_{-\infty}^{\infty} h(x,y)dy$ and $h(x|y)h(y) = h(x,y)$ (Bayes' rule).

Without additional information or assumptions, it is not possible to sign this expression. In particular, the first and second bracketed integration terms can be positive or negative. Nevertheless, two observations are worth emphasizing. First, the third bracketed integration term is non-negative, and therefore a positive or negative relationship between $E\left[\,(\pi_j - \bar{\pi}(\theta))^2\,\right]$ and θ becomes more positive (or less negative) as this term increases. Inspecting of the limits of integration, this term is relatively large when γ_0 and γ_1 are more negatively correlated, and relatively small when these variables are more positively correlated. Therefore temporal persistence in γ implies a stronger positive (or weaker negative) relationship between θ and RIV compared to the case of uncorrelated relative cost shocks. The second observation is with respect to the last term in the above expresion. $H(\gamma^*(\theta))\theta > 0$ if and only if $\theta > 0$. Furthermore, if search costs are sufficiently high that $\gamma_0^* > 0$ (i.e. there is some price dispersion at the mean value $E[\theta] = 0$ or above) then $\big[H(\gamma_0^*) - H(\gamma^*(\theta))\big]\gamma_0^* > 0$ if and only if $\theta > 0$. Therefore there is an asymmetry between the effects of positive and negative inflation shocks.

The degree of correlation of idiosyncratic cost differences over time determines whether the impact of unanticipated inflation on RIV is monotonically decreasing or U-shaped. For example, if γ_0 and γ_1 are drawn independently in each period from distribution H, the first and second bracketed integration terms in (5) sum to

$$2\big(1 - H(\gamma^*(\theta))\big)\int_{\gamma^-}^{\gamma^*(\theta)}\gamma_1 dH(\gamma_1)$$

and the third bracketed term is equal to

$$-2H(\gamma_0^*)\big(1 - H(\gamma^*(\theta))\big)\gamma_0^*.$$

Therefore equation (5) becomes

$$\frac{\partial E\big[\,(\pi_j - \bar{\pi}(\theta))^2\,|\theta\big]}{\partial\theta} = 2\big(1 - H(\gamma^*(\theta))\big)\int_{\gamma^-}^{\gamma^*(\theta)}\big(\gamma_1 - \gamma^*(\theta)\big)dH(\gamma_1) < 0.$$

Notice that this expression is identical to the marginal effect of θ on RPV in equation (3). However, the impact of θ on RIV is potentially very different when relative cost differences are persistent.

To see this, consider the other extreme where relative cost differences are constant over time ($\gamma_0 = \gamma_1 = \gamma$), then if $\theta \geq 0$ ($\gamma^*(\theta) \leq \gamma_0^*$), the first two bracketed integration terms of equation (5) sum to $2H(\gamma^*(\theta))\int_{\gamma^*(\theta)}^{\gamma_0^*}\gamma dH(\gamma)$ and the third integration term is equal to $-2\big(H(\gamma_0^*) - H(\gamma^*(\theta))\big)\gamma_0^*$. The relationship between positive inflation shocks and RIV is therefore given by

$$\begin{aligned}\frac{\partial E\big[\,(\pi_j - \bar{\pi}(\theta))^2\,|\theta \geq 0\big]}{\partial\theta} &= 2H(\gamma^*(\theta))\int_{\gamma^*(\theta)}^{\gamma_0^*}\big(\gamma - \gamma_0^*\big)dH(\gamma) + 2\big(1 - H(\gamma^*(\theta))\big)H(\gamma^*(\theta))\theta \\ &\geq 2H(\gamma^*(\theta))\theta \geq 0\end{aligned}$$

with strict inequalities if and only if $\theta > 0$. If instead $\theta \leq 0$ ($\gamma^*(\theta) \geq \gamma_0^*$), the first two bracketed integration terms in (5) sum to $2\big(1 - H(\gamma^*(\theta))\big)\int_{\gamma_0^*}^{\gamma^*(\theta)}\gamma dH(\gamma)$ and the third integration term equals zero. The relationship between positive inflation shocks and RIV becomes

$$\begin{aligned}\frac{\partial E\big[\,(\pi_j - \bar{\pi}(\theta))^2\,|\theta \leq 0\big]}{\partial\theta} &= 2\Big(1 - H(\gamma^*(\theta))\Big)\left(\int_{\gamma_0^*}^{\gamma^*(\theta)}\big(\gamma - \gamma_0^*\big)dH(\gamma) + H(\gamma^*(\theta))\theta\right) \\ &\leq -2\big(1 - H(\gamma^*(\theta))\big)H(\gamma_0^*)(\gamma^*(\theta) - \gamma_0^*) \leq 0\end{aligned}$$

with strict inequalities if and only if $\theta < 0$. Therefore when $\gamma_0 = \gamma_1 = \gamma$, the predicted relationship between θ and RIV is U-shaped with a turning point at $\theta = 0$.

The implied relationship between unanticipated inflation and RIV when idiosyncratic relative cost differences are persistent contrasts the monotonic negative relationship predicted when relative cost shocks are independent over time. These different relationships are depicted in Figure 1, where RIV is plotted against θ for different degrees of persistence in γ.

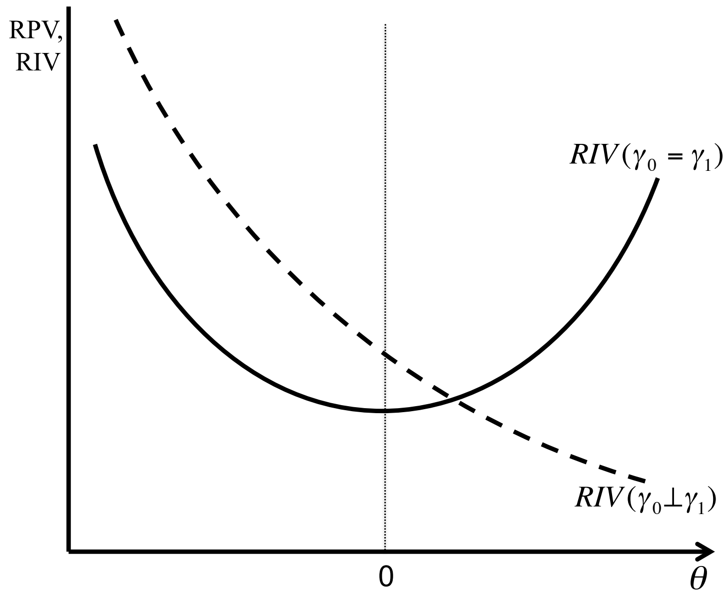

Figure 1: Effect of θ on RPV & RIV

The dashed curve representing RIV when γ_0 and γ_1 are independent and the solid line representing RIV when $\gamma_0 = \gamma_1 = \gamma$. When the idiosyncratic shocks are independent, the impact of θ on RIV is identical to the effect of θ on RPV, but when there these cost differences are highly persistent the model instead predicts a non-monotonic relationship between RIV and θ. If search costs have a significant influence price dispersion, this finding is relevant for empirical research on relative price variability. The expected marginal effect of unanticipated inflation on RIV is a reasonable approximation of the expected effect of inflation on RPV only in the special case of statistical independence of relative price differences. When this condition is violated, estimated relationships between inflation variability and unanticipated inflation is potentially uninformative with respect to the effect of unanticipated inflation on price dispersion.

5 Conclusion

We have shown that a standard search model can explain a discrepancy observed in some of the empirical literature on price dispersion and inflation: the dispersion of price levels is strictly decreasing in aggregate inflation shocks, but the dispersion of inflation rates is potentially a non-monotonic function, depending on the persistence of relative cost/price differences across sellers. In particular, when relative price differences across sellers are persistent, the standard search model predicts that both positive and negative shocks raise relative price dispersion. Our results are intended to stimulate a renewed interest in the theoretical foundations of empirical RPV/RIV models, in order to foster a better understanding of the processes driving price level and inflation rate dispersion. Modelling these processes is not of purely academic interest: for example, if aggregate inflation shocks impact on price level and inflation rate dispersion in different ways, then optimal monetary policy will require a thorough understanding of both.

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Appendices

A RPV in the Bénabou and Gertner Model

The environment studied by Bénabou and Gertner (1993) is similar to Reinganum (1979), but they focus on optimal pricing strategies in a duopolistic market where γ is interpreted as an idiosyncratic relative cost shock and where buyers observe one price and decide whether to incur a fixed cost of finding out the second price. As in Reinganum (1979), buyers adopt a reservation price search strategy, but Bénabou and Gertner (1993) show that, for sufficiently low search costs, an equilibrium exists in which a high-cost producer charges a price strictly between the monopoly price and the reservation price, and buyers observing this price actively search. When search costs are not too low, there is no active search and the equilibrium resembles Reinganum (1979). We derive similar relationships between unanticipated inflation and both RPV and RIV for Bénabou and Gertner's (1993) duopoly model in the equilibrium without active search.[2]

Beginning with the inflation-RPV relationship, the corresponding RPV measure for a duopoly market is:

[2]We suspect that the intuition from this exercise extends to the equilibrium with active search. However, it is not possible to derive an analytical solution to the sellers' pricing strategy for this case and therefore these relationships must be analyzed numerically for specific seller cost distributions and buyer welfare functions.

$$v = \sqrt{\frac{1}{2}\sum\nolimits_{1,2}(p_i - \bar{p})^2}$$

where $\bar{p} = 0.5(p_1 + p_2)$. Both sellers draw a relative cost parameter, γ_i, from a common distribution $H(\gamma)$. γ_j is known only to seller j but distribution is common knowledge to all market participants. As in Reinganum (1979), there is a cost threshold c^* above which a seller posts the reservation price p_r. The difference between seller i's price and the average price, given θ, γ_i and γ_j ($i \neq j$) is:

$$p(c_i) - \bar{p} = \begin{cases} \gamma_i - \frac{1}{2}(\gamma_i + \gamma_j) & = \frac{1}{2}(\gamma_i - \gamma_j) \quad \text{if} \quad \gamma_i \leq \gamma^*(\theta), \gamma_j \leq \gamma^*(\theta) \\ \frac{1}{2}(\bar{c} + \theta + \gamma_i + \mu) - \frac{1}{2}p_r & = \frac{1}{2}(\gamma_i - \gamma^*(\theta)) \quad \text{if} \quad \gamma_i \leq \gamma^*(\theta), \gamma_j > \gamma^*(\theta) \\ \frac{1}{2}p_r - \frac{1}{2}(\bar{c} + \theta + \gamma_j + \mu) & = \frac{1}{2}(\gamma^*(\theta) - \gamma_j) \quad \text{if} \quad \gamma_i > \gamma^*(\theta), \gamma_j \leq \gamma^*(\theta) \\ 0 & \quad \text{if} \quad \gamma_i > \gamma^*(\theta), \gamma_j > \gamma^*(\theta) \end{cases}$$

where $\gamma^*(\theta)$ is defined as before by equation (1). Therefore

$$\begin{aligned} 4 \times E[(p_i - \bar{p})^2 \,|\theta] = &\int_{\gamma^-}^{\gamma^*(\theta)} \int_{\gamma^-}^{\gamma^*(\theta)} (\gamma_i - \gamma_j)^2 h(\gamma_i) h(\gamma_j) d\gamma_i d\gamma_j \\ &+ \int_{\gamma^-}^{\gamma^*(\theta)} \left(\int_{\gamma^*(\theta)}^{\gamma^+} (\gamma_i - \gamma^*(\theta))^2 h(\gamma_j) d\gamma_j \right) h(\gamma_i) d\gamma_i \\ &+ \int_{\gamma^-}^{\gamma^*(\theta)} \left(\int_{\gamma^*(\theta)}^{\gamma^+} (\gamma_j - \gamma^*(\theta))^2 h(\gamma_i) d\gamma_i \right) h(\gamma_j) d\gamma_j. \end{aligned} \tag{6}$$

The derivative of (6) with respect to θ (multiplied by 4) simplifies to

$$\frac{\partial}{\partial\theta} E[(p_i - \bar{p})^2 \,|\theta] = \big(1 - H(\gamma^*(\theta))\big) \int_{\gamma^-}^{\gamma^*(\theta)} \big(\gamma_i - \gamma^*(\theta)\big) h(\gamma_i) d\gamma_i < 0$$

where we have made use of the fact that

$$\int_{\gamma^-}^{\gamma^*(\theta)} \big(\gamma_j - \gamma^*(\theta)\big) h(\gamma_j) d\gamma_j = \int_{\gamma^-}^{\gamma^*(\theta)} \big(\gamma_i - \gamma^*(\theta)\big) h(\gamma_i) d\gamma_i.$$

As in Reinganum's (1979) environment, there is also a negative, monotonic relationship between θ and RPV. The interpretation of this relationship is analogous to before, except that it is an expected relationship based on repeat sampling (rather than an outcome based on a large sample of sellers). Specifically, the integration term is the expected reduction in price dispersion resulting from a rise in one seller's monopoly price conditional on the other seller charging the no-search price p^* (that is, having an idiosyncratic cost shock of $\gamma^*(\theta)$ or above). The multiplicative term in front is simply the probability that the other seller has drawn an idiosyncratic shock $\gamma^*(\theta)$ or above.

If the variance of idiosyncratic marginal cost shocks is sufficiently high, the negative effect of θ on RPV again tends to be stronger for negative shocks compared with positive shocks. The second derivative of (3) with respect to θ is:

$$\frac{\partial E[(p_i - \bar{p})^2 \,|\theta]^2}{\partial \theta^2} = h(\gamma(\theta)) \int_{\gamma^-}^{\gamma^*(\theta)} \big(\gamma_i - \gamma^*(\theta)\big) h(\gamma_i) d\gamma_i + \big(1 - H(\gamma^*(\theta))\big) H(\gamma^*(\theta)).$$

B RIV in the Bénabou and Gertner Model

The RIV measure for a duopoly market is:

$$w = \sqrt{\frac{1}{2} \sum\nolimits_{1,2} (\pi_i - \bar{\pi})^2}$$

We begin by evaluating the case where a seller's idiosyncratic cost shocks are identically and independently drawn over time. To keep the analysis simple, we fix the initial idiosyncratic shock values γ_{0i} and γ_{0j} arbitrarily and evaluate expected changes in RIV due to a change in θ. (This relationship does not depend on γ_{0i} or γ_{0j}.) It is convenient to define $\delta_0 = \gamma_{0i} - \gamma_{0j}$ as the initial relative cost difference between sellers i and j. Between period 0 and period 1, the average inflation rate is $\bar{\pi} = 0.5(\pi_i + \pi_j)$ and the discrepancy between seller i's inflation rate and average inflation is $\pi_i - \bar{\pi} = 0.5(\pi_i - \pi_j)$. This difference, expressed as a function of θ, γ_{1i} and γ_{1j} ($i \neq j$), reduces to:

$$\pi_i - \bar{\pi} = \begin{cases} \frac{1}{2}(\gamma_{1i} - \gamma_{1j} - \delta_0) & \text{if} \quad \gamma_{1i} \leq \gamma^*(\theta), \gamma_{1j} \leq \gamma^*(\theta) \\ \frac{1}{2}(\gamma_{1i} - \gamma^*(\theta) - \delta_0) & \text{if} \quad \gamma_{1i} \leq \gamma^*(\theta), \gamma_{1j} > \gamma^*(\theta) \\ \frac{1}{2}(\gamma^*(\theta) - \gamma_{1j} - \delta_0) & \text{if} \quad \gamma_{1i} > \gamma^*(\theta), \gamma_{1j} \leq \gamma^*(\theta) \\ \frac{1}{2}(-\delta_0) & \text{if} \quad \gamma_{1i} > \gamma^*(\theta), \gamma_{1j} > \gamma^*(\theta) \end{cases}$$

The expected value of $(\pi_i - \bar{\pi})^2$ is:

$$\begin{aligned} 4 \times E[(\pi_i - \bar{\pi})^2 \,|\theta] = & \int_{\gamma^-}^{\gamma^*(\theta)} \int_{\gamma^-}^{\gamma^*(\theta)} (\gamma_{1i} - \gamma_{1j} - \delta_0)^2 dH(\gamma_{1i}) dH(\gamma_{1j}) \\ & + \int_{\gamma^-}^{\gamma^*(\theta)} \int_{\gamma^*(\theta)}^{\gamma^+} (\gamma_{1i} - \gamma^*(\theta) - \delta_0)^2 dH(\gamma_{1i}) dH(\gamma_{1j}) \\ & + \int_{\gamma^*(\theta)}^{\gamma^+} \int_{\gamma^-}^{\gamma^*(\theta)} (\gamma^*(\theta) - \gamma_{1j} - \delta_0)^2 dH(\gamma_{1i}) dH(\gamma_{1j}) \\ & + \int_{\gamma^*(\theta)}^{\gamma^+} \int_{\gamma^*(\theta)}^{\gamma^-} \delta_0^2 dH(\gamma_{1i}) dH(\gamma_{1j}) \end{aligned}$$

The derivative with respect to θ equals:

$$\begin{aligned} \frac{\partial}{\partial\theta} E[(\pi_i - \bar{\pi})^2 \,|\theta] = & 2\Big(1 - H(\gamma^*(\theta))\Big) \left\{ \int_{\gamma^-}^{\gamma^*(\theta)} \big(\gamma_{1i} - \gamma^*(\theta)\big) dH(\gamma_{1i}) \right. \\ & \left. + \int_{\gamma^-}^{\gamma^*(\theta)} \big(\gamma_{1j} - \gamma^*(\theta)\big) dH(\gamma_{1j}) \right\} < 0. \end{aligned}$$

Therefore there is a monotonic negative relationship between RIV and unanticipated inflation when relative cost differences across sellers are independent over time.

However, when relative cost differences are constant, the difference in inflation between sellers is simply

$$\pi_i - \bar{\pi} = \begin{cases} 0 & \text{if} \quad \gamma_i \leq \gamma_0^*, \gamma_j \leq \gamma_0^* \\ \frac{1}{2}\theta & \text{if} \quad \gamma_i \leq \gamma_0^*, \gamma_j > \gamma_0^* \\ -\frac{1}{2}\theta & \text{if} \quad \gamma_i > \gamma_0^*, \gamma_j \leq \gamma_0^* \\ 0 & \text{if} \quad \gamma_i > \gamma_0^*, \gamma_j > \gamma_0^* \end{cases}$$

The corresponding expected value of $(\pi_i - \bar{\pi})^2$ is

$$E[(\pi_i - \bar{\pi})^2 \,|\theta] = \frac{1}{2}\theta^2(1 - H(\gamma_0^*))H(\gamma_0^*)$$

resulting in a V-shaped relationship between RIV and θ:

$$\frac{\partial}{\partial\theta}E[(\pi_i - \bar{\pi})^2 \,|\theta] = \theta(1 - H(\gamma_0^*))H(\gamma_0^*).$$

Therefore RIV increases linearly for both positive and negative inflation shocks when relative cost differences are constant. Qualitatively, these relationships between unanticipated inflation and both RPV and RIV in the context of the Bénabou and Gertner (1993) model are similar to those derided for the environment considered by Reinganum (1979).