Bookworms versus Party Animals: An Artificial Labor Market with Human and Social Capital Accumulation

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Abstract

Data show that educated workers earn higher wages and experience lower unemployment rates. Some researchers believe this occurs because education improves a worker's productivity (or human capital), making them more desirable on the job market. Other researchers believe schooling improves a worker's network (or social capital), giving them more information about lucrative openings and more resources to secure a job (such as references from peers). Much of the research on human and social capital focuses on why schooling benefits workers, but often overlooks how education produces these outcomes. This paper develops an agent-based complex adaptive system that features human and social capital accumulation, formal schooling and on-the-job training, labor market search and durable employment contracts to explain the process linking education to labor market outcomes and economic performance. Sample simulations show that human capital accumulation explains many of the novel facts seen in the data while social capital alone is not enough.

Keywords: Agent-based modelling, human capital, social capital, macroeconomic impacts of education.

JEL: I23, E27
1. Introduction

Education is good. At the individual level, workers with more schooling tend to have higher wages and are unemployed less often. At the aggregate level, countries with large proportions of uneducated workers tend to have low GDP per capita growth rates. There are two popular explanations as to why this link between education and prosperity may exist. In one of these theories, education imparts new skills and improved abilities to workers (known as human capital accumulation). Increased productivity makes workers more desirable on the job market and increases overall output. In the other theory, education provides workers with the opportunity to network with others while in school (known as social capital accumulation). Workers with better networks gain more information about lucrative job openings, receive references from peers and are more likely to receive on-the-job mentorship. In both scenarios, educated workers have higher earnings, spend less time unemployed and produce more than uneducated workers.

Why education leads to these novel facts is a well-researched topic; how education generates those facts is often overlooked. This paper develops an agent-based complex dynamic system within a virtual environment to answer this question. Models of this sort are systemic in nature and can therefore explain how the empirical regularities associated with education are created. In this approach, artificially intelligent heterogeneous agents with private information and objectives to achieve are created within a computational simulation. The agents interact with each other and respond to the state of the environment as it evolves around them, often by following user-defined rules. Macroscopic phenomena are explained by the structure of the agents and the process guiding their interactions. Because these models are created computationally, they allow the researcher to include more complexity and realism than older approaches and also provide the researcher with a means for running experiments by generating ‘alternate realities’.

The model described in this paper is built upon both human and social capital accumulation. Features of the virtual world include formal schooling, on-the-job training, labor market search and durable contracts. Simulations show that the model is able to produce macroeconomic phenomena such as persistent economic growth, a negative relationship between growth and changes in unemployment rates (Okun’s law), and concurrent job vacancies and unemployment (Beveridge curves) under different specifications. Further, microeconomic phenomena such as wage distributions for educated and uneducated workers similar to those seen in the data, lower unemployment rates for workers with schooling, and “cycles” in school enrolment are also produced. Human capital is shown to be vital in generating many of these results. Social capital alone is often not enough. Institutional choices, specifically tuition levels and university acceptance rates, are built-in and allow for an assessment of higher education policy on these micro and macro phenomena.

The remainder for this paper reviews recent data and literature necessary for constructing and evaluating the computational model, describes the virtual framework, presents and discusses results, then concludes with a brief discussion of future research.

2. Education, Labor Market Outcomes and Economic Performance

The beneficial impact that education has on labor market outcomes and economic performance is well-known. In New Zealand, for example, a worker with either tertiary schooling (bachelors, masters or PhD) or vocational/trade schooling earns on average 77% more than an uneducated worker (see table 1). This result is not driven by a minority of extremely wealthy

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1 For New Zealand, high school achievement is categorized according to the National Certificate of Educational Achievement (NCEA) regime which began in 2004. A worker that has chosen not to seek higher education may either have no NCEA
individuals which happen to be educated as workers with more schooling appear frequently in
the higher end of the wage distribution (see figure 1). Schooling is also linked to better
outcomes at the aggregate level. In terms of GDP per capita growth, countries with a larger
population of uneducated workers tend to perform worse (see figure 2). Economists,
sociologists and higher-education institutions have sought to explain the link between
educational attainment and these outcomes so as to devise more effective tuition and
enrolment policies.

Economists have tended to focus on the role of human capital in explaining the benefits of
education. This strand of research, started by Becker (1962, 1975) and Ben-Porath (1967)
developed theoretical models of both formal education (chosen by workers) and on-the-job
training (chosen by firms) within the neo-classical paradigm. Although these ideas were
contested early on, specifically by Spence (1973) and his work on job market signalling, a vast
empirical literature flourished during the 1980s and 1990s to quantify the impact of human
capital accumulation on labor market outcomes.2

In the latter half of the 1990s, models of human capital development were applied to emerging
ideas about labor market frictions.3 Acemoglu (1996, 1997) and Brunello and Medio (2001)
showed, for example, that human capital externalities may develop naturally in the presence
of search frictions and result in both physical capital deepening and wage increases for all.

With or without labor market frictions, if education causes worker productivity to perpetually
rise then GDP per capita should rise as well. The idea that human capital influences the
economy as a while began with the work of Uzawa (1965) and was subsequently taken up by
macroeconomists (see, Topel (1999), for example). This literature used general-equilibrium
modelling to explore the links between improved technology, innovation and persistent GDP
growth.

Sociologists have tended to focus on the role of social capital in explaining employment
outcomes for educated workers (i.e. it’s not what you know but who you know). Lin, Ensel and
Vaughn (1981) and Lin and Dumin (1986) show that a worker’s social ties (in descending order
of ‘strength’: family, friends, and acquaintances) affect their access to employment
opportunities. This access can be in the form of knowledge about lucrative openings or in the
form of referrals from peers and results in lower hiring costs for firms, increased worker
productivity, and reduced turnover.

As noted by Montgomery (1991), the process of hiring workers can be extensive and expensive
for firms. A firm can use referrals as a screening device: workers are likely to refer people
similar to themselves, therefore it is in the firm’s best interest to request referrals from high
quality employees. This process increases both employment and wages for workers with well-
developed networks. Fernandez, Castilla and Moore (2000) add that hiring workers that
already have a contact within the firm can result in more productivity through informal
mentorship4 and reduced turnover (as having a friend on the job reduces the likelihood either
employee will quit). As intra-firm peer networks develop, the firm also saves costs on
monitoring workers (Kugler, 2003).

Tertiary schooling provides students with the opportunity to form networks either with peers
or with university services. Lee and Brinton (1996), for example, explore how human capital
and social capital interrelate within tertiary institutions. In this study, human capital
influences the creation of social capital. Students at top Korean universities (i.e. more ‘able’

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3 See Mortensen and Pissarides (1999) for a description of these models.
4 As noted by Coleman (1988), social capital can increase the rate of human capital accumulation – so friends who already have
experience in the company can give on-the-job advice.
students) rely more heavily on university-provided employment services (a type of social capital) when searching for a job. Actual employment at large firms (the high wage sector) depends on both the level of human capital the student has obtained and the social capital provided by the institution. We would naturally expect educated workers to have better networks and, therefore, obtain higher-paying jobs and become unemployed less often.

Be it from social capital accumulation or human capital accumulation, the link between education and prosperity plays a prominent role in policy discussions. Once the value of education is determined, a theory of optimal tuition can be developed (see Winston (1999) for a review). Rothschild and White (1995) first formalized higher education pricing. More recent work by Epple, Romano and Sieg (2006) develops a market model of higher education which incorporates enrolment and financial aid decisions, but also adds a notion of ‘peer-quality’. This latter model is used to predict both tuition levels and the distribution of students across colleges of different qualities. Neither of these studies extends the outcomes in the market for higher education to the student’s after-college labor market outcomes or to general economic performance.

Over the years, complications with the approaches used in this area of research have arisen. On the empirical side, researchers have found it difficult to account for all the important determinants of schooling and employment (such as inherent ability and personal history). As a result, increasingly complex econometric techniques have been developed in an attempt to adequately capture the relevant relationships. Further, while empirical studies have their uses, they are ultimately ‘black-box’ in nature. Statistics can describe but does not really explain (i.e. correlation does not imply causation).

On the theoretical side, many models involving markets for education and labor are steeped in the rational choice paradigm. This approach usually involves over-simplifying the economic environment and over-complicating human behaviour so that tractable equilibria can be derived. Once equilibria are found, they can be compared (i.e. comparative statics analysis can be performed – we can determine the economic outcomes at varying levels of education). While comparative statics may determine why higher education equals prosperity, it fails to fully explain the relationship – we don’t know how the equilibria are attained or sustained over time. Models of this sort project an “equilibrium systems” view of human interaction and ignore the evolutionary nature of society. As a result, their implications are non-durable.

To overcome some of the difficulties present in standard approaches, new methods have been developed using computation and simulation. One approach in particular, agent-based modelling (or ABM), treats markets like ecosystems built up by heterogeneous individuals. In this class of models, artificially intelligent virtual agents are provided with private, local information and a set of objectives to achieve. They are then allowed to interact with each other (and to the state of the world itself) within the virtual environment. Through their actions, aggregate outcomes are formed. These aggregate outcomes are fully explained by the characteristics of the agents, the systemic structure of their actions and the formation of the virtual environment. This generative approach to social science, as named by Epstein (1999), allows for a complicated and rich environment often unachievable in other paradigms. Further, since the agents and the virtual world are constructed by the researcher, a variety of ‘experiments’ can be performed by augmenting initial conditions or turning on and off different features of the model. With full information available to the researcher, no part of the model is a ‘black box’. Tesfatsion (2002, 2007), Macy and Willer (2002), Mathieu, Beaufils and Brandouy (2005), Tesfatsion and Judd (2006), and Pyka and Fagiolo (2007) provide a description of the applicability of this type of modelling to economics and other social sciences.

This paper develops an ABM of education and economic performance. It is not, however, the

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5 For example, we often ask an agent to consider his entire discounted lifetime income when choosing how much schooling to buy – a bit much to ask for from an agent who is currently uneducated.
first to do so. Tesfatsion (2001) constructs a model with job search costs, wage determination via bargaining and network formation between workers and firms to evaluate the impact of job capacity (the ratio of potential job openings to potential work offers) and job concentration (the ratio of workers to employers) on the employer’s ability to exercise market power in wage determination. Delre and Parisi (2007) model search frictions and social network formation (networks between workers) to create and analyze labor markets with collective bargaining and information transfer. Tassier and Menczer (2001) generate an artificial labor market with referral networks and search costs to analyze the impact that a worker’s job search strategies (finding jobs via friends or private search) has on employment. Similarly, Gemkow and Neugart (2011) focus on referral hiring and social networks between workers, but evaluate their impact on employment rates at the aggregate level. Fagiolo, Dosi and Gabriele (2004, 2005) construct a model with job search and wage determination via bargaining to generate other macroeconomic features, such as Beveridge curves (the negative relationship between posted job vacancies and unemployment rates) and Okun’s law (the negative relationship between GDP growth and unemployment rates).

None of the aforementioned models explore the role of education in the economy, which is the niche this project fills. The ABM presented here builds upon this past literature, however, by incorporating notions of job search, matching and by focusing on both ‘microeconomic’ (wage distributions and unemployment rates) and ‘macroeconomic’ (economic growth, Beveridge curves and Okun’s law) phenomena. Human capital accumulation is added through both formal schooling and on-the-job training. Social capital is accumulated through formal schooling, and further is only accumulated by students who choose to spend their education time socializing as opposed to studying. Scope for policy discussion is incorporated by adding rules which guide tuition pricing and enrolment. The model aids in explaining how persistent economic growth is achieved, how wage distributions are set, how education generates a wage gap between educated and uneducated workers, and how those who spend more time studying in college (‘bookworms’) become better off in terms of lifetime income than those who focus on social activities (‘party animals’).

3. An Agent-based Model of Human and Social Capital

The ABM constructed in this study is described in two phases. First, the set-up (or simulation period 0) is described followed by an outline of a typical simulation period (or simulation period t). In describing simulation period 0 and simulation period t, the individual features of the agents, how they interact with each other, their relationship to the virtual world itself, and the formation of the macroscopic endogenous variables are fully characterized.

3.1. Simulation Period 0 – Set-up

Consider a virtual world populated by a user-defined number of initial worker agents indexed by i. The experiments that follow calibrate the size of this initial population to 300. Each worker is assigned a random age (A\textsubscript{i,0}) and a user-specified amount of starting wealth (W\textsubscript{i,0}). A starting wealth of 1000 will be considered in the actual simulations. Workers have different abilities which are measured by “skill levels”. Define an agent’s ability as a vector of N skills: \( \mathbf{S}_{i,0} = [s_{i,1,0}, s_{i,2,0}, ..., s_{i,N,0}] \) where each \( s_{i,n,0} \) represents agent i’s skill level in skill \( n \in \{1, ..., N\} \). The user can choose the number of available skills, N, and the maximum initial skill level, \( \mathbf{S}_{i,n,0} \), that any member of the starting population may achieve. In the following experiments, both N and \( \mathbf{S}_{i,n,0} \) are set to 20. Skill levels are assigned to each worker by pulling a random draw for each skill from a uniform distribution with a minimum of 0 and a maximum of \( \mathbf{S}_{i,n,0} \). It is assumed that society places a value of $1 for on each level of each skill, making the agents overall ‘worth’ (\( \omega_{i,0} \)) in the initial period:
\[ \omega_{i,0} = \sum_{n=1}^{N} s_{i,n,0} \]  

Equation (1)

All members in the initial workforce are labelled as “uneducated” and “unemployed”. At the start, it is assumed that no networks have formed between workers. The user can select a minimum amount of wealth that workers must eat in each period (E) and a percentage of the agent’s earned income that is also consumed (\(\epsilon\) - the marginal propensity to consume). In the simulations below, E is set to 50 while \(\epsilon\) is set to 0.8.

The virtual world is also populated by a user-defined number of initial firm agents indexed by \(j\). In the experiments below, the number of starting firms is set to 50. Each firm is assigned a random number of available vacancies such that the total number of available jobs is approximately equal to the size of the working population. Each initial firm is assigned a skill-requirement vector which describes the amount of each of the \(N\) skills the firm needs to produce 1 unit of output: \(S_{j,0}=[s_{j,1,0}, s_{j,2,0}, \ldots, s_{j,N,0}]\). As with the agents, skill requirements are assigned to each firm by pulling a random draw for each skill from a uniform distribution with a minimum of 0 and a maximum of \(s_{i,n,0}\). Since society values each skill at $1, the value of the output produced by a firm (\(\omega_{i,0}\) - the ‘worth’ of a unit of firm \(j\)’s output) is defined as:

\[ \omega_{j,0} = \sum_{n=1}^{N} s_{j,n,0} \]  

Equation (2)

Within the virtual world there is also the education sector which is fully controlled by the user. A variety of policy variables are available for the user to choose. These include defining how tuition is set, how many students are accepted into schooling each year, and how long a student must spend in school to achieve their degree (M). The user can also choose the rate at which schooling improves a worker’s skills and what proportion of incoming students are socialites.

### 3.2. Simulation Period \(t\) – a typical period

The algorithm followed during simulation is depicted in figure 4 and is described as follows:

1. At the start of a simulation period, \(t\), college tuition (\(T_t\)) is set. To determine tuition, colleges follow a rule involving economy-wide outcomes occurring in the previous period and parameters of the model. Tuition policies will be elaborated on when experiments are discussed in the next section.

2. Students decide whether or not to apply for schooling. Schooling is assumed to improve each skill by a factor of \((1+\theta_b)\), where \(\theta_b\) is the user-defined skill improvement rate for a serious student. In the simulations below, \(\theta_b\) is set to 20%. Given tuition, their private abilities and economy-wide outcomes from the previous period, agents choose to apply for college if the following two conditions are satisfied:
   a. \(A_{i,t} < 2\). The agent is young.
   b. \(W_{i,t} > M(T_t + E)\). The agent’s wealth must be greater than the advertised annual tuition level plus the minimum consumption amount multiplied by the number of years in school.
   c. \((1+\theta_b)\omega_{i,t} > \bar{w}_{t-1}^u\) where \(\bar{w}_{t-1}^u\) is defined as last period’s average wage for an uneducated worker. The agent’s post-education worth must be sufficiently large (in this case, larger than the most recent measure of average earnings for unemployed workers).

Condition (a) limits education to the young, an assumption that allows us to calibrate the model using demographic data. Condition (b) represents a wealth constraint. Only
those that can afford to go to college bother with applying. Condition (c) is an incentive constraint. Agents who believe their natural abilities will not be enhanced enough to earn wages higher than what they could be earning now will not apply. Both conditions (b) and (c) limit college attendance to the sufficiently wealthy and the sufficiently talented.

3. The university accepts a user-defined proportion of applicants ($\alpha$). Acceptance is based on a random draw and is independent from the applicant’s skill level. Accepted students pay the full amount of tuition ($M \times T_t$) to the college up front and spend the next $M$ years “in school” (out of the labor force). Non-applicants and non-accepted applicants are sent to the labor market as uneducated workers.

4. A user-defined proportion of accepted students ($\beta$) become serious students (‘bookworms’) while the remainder become socialites (‘party animals’). Serious students spend their time studying and improve their abilities by the full factor $(1 + \theta_b)$. Socialites improve their skills by a factor $(1 + \theta_{pa})$ where $0 \leq \theta_{pa} \leq \theta_b$. Although they gain less human capital from education, party animals do build social networks. These networks are represented as links to all other party animals currently in college (regardless of how long they have been there).

5. Once education decisions are made, the primary labor market opens. This market proceeds as follows:

   a. Firms with openings advertise vacant positions. They post the skill requirements and offer a wage equal to the value of the skills needed to produce the product (see equation (2)). They note that workers who fall short of the skill requirement may still be hired, but must undergo on-the-job training to improve their abilities. The cost of training for worker $i$ and firm $j$ ($c_{i,j,t}$) equals the value of the worker’s deficiency:

   \[ c_{i,j,t} = \sum_{n=1}^{N} (s_{j,n,t} - s_{i,n,t}) \text{ if } s_{j,n,t} > s_{i,n,t} \\
   0 \text{ if } s_{j,n,t} \leq s_{i,n,t} \]

   Because skills in the model are ‘general’, the worker is responsible for absorbing the training costs. The wage offer ($w_{i,j,t}$) is therefore: $w_{i,j,t} = \omega_{j,t} - c_{i,j,t}$. Note that workers who are over-qualified are not paid extra by the firm.

   b. Unemployed workers rank the posted vacancies according to the wage offerings. They apply to the top 20% most lucrative openings by showing their skill set to the firm.

   c. The firm receives applications and make offers to fill the open vacancies, starting with the most qualified workers.

   d. Workers receive job offers and accept the offer with the highest wage. The worker is then labelled as “employed” at that wage.

   e. For each accepted offer, the firm closes a vacancy.

   f. Durable contracts may be offered when a worker is hired. The probability a worker receives one of these contracts is denoted by $\pi$. $\pi$ can be chosen by the researcher. The duration of the contract, should it be offered, is a random integer draw from a uniform distribution with a user-defined maximum, $Y$.

The sequence of events described in (a) – (f) repeat a specified number of times ($L$). When $L$ is low, the labor market is slow and simultaneous vacancies and unemployment can exist. A calibration for $L$ that accomplishes this (which is the calibration used the experiments below) is to set $L$ equal to 2 plus an additional round for every 1000 workers plus an additional round with probability 0.5. Note, in particular, that as the sequence repeats matching becomes imperfect; a firm might not
hire the best-fitting worker for them.

6. After the primary labor market closes, a secondary labor market opens. This market is only for workers with a social network and operates as follows:

a. Educated but unemployed workers with a social network ask their employed friends about potential openings at their firm. If an opening exists, the worker asks the firm for a job.

b. For each job inquiry, the firm sends to the worker a wage offer. As in the primary labor market, the worker engages in on-the-job training their skills fall short of the firm’s requirements at their own cost.

c. The worker responds to the top 20% of wage offers by indicating their skills to the firm.

d. Firms make employment offers to fill open vacancies starting with the most qualified workers.

e. Workers receive job offers and accept the offer with the highest wage net of on-the-job training costs. The worker is labelled as employed at this net wage.

f. For each accepted offer, the firm closes a vacancy.

g. As in the primary labor market, a durable contract may then be offered.

This market operates only once. As such, it may be possible for a networked worker to remain unemployed after both the primary and secondary labor markets close.

7. Once hiring has taken place, training and production occur. Worker’s skills are adjusted to account for the training they receive:

\[
s_{i,n,t} = \begin{cases} 
    s_{i,n,t} + \theta_b s_{j,n,t} & \text{if } s_{j,n,t} > s_{i,n,t} \\
    s_{i,n,t} & \text{if } s_{j,n,t} \leq s_{i,n,t}
\end{cases}
\]

(4)

Note that workers improve their skills by learning a fraction of the skill-level required by the firm. This fraction is equal to the rate of skill improvement of a studious college student. Workers are not required to improve all of their skills, only those in which they are deficient.

Based on the average skills of the workers hired, the skill requirements set by the firm are also assumed change (after training has taken place). This captures a notion of “technological evolution” in production:

\[
s_{j,n,t} = \begin{cases} 
    s_{j,n,t} & \text{if } s_{j,n,t} > \bar{s}_{n,t} \\
    \bar{s}_{n,t} & \text{if } s_{j,n,t} \leq \bar{s}_{n,t}
\end{cases}
\]

(5)

where \( \bar{s}_{n,t} \) is the average skill level for skill \( n \) of all \( i \) employed at \( j \) in period \( t \):

\[
\bar{s}_{n,t} = \frac{1}{\text{all } i \text{ employed at } j} \sum_{i \text{ employed at } j} s_{i,n,t}
\]

(6)

For any skill, \( n \), if the average ability of the workforce exceeds the firm’s current requirement, the firm revises their skill demands upwards. Otherwise, their skill demands remain unchanged.

8. Employed workers are paid their wage net of training costs \( (w_{i,t} = w_{i,j,t} \text{ where } j \text{ indexes the worker’s employer}) \) while unemployed workers are paid \( w_{i,t} = 0 \). Wages are added to the worker’s lifetime wealth \( (W_{i,t}) \). Workers then eat a constant amount plus a fraction of their period earnings:
\[ W_{i,t} = W_{i,t-1} - E + (1 - \epsilon)w_{i,j,t} \]  

(7)

9. Economy-wide data is collected. This includes average wages for all workers (including unemployed workers), for employed workers, for educated workers, and for uneducated workers. The wage growth rate is also calculated for each of these. Unemployment rates for all workers, for educated workers and for uneducated workers are also derived in addition to the vacancy rate for firms (measured as the percentage of total openings which are unfilled). Distributions for the total worth of both workers and firms (see equation 1 and 2) and for worker’s total wealth (see equation 6) are derived. The number of agents in college and the posted tuition rate are also recorded.

10. Agents with durable contracts decide whether or not to break their agreement following a simple rule (described below). Agents holding active contracts do not go to the labor market in the next round.

11. Agents die. Workers die if old (\(A_{i,t} > 100\)) or by accident (approximately 1% of agents randomly die). Workers also die if they cannot afford to eat the minimum amount (i.e. if equation (6) generates \(W_{i,t} < 0\)). ‘Friends’ of dead agents are notified and networks are updated. Firms that fail to fill any open positions for more than 20 periods also die.

12. Agents reproduce. The population of workers grows at a rate of 2%. Workers are randomly selected to reproduce. If selected, an agent produces another agent (a ‘hatchling’) nearly identical to themself. The hatchling has the same skills as the parent, however their age is set to 1 and are listed as being uneducated, unemployed and having no social network. Parents bequest a fixed proportion of their wealth, set at 30%, to their hatchling. The hatchling also receives additional starting income (which is meant to reflect socially-provided wealth and grows each period at the current wage growth rate). Firms reproduce at a rate equal to the current growth rate of the economy if the number of workers exceeds job availability (indicating a hot labor market). Like workers, firms are randomly selected to clone themselves. The firm’s hatchling (or branch) has the same skill demands as the parent, but is given a randomly chosen number of open positions to fill. If the size of the workforce is larger than the total number of vacancies in the economy (indicating overinvestment), firms do not open new branches.

13. The economy is reset. All workers with no contracts (or with expired contracts) are labelled as “unemployed”. The number of available vacancies in each firm is reset as the total number of open vacancies net of the number of workers currently holding contracts with the firm.

This process then repeats. The model is programmed into Netlogo, a program specifically designed to accommodate agent-based modelling. Netlogo has many convenient features related to network formation and population growth. Further, the language is flexible and easy to learn. The Netlogo program is available upon request.

4. Experiments

4.1. Experiment validation

Usually, a researcher compares results from the simulated environment to real-world data to make the arguments embodied in the model more convincing. In the case of agent-based modelling, this has been shown to be a difficult exercise since both the agent’s decisions and the state of the virtual economy are history dependent (or ‘path dependent’). Sargent (2005), Law (2005), Boero and Squazzoni (2005) and Windrum, Fagiolo, Moneta (2007) describe the
current controversies associated with the empirical validation of agent-based models and suggest appropriate strategies to deal with them.

In many descriptions of validation (in particular, Windrum, Fagiolo and Moneta (2007)), multiple monte-carlo simulations are performed and then the resulting data are analysed. Unfortunately, Netlogo is limited in its ability to run repeated simulations. A more robust result-generating process along these lines is left for future research. However, many interesting implications about the potential outcomes a repetitive simulation may produce can be made by looking at a single simulation. In the experiments that follow, a single 100-period run of the algorithm described in section 3 is performed. Periods 1 through 50 are dropped from the analysis to filter out the effects of the initial population. Experiments involving different calibrations of the model are described below.

4.2. Experiment 1: on-the-job training and durable contracts

What does the simulated economy look like when there is no formal education? Is on-the-job training enough to account for any stylized facts? What is the role of durable contracts in the economy? To answer some of these questions, I set the college acceptance rate ($\alpha$) equal to zero and run the program with different specifications for the frequency and duration of durable contracts.

Figure 5 shows selected results when there are no durable contracts offered. These results are compared to real New Zealand economy described in figure 6. The model succeeds in producing business cycle fluctuations (in both average income and unemployment rates), persistent unemployment and Okun’s law. However, the average growth rate of income in the simulation is approximately 0.19% [st.dev. 1.76%], far below empirical estimates for New Zealand (1.39% [st.dev. 2.27%]). Although there is a short-term transition period characterized by increasing average wages, the model fails to produce exponentially increasing average wages.$^7$

Table 2, columns (1) and (2), inspects the distribution of wages amongst the population, both for New Zealand and for the simulated economy, more closely. The simulated model generates extremely ‘narrow’ wage quintiles – earnings are generally equal across the entire population. On-the-job training coupled with repetitive redistribution of workers into different jobs builds up everyone’s skills to nearly the same level.

Can adding durable wage contracts improve some of the shortcomings found in this experiment? Will some workers earn dramatically more than others due to enhanced job security? With on-the-job training in place, will workers that have long-term employment become higher-quality employees? Further, would a ‘loose’ labor market (many advertised vacancies at the start of the simulation) be correlated with a lower end-of-simulation unemployment rate (i.e. will Beveridge curves emerge)? To explore these questions, contracts are allowed to be long-lived. I look at both high (100%) and low (40%) probabilities of being offered a contract and high (10) and low (5) maximum contract lengths. I assume that workers break their contract if the contracted wage falls below the average economy-wide wage.

Figure 7 shows the impact of durable contracts on wage growth. The figure suggests that contract duration can generate persistent economic growth (independent of the frequency that contracts are offered). Table 2 describes the wage distributions for different combinations of contract duration and frequency. These results indicate long contracts also contribute to increasing wage inequality. We might explain the connection between long contracts and both wage growth and inequality by the structure of on-the-job training in the virtual world.

$^6$ Data on the New Zealand economy includes a particularly deep recession from 1990 – 1992. Limiting the sample to 1993 – 2007 brings the average growth rate of GDP per capita to 2.39% [st.dev. 1.26%].

$^7$ When there is no schooling, the maximum achievable average wage is $N\times s_{\alpha,0}=400$ as calibrated.
Workers that stay in jobs longer become more likely to meet or exceed the firm’s skill requirements as they train over time. These workers receive higher wages when they move on to their next employer. New workers are more likely to be trained, since the firm’s expectations are enhanced by ‘experienced’ workers. As a result, wages are higher for everyone (growth), but particularly larger for those who have had durable contracts (inequality).

Figure 8 describes the model’s ability to reproduce Beveridge curves. The correlation between vacancy rates and unemployment rates in New Zealand data is unclear, particularly during the outset of the most recent recession (2007), but is statistically negative. The model can generate a negative relationship between start-of-period vacancy rates and end-of-period unemployment rates when the duration of employment contracts is low. This relationship is less obvious when contract duration is high. Since the data is also unclear, a poor fit on this dimension is not necessarily indicative the model is performing inadequately.

4.3. Experiment 2: formal education

When the option to attend college is ‘switched on’, the simulation program identify the effects of formal education on labor market outcomes and economic performance. To do this, enrolment and admittance policies must be identified. I start with a simple case where tuition is set to be a fixed fraction of the uneducated average wage. During 2009, average annual wage for an uneducated worker in New Zealand was approximately NZ$ 25,500 (see table 1) while the total revenue per full time student enrolled in college was approximately NZ$ 21,975 (Ministry of Education – New Zealand, 2009a). I calibrate tuition, therefore, to be 86% of the average uneducated wage in the simulation program.

I use schooling participation rates in New Zealand to calibrate the proportion of applications accepted in the model. Data suggests approximately 23.7% of New Zealand youth between the ages of 18 and 19 were enrolled in bachelor’s degree programs in 2009 (Ministry of Education – New Zealand, 2009b). Given that the program does not assume all young people will apply for college, a premium will be added to this participation rate to calibrate the acceptance rate in the simulation at 35%. The participation rate derived during the simulation will be checked against New Zealand data to ensure the model mimics the New Zealand education system.

The following sub-experiments are conducted (all of which involve non-durable wage contracts):

A. Human capital only ($\beta = 1$)
B. Social capital only ($\beta = 0, \theta_{pa} = 0$)
C. Social capital with mild skill improvement ($\beta = 0, \theta_{pa} = 0.05$)

4.3.A. Human capital

Can education induce persistent economic growth? Doe educated workers earn more (even when on-the-job training is present)? Are educated workers employed more often? To answer these questions, I turn on the education system with the aforementioned tuition and acceptance policies in place and run the simulation. Results for this version of the model are illustrated in figure 9.

The model produces persistent wage growth…and not just for educated workers! College students exit schooling with high skill levels relative to the rest of the population. As firms hire educated workers, their skill expectations start to increase. With higher skill demands from firms, uneducated workers become more likely to receive on-the-job training. Skilled parents have skilled offspring and the process repeats. As skill levels rise in the economy in general, average wages for all worker types increases. The result: persistent economic growth.

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* Recall that colleges accept a fixed fraction of applicants in the model. This acceptance policy implies that fluctuations in enrolments are driven principally by fluctuations in applicants and not the college’s willingness to admit students.
High-skill, educated workers are more attractive on the job market in the model. During the last 30 periods of the simulation, workers who have had formal schooling earn on average 63.4% [st.dev. 4.5%] more than workers who have not. This result is similar to the New Zealand economy, where educated workers earn approximately 76.6% more than uneducated workers. When comparing the wage distribution workers produced by the simulation with those in New Zealand, we can see a fairly reasonable reproduction of the wage quintiles for educated and uneducated workers (see columns (1) and (2) of table 3). Further, educated workers have a tendency (but not a strict tendency) to experience lower rates of unemployment.

The model can also produce interesting schooling trends. Data on participation in Bachelor degree programs suggests that 3.6% of the entire New Zealand population was enrolled in such programs in 2009 (Ministry of Education – New Zealand 2009b). During the final 50 periods of the simulation, 3.1% [st.dev. 0.6%] of the virtual population was enrolled in formal schooling on average in any given period. In the model, college participation fluctuates regularly (i.e. ‘education cycles’). Since the decision to apply to college depends on the workers outside option, namely the average wage for uneducated workers, periods of low unemployment (and thus high average wages) correspond to periods in which relatively few young agents apply to college. As a result, participation is low. As young uneducated workers enter the market, unemployment for the unskilled rises (and thus average wages for this group fall), providing incentive for future generations to apply for schooling.10

As in the simulation with on-the-job training alone, the virtual economy produced in this experiment also generates business cycles and Okun’s law. These macroeconomic phenomena seem to be inherent features of the framework.

4.3.B. Social capital

Is ‘who you know’ more important than ‘what you know’? What is the influence of social networking on labor market performance? To address this, I allow students to believe they will become serious students when they apply for college, but become socialites when they actually arrive (β). At school, it is assumed that this group receives no skill improvement (θpa = 0), but do exit college with networks that they can use in the future to obtain jobs. All ‘learning’ in the model is via on-the-job training as a result.11

Results are shown in figure (10). The macroeconomic phenomena produced by the virtual economy created during this experiment do not differ substantially from those produced by the economy with no formal education system (on-the-job training without durable contracts). Business cycles and Okun’s law are present, but growth is not persistent.

At the micro level, results for educated workers are not as good as we would expect them to be given the role of networks in the virtual world. Because schooling takes a worker out of the work force for several periods, during which time their skills do not improve via on-the-job training, educated workers enter the workforce with fewer abilities than experienced, uneducated workers. They receive lower wages and are unemployed more often as a result. Further, when they do gain employment via their network chances are the job does not pay well (as their peers have also spent time outside the labor market and thus have relatively lower skills than the rest of the population). Over time, on-the-job training pulls up everyone’s skills, regardless of the worker’s education history, and average wages for both groups converges. As a result, wages across worker types are evenly distributed (see column (3) of

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9 The wage gap worsens as the economy develops and the effects of the initial population are ‘filtered out’. This process however takes 70 periods. For the ending periods of the program, the wage gap fluctuates around a stable level of 63.4%.

10 This result is supported by the relationship between education participation rates and the ratio of tuition to the average uneducated wage produced by the simulation. The correlation between these two variables is −0.53, indicating the ‘demand for education’ is negatively sloped. There is insufficient data on schooling participation to validate this result empirically.

11 As before, workers engaged in on-the-job training learn at the rate θb = 0.20.
4.3.C. Social capital with mild skill improvement

College isn’t all fun and games. Even the most socially active students may accumulate some small amounts of human capital during their matriculation. Is it possible to add a measure of human capital accumulation to the social capital experiment described in the previous subsection to achieve improved results? If so, how much human capital is necessary to make formal education beneficial? To address these questions, repeat the experiment with social capital but allow students to acquire human capital at a small rate: $\theta_{pa} = 0.05$. Results are shown in figure 11.

As it turns out, a little human capital goes a long way. The mild amount of human capital accumulation that occurs while in school is enough to generate persistent economic growth, a gap between the average wages of educated and uneducated workers (educated workers earn approximately 7.38% [st.dev. 0.77%] more than uneducated workers on average), and a distribution of wages across the population that matches empirical findings (see column (4) of table 3). As in the other models, business cycles and Okun’s law also occur. The only shortcoming produced by the experiment is the relationship between schooling and unemployment rates which, as in the model with social capital only, is unclear. Educated workers in the model are occasionally employed more often, but not always.

5. Auxiliary Analyses

The agent-based model described in section 3 can be useful when discussing the impact that a variety of different education policies have on schooling participation, labor market outcomes and economic performance. In this section, I focus on two in particular:

1. The Balanced College Experience
2. Value-based Tuition Pricing

5.1. The Balanced College Experience

In the experiments conducted in section 4, all students were either bookworms or party animals. In reality, however, colleges offer both skill development services and an environment where networking can occur. Perhaps they do so to increase the number of applicants (as college students are often interested in both high-quality education and social activities) or to increase the price of tuition. Perhaps students themselves form the social environment once they enrol, and the university contributes funding to exercise control over the student body. Whatever the reason, we might be interested in the outcomes that occur when both bookworm and party animal types are present. To pull these out, I simulate the model with $\beta = 0.5$. In this calibration, both studious and social students exist simultaneously. I run two separate experiments: one in which the rate at which party animals improve their skills is mild ($\theta_{pa} = 0.05$) and one in which this rate is strong ($\theta_{pa} = 0.20$). Results are reported in figure 12.

Serious students earn, on average, 14.63% [st.dev. 2.65%] more than social students with mild skill improvement. This amount is approximately equal to the difference in skill improvement experienced by the two groups when in college. The growth rate of the average wage (for all virtual agents) is on average 1.83% [st.dev. 0.67%]. Again, the relationship between education, student type and unemployment is unclear. When the rate of human capital accumulation for party animal students is high, however, the wage gap between studious and social student types varies with bookworms earning as much as 8% more in early periods to earning as much

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12 The unemployment rate for bookworms is on average 0.25% [st.dev. 0.87%] less than party animals in this simulation which is not statistically different from 0.
as 12% less in later periods. The growth rate of average wages is much higher (on average 3.41% [st.dev. 1.23%] since there is more skill improvement in general. As before, the unemployment experience of each worker type is ambiguous.13

These results suggest that increasing the rate of skill improvement generates not only higher wages for the student, but faster growth in the economy in general. Even when social capital is an ‘add-on’ for an already strong academic program, as in the experiment with strong skill improvement, the benefits to students may take some time to appear (however, when they do appear, they may be significant). A more complex social structure may be worth investigating in future work, but the results generated in this study suggest that what you know is a bit more important than who you know in terms of economic performance.

5.2. Value-based Tuition Pricing

In the model so far, tuition is set as a fixed fraction of the uneducated wage. This assumption provided a convenient way to calibrate the model using data for the New Zealand tertiary education system, but does not take into account the value of education to the worker. We can relax this assumption consider an alternative tuition setting rule in which universities charge a mark-up on the benefits to education:

\[ T_t = x(\bar{w}_t^c - \bar{w}_t^u) \]  

(7)

where \( \bar{w}_t^c \) and \( \bar{w}_t^u \) are the average educated and uneducated wage rates that prevailed in the previous period. Tuition is set as a mark-up, \( x \geq 1 \), over the wage differential between educated and uneducated workers. Under this specification, colleges respond to an increase in the “value” of education (as measured by a widening in the wage gap) by raising tuition prices. Holding everything else constant, an increase in \( x \) will change the total number of applications a college receives in addition to the revenue received from each enrolled students.

All parameters in the model other than \( x \) are calibrated to be the same as in the experiment with human capital accumulation only (experiment A in section 4). For \( x \), two different values are considered. In 2009, tuition was approximately 87% of the uneducated average wage and that educated workers earned approximately 77% more than uneducated workers in New Zealand. This suggests an appropriate value for \( x \) may be 1.117. Results for this simulation are shown in figure 13. The model generates persistent growth in average wages (at 2.52% [st.dev. 1.29%] on average) and reasonable values for schooling participation rates (2.58% [st.dev. 0.46%]). However, the model produces a low ratio of tuition to uneducated wages (38% [st.dev. 6.27%] in the model versus 87% in the data) and a small wage gap (34.56% [st.dev. 5.33%] in the model versus 77% in the data).

To achieve a better fit to the data, I experiment with a higher value of \( x (= 1.5) \). Results from this simulation suggest that although tuition is now more expensive, participation rates have not been affected (recall that colleges are assumed to accept a fixed fraction of applicants) and the growth rate of wages on average is still persistently positive (2.66% [st.dev. 1.28%]). The higher mark-up generates higher wages on average for educated workers (the wage gap produced by the simulation is 58.33% [st.dev. 4.67%]) and therefore a more realistic ratio of tuition to the uneducated wage (86.3% [st.dev. 7.13%]).

A noteworthy result is that the average wages of uneducated workers also seem to increase. This may be due to the impact of relatively higher tuition prices during early phases of the simulation. Recall that an agent will apply to college only if they can afford it. Further, the amount of wealth an agent has to spend on college is, in part, dependent on the bequest from their parent. Agents also receive inherent skills from their parent when they are born.

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13 The unemployment rate for bookworms is on average 0.58% [st.dev. 1.67%] greater than party animals in this simulation which is not statistically different from 0.
Therefore, agents with rich parents may also be highly skilled before applying to college since earnings are linked to a worker’s abilities. Therefore, a university can ‘screen’ for high-ability applicants by charging high tuition. When high-ability agents acquire skill improvement, they in turn earn higher wages (which results in faster wage growth for the educated) and spread their ability via on-the-job training through firm skill demands (which results in faster wage growth for the uneducated as well). Unfortunately, this result suggests an overall economic benefit to incorporating a certain degree of elitism in education pricing and warrants further investigation.

6. Conclusion

As economic theory suggests, the model produced in this paper shows that human capital accumulation in formal schooling is essential to replicating the improvements that education makes on labor market outcomes and economic performance. Wages are generally higher for those who improve their abilities while in school. The gap generated between average educated and uneducated wages is close to empirical estimates. As the skills for educated workers improve they “spill over” into the rest of the uneducated population via on-the-job training. Persistent economic growth occurs for the economy as a whole. A similar effect can occur without formal schooling, but involves durable labor contracts. When these are present, a negative relationship between the economy-wide vacancy rate and the unemployment rate (Beveridge curves) can occur. Business cycles and Okun’s law are produced by the functioning of the labor market within the framework and are both independent of how human capital accumulates.

Social capital within the model is not strong enough on its own to account for many stylized facts. As a result, the model recommends that tertiary institutions focus on skill improvement as opposed to network opportunities. Looking at more complex social networking systems may reverse this result. When social capital accumulation is linked to human capital development, this result may be reversed. As noted by Coleman (1988), networking may facilitate learning. If so, colleges may wish to consider stronger support for social activities. This is a topic left for future studies.

In addition to incorporating more complex social networking regimes, adding a variety of other labor market features (such as wage negotiation, signalling or unionization) can also be accomplished. Case-specific agent-based simulations can serve as an alternative method for providing policymakers insight on the impact of financial-aid regimes, performance-based admission policies or educational standards on both the institution and the economy as a whole.

Agent-based modelling, which features more realism and complexity than standard approaches, suggests intriguing new directions for future research explaining the subtle relationships between educational attainment, labor market outcomes and economic performance. The initial exploration done in this paper is an example of how virtual economies can aid our understanding of these linkages (which are often hidden or over-generalized by mainstream empirical and theoretical studies).
7. References


8. Tables and Figures

8.1. Tables

Table 1: Average Earnings for New Zealand Workers by Highest Qualification, 2009a

<table>
<thead>
<tr>
<th>Highest Qualification</th>
<th>Average Annual Incomeb</th>
<th>Population (1,000s)</th>
<th>% Sample</th>
<th>% Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No qualification</td>
<td>$23,764</td>
<td>844.1</td>
<td>51%</td>
<td>25%</td>
</tr>
<tr>
<td>School certificate / NCEA level 1</td>
<td>$28,548</td>
<td>286.8</td>
<td>17%</td>
<td>9%</td>
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<tr>
<td>Sixth form / NCEA level 2</td>
<td>$28,652</td>
<td>202.5</td>
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<td>6%</td>
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<tr>
<td>Higher school / NCEA level 3</td>
<td>$26,988</td>
<td>167.2</td>
<td>10%</td>
<td>5%</td>
</tr>
<tr>
<td>Other school</td>
<td>$23,348</td>
<td>138.6</td>
<td>8%</td>
<td>4%</td>
</tr>
<tr>
<td>TOTAL</td>
<td>$25,499c</td>
<td>1,639.2</td>
<td>100%</td>
<td>49%</td>
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</table>

Table 2: Wage quintiles (as a proportion of the lowest quintile)

<table>
<thead>
<tr>
<th>Quintile</th>
<th>New Zealand Data 2009a</th>
<th>No Education, No Durable Contracts</th>
<th>No Education, Durable Contracts</th>
<th>40% Probability</th>
<th>100% Probability</th>
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</thead>
<tbody>
<tr>
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<td>0 – 1</td>
<td>0 – 1</td>
<td>0 – 1</td>
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<td></td>
</tr>
<tr>
<td>2</td>
<td>1 – 2.05</td>
<td>1 – 1.01</td>
<td>1 – 1.008</td>
<td>1 – 1.10</td>
<td>1 – 1.11</td>
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<tr>
<td>3</td>
<td>20.5 – 3.58</td>
<td>1.01 – 1.02</td>
<td>1.008 – 1.013</td>
<td>1.005 – 1.008</td>
<td>1.05 – 1.11</td>
</tr>
<tr>
<td>4</td>
<td>3.58 – 5.47</td>
<td>1.02 – 1.03</td>
<td>1.013 – 1.019</td>
<td>1.008 – 1.013</td>
<td>1.11 – 1.24</td>
</tr>
<tr>
<td>5</td>
<td>&gt; 5.47</td>
<td>&gt; 1.03</td>
<td>&gt; 1.019</td>
<td>&gt; 1.013</td>
<td>&gt; 1.24</td>
</tr>
</tbody>
</table>

Table 3: Wage quintiles by educational attainment

<table>
<thead>
<tr>
<th>Quintile</th>
<th>New Zealand 2009</th>
<th>A. Human capital only</th>
<th>B. Social capital only</th>
<th>C. Social capital with mild skill improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uneraducated</td>
<td>Educated</td>
<td>Uneducated</td>
<td>Educated</td>
</tr>
<tr>
<td>1</td>
<td>27.89</td>
<td>12.76</td>
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<td>2</td>
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<tr>
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<td>18.05</td>
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<tr>
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<td>23.59</td>
<td>17.11</td>
<td>28.20</td>
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<tr>
<td>5</td>
<td>9.84</td>
<td>29.80</td>
<td>10.05</td>
<td>48.26</td>
</tr>
</tbody>
</table>

Source: Statistics New Zealand (2009)
8.2. Figures

Figure 1: Annual Income Distribution (by quintile) of New Zealand Population by Highest Educational Attainment, 2009

![Image of income distribution by quintile and highest educational attainment.](image)

Source: Statistics New Zealand (2009). Annual measures are calculated from weekly data.

Figure 2: Annual Unemployment Rates (%) for New Zealand by Highest Educational Attainment, 1991 – 2008

![Image of unemployment rates by highest educational attainment from 1991 to 2008.](image)

Source: Ministry of Education – New Zealand (2010).
Figure 3: Average Annual Growth Rate of Real GDP Per Capita (PPP Adjusted) 1995 – 2005 and Percentage of Population with No Schooling, Primary Schooling (First Level), Secondary Schooling (Second Level) and Tertiary Schooling for 2000, Highest Educational Attainment

Sources: Barro and Lee (2000) and Heston et al. (2011). Countries represented in the sample include Algeria, Argentina, Australia, Austria, Bahrain, Bangladesh, Barbados, Belgium, Benin, Bolivia, Brazil, Bulgaria, Cameroon, Canada, Central African Republic, Chile, China, Colombia, Costa Rica, Cuba, Cyprus, Czech Republic, Denmark, Dominican Republic, Ecuador, Egypt, El Salvador, Fiji, Finland, France, Germany, Ghana, Greece, Guatemala, Haiti, Honduras, Hong Kong, Hungary, Iceland, India, Indonesia, Iran, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kenya, Republic of Korea, Kuwait, Lesotho, Malawi, Malaysia, Mali, Malta, Mauritius, Mexico, Mozambique, Nepal, Netherlands, New Zealand, Nicaragua, Norway, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Romania, Senegal, Sierra Leone, Singapore, Slovak Republic, Slovenia, South Africa, Spain, Sri Lanka, Swaziland, Sweden, Switzerland, Syria, Taiwan, Thailand, Togo, Trinidad and Tobago, Tunisia, Turkey, Russia, Uganda, United Kingdom, United States, Uruguay, Venezuela.

Notes: The data suggests that annual per-capita real GDP growth is negatively correlated with the percent of the population with no schooling (-0.34) but positively correlated with the percent of the population with primary schooling (0.23), with secondary schooling (0.29) and with tertiary schooling (0.16).

Figure 4: The agent-based model of schooling and employment
Figure 5: No formal education and no durable contracts

Figure 6: Annual Growth Rates of Real GDP per Capita and Unemployment Rates for New Zealand, 1990 - 2009

Source: OECD (2010)
Figure 7: No education and durable contracts – average wages

Figure 8: No education and durable contracts – Beveridge curves

Source: Vacancies: New Zealand Department of Labour [NZDOL] (2007). Employment and Unemployment: OECD (2010). Monthly vacancy data was seasonally adjusted (X-11), then converted to quarterly frequency (averaged). Total openings is calculated as vacancies plus employment, then vacancy rates (vacancies/openings) was computed.
Figure 9: Education – human capital only
Figure 10: Education – social capital only
Figure 11: Education – social capital with mild skill improvement
(A) Human and social capital accumulation with mild skill improvement ($\theta_{pa} = 0.05$)

(B) Human and social capital accumulation with strong skill improvement ($\theta_{pa} = \theta_b = 0.2$)
Figure 13: Marked-up tuition