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Endogenous Labor Supply, Heterogeneous Firms and International Business Cycles*

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Abstract

This paper examines employment dynamics and international business cycle transmission within a two-country dynamic stochastic general equilibrium model featuring an endogenously determined trade pattern. In contrast to existing literature, this model allows for the household's labor supply to be determined endogenously, producing fluctuations in employment as business cycles are transmitted from one country to another. The model is able to generate pro-cyclical domestic employment as well as positive correlations of employment and output across countries. In addition, previous studies have difficulty generating international correlations of consumption and investment. This model replicates these stylized facts by strengthening frictions in international asset markets. The structure of labor supply is shown to be vital for the transmission of business cycles through trade linkages.

Keywords: International Real Business Cycles, Labor Supply, Imperfect Competition, International Trade.

JEL: F12, F16, F21, F41.

1 Introduction

Many would agree that trade and financial linkages are important to the transmission of economic fluctuations across countries. Past international real business cycle (IRBC) models featuring trade in goods and assets have had difficulty generating realistic international correlations of output, consumption, investment and employment. These models¹, which extend the closed economy models of Kydland and Prescott (1982) and Long and Plosser (1983) to multiple countries, assume a fixed pattern of trade. An emerging class² of IRBC models draws upon ideas from trade theory and allows for an endogenously determined trade pattern. Featuring monopolistic competition, love of variety and intra-industry trade, these models allow firms to choose when to enter the market and whether or not to export.

One such model, proposed by Ghironi and Melitz (2005) [GM], has recently become quite popular as a bridge between traditional real business cycle theory and new models of international trade. The GM model reproduces a variety of the empirical regularities seen in international data on multiple levels. For example, the model generates deviations from purchasing power parity, persistent trade deficits, the Harrod-Balassa-Samuelson³ effect and an environment in which only the most productive firms become exporters⁴. Calibrated results suggest that changes in the trade pattern over time are important to business cycle persistence and international co-movement.

While the model is able to match many of the stylized facts associated with international business cycles, it oversimplifies the labor market by assuming inelastic labor supply. Because labor market dynamics are suppressed, the basic model cannot predict how downturns in the economies of our trading partners affect our employment rate (the main concern for policy-makers). Also, since the model is "Ricardian" in spirit⁵, the labor market may affect business cycle co-movement across countries via changes in the determinants of the trade pattern. I contribute to the understanding of international employment dynamics and business cycle correlation by adding a richer theory of the labor market to the original GM framework.

The structure of labor supply is shown to be pivotal in simulating more realistic output volatility, pro-cyclical employment and positive international correlation of employment across countries. A positive shock to aggregate productivity in one country increases output and consumption. There is an increase in the demand for imported goods coupled with a contraction of the domestic export sector. The profitability

¹Mendoza (1991) and Backus, Kehoe, and Kydland (1992, 1994) are seminal in this strand of literature.

²Examples of models in this class include Head (2002), Cook (2002) and Ghironi and Melitz (2005).

³The Harrod-Balassa-Samuelson effect states that prices are generally higher in wealthier (or more productive) countries.

⁴Which is consistent with firm-level data.

⁵In the GM model, countries trade intermediate products for which labor is the only input. Effectively, the countries trade labor services.

of local production rises, leading to firm entry. As firms enter, the demand for labor rises. When wealth effects are absent from labor supply, the model predicts higher wages and pro-cyclical employment. When mild wealth effects are introduced, the productivity shock leads to a reduction in labor supply but an overall increase in wages and employment. Boosts to employment feed back into production, leading to further output gains. Employment is pro-cyclical and output volatility is increased.

Since it becomes more profitable to invest in the country that receives the positive productivity shock, open bond markets lead to an exodus of investment funds out of less productive countries. As foreign agents switch investments towards the purchase of traded bonds, firm construction in their own country falls. Firm attrition in the less productive economy leads to a fall in labor demand, which is partially (but not fully) offset by an expansion of their export sector (due to booming demand abroad). As the home country's export sector contracts, fewer varieties are imported into the foreign country resulting in reduced consumption. When wealth effects are too mild or absent from labor supply, the reduction in labor demand dominates the foreign labor market leading to reductions in wages and employment. Simulated international correlations of employment, output and consumption become low. When stronger wealth effects are imposed, reductions in foreign consumption leads to strong increases in labor supply. Foreign employment rises, which boosts foreign output. Simulated results for the correlation of output and employment across countries are improved.

To strengthen international correlations further, past research suggests imposing asset market frictions. With higher foreign bond adjustment costs, there is less investment exodus to countries that receive favorable productivity shocks. Output, consumption and investment all become more correlated across countries. International correlations are particularly strong when labor supply is endogenous but wealth effects are absent. Positive correlation of consumption that arises from changes in the trade pattern cause a reduction in labor supply when wealth effects are present, reducing employment correlation across countries. Further, since the purchase and sale of foreign assets allow a country to run a trade deficit or surplus, asset market frictions of this sort result in reduced volatility of imports, exports and the trade balance as each country attempts to maintain more balanced trade. Imposing strong asset market frictions improves international correlations at the expense of the international market and potentially the labor market.

The remainder of this paper is as follows. In section 2, I review the data of international business cycles. I construct the model in section 3 and present and discuss a series of numerical experiments in section 4. Section 5 presents a short conclusion.

2 Stylized Facts

When evaluating the success of an international real business cycle model, many researchers refer to the econometric analysis performed by Backus, Kehoe, and Kydland (1992). This analysis looks at quarterly data from 1954q1 to 1983q4 for domestic (U.S.) business cycle statistics and transformed⁶ quarterly data from 1970q1 to 1986q4 for international⁷ business cycle correlations. Stock and Watson (2005) note that both domestic business cycle volatility and the correlation of business cycle movements between the U.S. and several European countries have changed since the 1980s. U.S. business cycles have become more moderate and less synchronized with Europe⁸. Because nearly 20 years of additional data are now available and the interrelationships between business cycle fluctuations have potentially changed, it is worthwhile to update parts of the Backus, Kehoe and Kydland analysis.

I organize the data on international co-movement into three categories: domestic business cycle statistics, international correlations and international market statistics. Table 1 reports domestic business cycle statistics for the United States, 1957q1-2007q1. As is commonly found, consumption, investment and employment are pro-cyclical. Investment is more volatile than GDP, while both consumption and employment are less volatile than GDP. Table 2 reports international business cycle co-movement between the US and Europe, 1970-2004⁹. The statistics suggest that U.S. and European business cycles are positively correlated.

Business cycle co-movement, however, proves to be time-dependant. In the original analysis, Backus, Kehoe and Kydland find that output is more correlated across countries than consumption (with correlation coefficients 0.70 and 0.46 respectively). Many researchers have difficulty constructing a model that can replicate this relationship (dubbed the "consumption-output anomaly"). Table 2 confirms that this relationship is much weaker, as predicted by Stock and Watson (2005), when recent data is considered. Figure 1 shows how the correlation coefficients described in table 2 change over time by looking at 5-year and 10-year rolling window estimates. When looking at the 10-year rolling window for the Backus, Kehoe and Kydland sample (1970 - 1985), output correlation is significantly higher than consumption correlation. However, consumption is more correlated than output in later years (1985 - 2004). In general, we can say that output correlation and consumption correlation are "close" and generally positive.

⁶Deflated quarterly data is transformed into comparable terms by comparing the 1985 annual average to the Penn World Table 1985 value.

⁷Business cycle correlations are between the U.S. and a "European Aggregate" which consists of Austria, Finland, France, Germany, Italy, Switzerland and the United Kingdom.

⁸According to Stock and Watson (2005), this is due to the reduction in "common shocks", i.e. oil price shocks and commodity price shocks

⁹Unlike Backus, Kehoe, and Kydland (1992), the annual data from the Penn World Tables are considered directly.

Table 1:

U.S. Business Cycle Statistics (1957q1-2007q1)			
Volatility (Std. Dev.)		Correlation with GDP	
$\sigma_Y(\%)$	1.54	$\rho_{Y,Y}$	1.00
σ_C/σ_Y	0.75	$\rho_{C,Y}$	0.88
σ_I/σ_Y	3.41	$\rho_{I,Y}$	0.90
σ_L/σ_Y	0.61	$\rho_{L,Y}$	0.81

Table 1: U.S. quarterly data (1957q1-2007q1) for output (Y), consumption (C) and investment (I) is extracted from the International Financial Statistics maintained by the International Monetary fund. Data for investment includes gross fixed capital formation plus changes in inventories. Labor data is generated from civilian employment measures from the OECD.Stat database. Logs of deflated measures are de-trended using the HP filter.

Table 2:

U.S. - European Co-movement (1970-2007)	
Correlation	
ρ_{Y,Y^*}	0.56
ρ_{C,C^*}	0.45
ρ_{I,I^*}	0.37
ρ_{L,L^*}	0.39

Table 2: Annual data for output (Y), consumption (C) and investment (I) for the U.S. and a "European Aggregate" consisting of Austria, Finland, France, Germany, Italy, Switzerland and the United Kingdom are taken from the Penn World Tables maintained by Heston, Summers, and Aten (2006). Annual labor (L) data is taken from the OECD.Stat database and omits Finland due to missing data. Data availability for Germany restricts the sample to 1970-2004. Logs for deflated variables are de-trended using the HP filter.

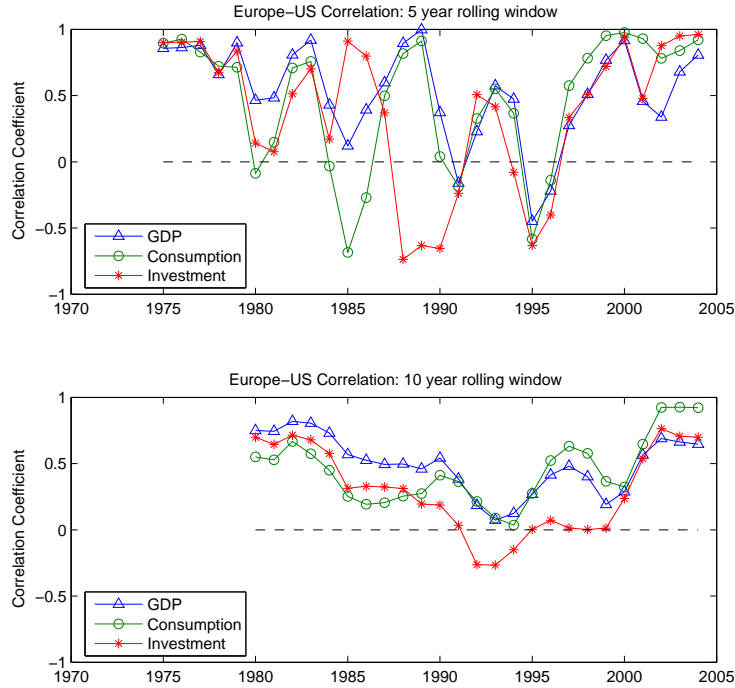


Figure 1: U.S. - European Co-movement (1970-2004) - Rolling Window

Figure 1: At any time, t , the figure plots the international correlations of GDP, consumption, and investment using data from the preceding j years, where $j=5$ and 10 respectively

Table 3:

U.S. International Market Statistics (1957q1-2007q1)			
Volatility (Std. Dev.)		Correlation with GDP	
σ_{IM}/σ_Y	3.31	$\rho_{IM,Y}$	0.6
σ_{EX}/σ_Y	3.74	$\rho_{EX,Y}$	0.28
$\sigma_{NX/Y}$	0.4	$\rho_{NX/Y,Y}$	-0.36
σ_{TOT}/σ_Y	1.69	$\rho_{TOT,Y}$	0.07
σ_Q/σ_Y	3.37	$\rho_{Q,Y}$	-0.18

Table 3: U.S. quarterly data (1957q1-2007q1) for imports (IM), exports (EX) the trade balance relative to output (NX/Y), the terms of trade (TOT) (index) and the real exchange rate (Q) (index) is extracted from the International Financial Statistics database maintained by the World Bank. Logs of deflated measures are de-trended using the HP filter, with exception to the trade balance which is de-trended in levels.

A rich set of statistics pertaining to the international market are also considered. These measures include imports (IM), exports (EX), the trade balance as a fraction of output (Net Exports/Output = NX/Y) the terms of trade (TOT, measured as the ratio of import prices to export prices) and the real exchange rate index (Q) and are reported for the U.S. in table 3. The data suggests high volatility of quantities (imports and exports) as well as prices (the terms of trade and the real exchange rate). Exports and imports are both pro-cyclical, net exports are counter-cyclical and prices are generally a-cyclical. Exports tend to be more volatile than imports, and less correlated with output. This set of data will be used to test how well the model captures the behavior of trade linkages (an important source of business cycle transmission across countries) over the business cycle.

3 The Model

The theoretical model is heavily influenced by Ghironi and Melitz (2005). I diverge, however, from their framework by including endogenously determined labor supply in the household's decision. The model described here considers two countries, home and foreign (denoted by *). Both countries are large and assumed to be structurally identical. That in mind, I construct the framework for one country (the home country) knowing that a symmetric framework exists for the foreign counterpart.

3.1 The Consumer's Problem

The most relevant changes I make to the original framework are in the household's problem. I assume that there exists a representative household that chooses to work, consume and save. A variety of products are available to the household. Denote the universe of intermediate input varieties available to both countries as Ω . At any time, t , a subset $\Omega_t \subseteq \Omega$ is actually available to home consumers. Ω_t contains both domestically produced and imported goods. It need not be the case that $\Omega_t = \Omega_s$ for $t \neq s$ nor $\Omega_t = \Omega_t^*$. Composite consumption, C_t , is produced using Dixit-Stiglitz technology:

$$C_t = \left(\int_{\omega \in \Omega_t} c_t(\omega)^{(a-1)/a} d\omega \right)^{a/(a-1)}$$

where $c_t(\omega)$ denotes the quantity of variety ω used in the production of the composite, and $a > 1$ denotes the elasticity of substitution across varieties. The framework suggests that, ceteris paribus, the amount of composite commodity produced is increasing in the number of available varieties (referred to in the literature as "love of variety" (Dixit and Stiglitz (1977)). This set-up allows me to clearly incorporate imperfect competition on the producer's side of the model. Recent advancements in the trade literature

with respect to imperfect competition and trade patterns make this formulation desirable¹⁰.

Denote $P_t(\omega)$ as the price for variety, ω , and P_{Ct} as the price index for consumption. It is straightforward to construct conditional demand equations for individual varieties, c_t^{11} :

$$c_t(\omega) = C_t(P_t(\omega)/P_{Ct})^{-a} \quad (1)$$

I also construct the associated price index:

$$P_{Ct} = \left(\int_{\omega \in \Omega_t} P_t(\omega)^{1-a} d\omega \right)^{1/(1-a)} \quad (2)$$

The household is endowed with 1 unit of time that it can divide between labor, l_t , and leisure. Labor earns the real wage, w_t . The household can choose to save by purchasing bonds, B_{t+1} , which each cost one unit of consumption, but yield $(1 + r_{t+1})$ units in the next period. Both domestic and foreign bonds (B_*) are available to the household and are subject to a quadratic transaction cost which is rebated, lump-sum (Γ_t), to the household. The household can also purchase shares of a mutual fund, x_{t+1} , which entitle the owner to a fraction of the profits (\tilde{d}_t) of the producing firms. Arbitrage prices the shares at the firms expected discounted value ($N_{ht}\tilde{v}_t$ for new shares and $N_{dt}\tilde{v}_t$ for old shares, where N_{ht} is the number of existing firms, N_{dt} is the number of producing firms, and \tilde{v}_t is the average value of a firm). Households are obligated to hold bonds and stocks for only one period before they are resold in asset markets¹². The consumer's real period budget constraint is then given by:

$$C_t + B_{t+1} + Q_t B_{*t+1} + \frac{n_1}{2}(B_{t+1}^2) + \frac{n_2}{2}(Q_t B_{*t+1}^2) + (N_{ht}\tilde{v}_t)x_{t+1} = \\ w_t l_t + (1 + r_t)B_t + Q_t(1 + r_t^*)B_{*t} + \Gamma_t + x_t(N_{dt}\tilde{v}_t + N_{dt}\tilde{d}_t)$$

where Q_t is the real exchange rate, $n_2 \geq n_1 > 0$ are the scale parameters on foreign and domestic bond adjustment costs respectively, and the lump-sum rebate $\Gamma_t = \frac{n_1}{2}(B_{t+1}^2) + \frac{n_2}{2}(Q_t B_{*t+1}^2)$ in equilibrium. Allowing for differences in domestic and foreign bond adjustment costs allow me to adjust the magnitude of asset market frictions in international bond markets. As n_2 increases, domestic consumers will be less willing to adjust their holdings of foreign bonds which constrains the extent to which a country is willing to finance a trade deficit.

The agent's maximization problem can then be written as:

$$\max_{\{C_s, l_s, B_{s+1}, B_{*s+1}, x_{s+1}\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t} [U_s(C_s, l_s)] \quad s.t. \\ C_s + B_{s+1} + Q_s B_{*s+1} + \frac{n_1}{2}(B_{s+1}^2) + \frac{n_2}{2}(Q_s B_{*s+1}^2) + (N_{hs}\tilde{v}_s)x_{s+1} = \\ w_s l_s + (1 + r_s)B_s + Q_s(1 + r_s^*)B_{*s} + \Gamma_s + x_s(N_{ds}\tilde{v}_s + N_{ds}\tilde{d}_s)$$

¹⁰ Helpman and Krugman (1987) illustrate the emergence of this strand of literature.

¹¹ We solve a series of cost-minimization problems to construct demand equations and price indices.

¹² All savings instruments are denoted with the time subscript in which they yield a return.

I will construct four different formulations for the agent's instantaneous utility function. When considering perfectly inelastic labor supply, I assume the period utility function is of the form¹³:

$$U_t(C_t, l_t) = \log C_t$$

The labor supply equation associated with this utility function is $l_t = 1$, workers contribute their entire time allocation to working regardless of the real wage.

When incorporating endogenously determined labor supply, I will first consider a fairly simple and common¹⁴ utility function:

$$U_t(C_t, l_t) = \log C_t - \frac{l_t^{1+\lambda}}{1+\lambda}$$

This utility function generates a well-known labor supply equation:

$$l_t = C_t^{-1/\lambda} w_t^{1/\lambda} \quad (3)$$

λ is the inverse elasticity of labor supply, which can be calibrated using empirical measures. The presence of the term C_t in the labor supply curve indicates the existence of "wealth effects" on labor supply. An increase in the consumption level, *ceteris paribus*, will induce a leftward shift of labor supply. The marginal rate of substitution associated with this formulation is:

$$\frac{\mu_t}{E_t \mu_{t+1}} = \left(\frac{C_t^{-1}}{E_t C_{t+1}^{-1}} \right) \quad (4)$$

where μ is the shadow price of consumption. A convenient feature of this common utility function is the absence of labor in the marginal rate of intertemporal substitution. Savings and investment decisions are independent of the amount of time spent working, which makes the model relatively easy to solve.

It may be worthwhile to eliminate the wealth effect on the labor supply decision. To do this, I will use the non-separable utility function proposed by Greenwood, Hercowitz, and Huffman (1988) (often referred to as GHH preferences):

$$U_t(C_t, l_t) = \frac{(C_t - A \frac{l_t^{1+\lambda}}{1+\lambda})^{1-\gamma} - 1}{1-\gamma}$$

The labor supply equation and marginal rate of intertemporal substitution are:

$$l_t = \left(\frac{w_t}{A} \right)^{1/\lambda} \quad (5)$$

¹³In their original paper, Ghironi and Melitz use a CRRA utility function: $U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$ with $\gamma = 2$. I have chosen log consumption ($\gamma = 1$) to ensure balanced growth properties of the model when endogenous labor supply is considered.

¹⁴This utility function is chosen to ensure the model exhibits the proper balanced growth properties, as described by King, Plosser, and Rebelo (1988).

$$\frac{\mu_t}{E_t\mu_{t+1}} = \frac{(C_t - A\frac{l_t^{1+\lambda}}{1+\lambda})^{-\gamma}}{E_t(C_{t+1} - A\frac{l_{t+1}^{1+\lambda}}{1+\lambda})^{-\gamma}} \quad (6)$$

C_t is absent from the labor supply equation, indicating there is no wealth effect. However, the amount of labor chosen by the agent affects the rate at which consumption today is traded for consumption tomorrow. Ceteris paribus, an increase in the amount of time spent working today will lead to an increase in the amount of future consumption the agent must receive when reducing consumption today in order to keep utility constant¹⁵. The amount of labor supplied to the market thus affects the agent's willingness to save and invest.

Finally, I will consider the same utility function used by Backus, Kehoe, and Kydland (1992) which includes both a wealth effect and labor in the *MRIS*:

$$U_t(C_t, l_t) = \frac{(C_t^\nu(1-l_t)^{1-\nu})^{1-\gamma}}{1-\gamma}$$

The associated labor supply equation and marginal rate of intertemporal substitution are:

$$l_t = 1 - \frac{1-\nu}{\nu} \left(\frac{C_t}{w_t}\right) \quad (7)$$

$$\frac{\mu_t}{E_t\mu_{t+1}} = \frac{C_t^{\nu(1-\gamma)-1}(1-l_t)^{(1-\nu)(1-\gamma)}}{E_t C_{t+1}^{\nu(1-\gamma)-1}(1-l_{t+1})^{(1-\nu)(1-\gamma)}} \quad (8)$$

The presence of C_t in the labor supply equation indicates the presence of wealth effects. The non-separability of the utility function indicates that the amount of labor supplied to the market affects the marginal rate of intertemporal substitution. This framework is a hybrid of the two proceeding models and is common in the IRBC literature.

To finish the consumer's problem, a series of Euler equations for the assets of the model are generated. The Euler equation for domestic bonds is:

$$\mu_t(1 + n_1 B_{t+1}) = (1 + r_{t+1})\beta E_t \mu_{t+1} \quad (9)$$

Similarly, the Euler equation for foreign bonds is:

$$Q_t \mu_t (1 + n_2 B_{*t+1}) = (1 + r_{t+1}^*)\beta E_t Q_{t+1} \mu_{t+1} \quad (10)$$

The Euler equation for stocks is:

$$\mu_t \tilde{v}_t = \beta(1 - \delta) E_t m_{t+1} (\tilde{d}_{t+1} + \tilde{v}_{t+1}) \quad (11)$$

¹⁵When the marginal rate of intertemporal substitution (*MRIS*) rises, ceteris paribus, the current interest rate is no longer sufficient to compensate the consumer for his savings. The consumer consequently chooses to save less.

3.2 The Firm's Problem

I now consider the problem faced by a typical intermediate good producer in the home country. I assume that there exists an unbounded mass of firms in the economy that may begin production at any time. As in Ghironi and Melitz (2005), these firms are monopolistically competitive and produce for domestic and foreign markets separately (i.e. markets are segmented). Given the demand for their products derived in the previous section, the firm's problem is as follows:

Step 1: Decide whether or not to enter. Step 2: Upon entry, choose how much output to produce and what price to set. Step 3: Death or exit.

In practice, firms will first derive the solutions to step 2 and step 3 before choosing whether or not it's worth it to enter the industry.

Consider a potential domestic producer, j . If firm j decides to enter the market, they must pay a one-time entry fee in terms of effective units of labor. Once the entry cost is paid, they receive a firm-specific productivity draw, z_{jt} . The firm then produces output according to the production function:

$$y_{jt} = Z_t z_j L_{jt}$$

where y_{jt} denotes the quantity of output firm j produces, Z_t denotes an economy-wide technology variable, and L_{jt} is the quantity of labor firm j hires in the production of output. The firm will choose how much to produce for the domestic market (c_{djt}) and how much to produce for export (c_{xjt}). Firms that choose to export must pay a per-period export fee in terms of effective units of labor. Exports are subject to an iceberg cost, $\tau \geq 1$. Thus, total output for the firm is given by $y_{jt} = c_{djt} + \tau c_{xjt}$.

The firm's expenses include a wage bill, a one-time start-up cost, and exporter fees. The wage bill is simply $w_t L_{jt}$. Start-up costs are a fixed cost paid in terms of labor once during the period the firm begins production. The "production" of entry is given by $F_{Et} = Z_t L_{Ejt}$ which suggests a total fixed cost of $w_t F_{Et}/Z_t$ ¹⁶. Exporter fees are paid each period the firm chooses to export ($c_{xjt} > 0$). The production of the exporter fee is given as $F_{Xt} = Z_t L_{Xjt}$. Thus, the per-period exporter cost is $w_t F_{Xt}/Z_t$. Dropping the start-up cost, I form the firm's real instantaneous profit maximization problem:

$$\max_{c_{xjt}, c_{djt}} \frac{\Pi_{jt}}{P_{Ct}} = \frac{P_{djt}}{P_{Ct}} c_{djt} + \epsilon_t \frac{P_{xjt}}{P_{Ct}} c_{xjt} - \frac{w_t}{Z_t z_j} (c_{djt} + \tau c_{xjt}) - I_{xt} \frac{w_t F_{Xt}}{Z_t}$$

where $I_{xt} = 1$ if $c_{xjt} > 0$, else $I_{xt} = 0$. ϵ_t denotes the nominal exchange rate. It is important to note that P_{xjt} is measured in terms of foreign currency. For any variable, X , we denote *real* prices as $\rho_X = P_X/P_C$ and substitute the demand functions from the previous section into the firm's problem (as is standard

¹⁶The presence of a fixed entry fee induces constantly falling average cost, ensuring each firm produces a unique variety.

with monopolistic competition) to generate the first order conditions for the firm¹⁷:

$$\begin{aligned} c_{djt} &= C_t \left(\frac{a}{a-1} \frac{w_t}{Z_t z_j} \right)^{-a} \\ c_{xjt} &= C_t^* \left(\frac{a}{a-1} \frac{w_t}{Z_t z_j} \frac{\tau}{Q_t} \right)^{-a} \end{aligned}$$

where the real exchange rate reflects the relative price of consumption between the home and foreign countries, $Q_t = \epsilon_t P_{C_t}^* / P_{C_t}$. For simplicity, it is assumed that $\epsilon_t = 1$. The demand equations are used to find the optimal real prices charged by the firm:

$$\rho_{djt} = \frac{a}{a-1} \frac{w_t}{Z_t z_j} \quad (12)$$

$$\rho_{xjt} = \frac{a}{a-1} \frac{w_t}{Z_t z_j} \frac{\tau}{Q_t} \quad (13)$$

The prices suggest a constant markup over marginal cost ($w_t / Z_t z_j$).

Profits for the firm can be divided into "domestic production profits" and "export profits". They are given as:

$$d_{djt} = \frac{1}{a} C_t \rho_{djt}^{1-a} \quad (14)$$

$$d_{xjt} = \begin{cases} \frac{1}{a} C_t^* Q_t \rho_{xjt}^{1-a} - \frac{w_t F_{X_t}}{Z_t} & \text{if } c_{xjt} > 0 \\ 0 & \text{if } c_{xjt} = 0 \end{cases} \quad (15)$$

For a continuum of potential firms, there exists some productivity draw, z_{xt} , such that the firm with that draw earns zero profits whether it exports or not (the exporter cutoff). For this firm, $\frac{1}{a} C_t^* Q_t \rho_{xjt}^{1-a} = \frac{w_t F_{X_t}}{Z_t}$.

Without knowing their productivity draw prior to entry, firms form an expectation of their profits before deciding to enter the market. To do this, I employ the "special averages" developed by Melitz (2003). First, it is assumed that the productivity draws follow a Pareto distribution with CDF $G(z) = 1 - (z_{\min}/z)^k$ and PDF $g(z) = k z_{\min}^k z^{-k-1}$ where k denotes the shape parameter of the distribution. Next, I define the special productivity averages as:

$$\begin{aligned} \tilde{z}_d^{1-a} &= \int_{z_{\min}}^{\infty} z_j^{1-a} g(z) dz \\ \tilde{z}_{xt}^{1-a} &= \int_{z_{xt}}^{\infty} z_j^{1-a} \frac{g(z)}{1-G(z_{xt})} dz \end{aligned}$$

Finally, I substitute these definitions in constructing average prices: $\tilde{\rho}_{dt}^{1-a} = \int_{z_{\min}}^{\infty} \rho_{djt}^{1-a} g(z) dz = \left(\frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_d} \right)^{1-a}$ and $\tilde{\rho}_{xt}^{1-a} = \int_{z_{xt}}^{\infty} \rho_{xjt}^{1-a} \frac{g(z)}{1-G(z_{xt})} dz = \left(\frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_{xt}} \frac{\tau}{Q_t} \right)^{1-a}$. We define N_{dt} as the number of firms

¹⁷It is assumed that firms do not observe the impact of price setting on the aggregate price level when solving the firm's maximization problem.

that produce for the domestic market and N_{xt} as the number of producing firms that produce for export. The model is recast in terms of average prices:

$$\tilde{\rho}_{dt} = \left(\frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_d} \right) \quad (16)$$

$$\tilde{\rho}_{xt} = \left(\frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_{xt}} \frac{\tau}{Q_t} \right) \quad (17)$$

$$\tilde{d}_{dt} = \frac{1}{a} C_t \tilde{\rho}_{dt}^{1-a} \quad (18)$$

$$\tilde{d}_{xt} = \frac{1}{a} C_t^* Q_t \tilde{\rho}_{xt}^{1-a} - \frac{w_t F_{Xt}}{Z_t} \quad (19)$$

Completing the integral for the special productivity averages suggests that $\tilde{z}_d = \left(\frac{k}{k+1-a} \right)^{1/(a-1)} z_{\min}$ and $\tilde{z}_{xt} = \left(\frac{k}{k+1-a} \right)^{1/(a-1)} z_{xt}$ where $k > a - 1$ for boundedness. Knowing that all existing firms produce for the domestic market, and a fraction of those become exporters, the expected per-period profit for a potential firm, on average, is:

$$\tilde{d}_t = \tilde{d}_{dt} + (1 - G(z_{xt})) \tilde{d}_{xt} \quad (20)$$

There are two important features guiding the expected value of the firm's lifetime profits. The first is a lag in production. A firm that enters in period t starts producing at period $t + 1$. The entering firm, however, is still *counted* as a firm in period t . The total number of firms that exist at period t , N_{ht} , is given by the number of producing firms that already exist plus the number of new firms, N_{et} .

$$N_{ht} = N_{dt} + N_{et}$$

Second, firms are subject to an exogenous exit shock. The number of firms that "survive" to produce in period $t + 1$ is given by:

$$N_{dt+1} = (1 - \delta) N_{ht} = (1 - \delta) (N_{dt} + N_{et}) \quad (21)$$

Therefore, the expected value of the firm's lifetime profit stream, \tilde{v}_t , is given by:

$$\tilde{v}_t = E_t \sum_{s=t+1}^{\infty} [1 - \delta]^{s-t} \left[\beta^{s-t} \left(\frac{m u_s}{m u_{t+1}} \right) \right] \tilde{d}_s$$

where $\left[\beta^{s-t} \left(\frac{m u_s}{m u_{t+1}} \right) \right]$ is the stochastic discount factor of the household and $(1 - \delta)$ is the firm's survival probability. Notice that repetitive forward substitution of the Euler equation for stocks generates the expected value of the firm's lifetime profit stream. Firms continue to enter as long as the discounted value of their profit stream exceeds the cost of entry. Therefore, the entry cutoff is determined by:

$$\tilde{v}_t = w_t F_{Et} / Z_t \quad (22)$$

3.3 Market Clearing and Equilibrium

I now turn to important market clearing conditions implied by the model. The most important of these conditions for this discussion is labor market clearing. As determined in the consumer's problem, labor supply is given as $l_t = C_t^{-1/\lambda} w_t^{1/\lambda}$ when labor supply is endogenous and $l_t = 1$ when labor is supplied inelastically. Labor demand comes from three sources: production of intermediate inputs, exporter costs and start-up costs. Labor demand for intermediate goods production is derived from the labor requirement curve, the demand equations and average prices. I calculate total labor demand for production of goods as $N_{dt} \tilde{d}_{dt} (\frac{a-1}{w_t}) + N_{xt} \frac{a-1}{w_t} \tilde{d}_{xt} + N_{xt} (a-1) F_{Xt} / Z_t$. Labor demand from the payment of exporter fees is simply $N_{xt} F_{Xt} / Z_t$. Labor demand for the payment of start-up fees is $N_{Et} F_{Et} / Z_t$. Total labor demand is then:

$$L_t = \frac{a-1}{w_t} (N_{Dt} \tilde{d}_{dt} + N_{xt} \tilde{d}_{xt}) + (aN_{xt} F_{Xt} + N_{Et} F_{Et}) / Z_t \quad (23)$$

When labor is supplied inelastically, the labor market clearing condition is:

$$1 = \frac{a-1}{w_t} (N_{Dt} \tilde{d}_{dt} + N_{xt} \tilde{d}_{xt}) + (aN_{xt} F_{Xt} + N_{Et} F_{Et}) / Z_t \quad (24)$$

The equation that governs equilibrium in the labor market when labor supply is endogenously determined depends on the formulation of the utility function. If we denote aggregate labor supply as l_t , the labor market clearing condition is:

$$L_t = l_t \quad (25)$$

The second market clearing condition ensures market clearing in the mutual fund market (shares of the portfolio sum to unity):

$$x_{t+1} = x_t = 1 \quad (26)$$

The third market clearing condition ensures zero net supply of bonds. The representative agent in each country is either a borrower or a lender, but not both. If the agent produces bonds to sell, they must be purchased by agent in the foreign country: $B_{t+1} = -B_{t+1}^*$. Thus, the zero net supply conditions are:

$$\begin{aligned} B_{t+1} + B_{t+1}^* &= 0 \\ B_{*t+1} + B_{*t+1}^* &= 0 \end{aligned} \quad (27)$$

The presence of adjustment costs ensure that the unique steady state bond issuance is zero ($B = 0$)¹⁸.

¹⁸As in Chapter 2, this can be seen by comparing the home agent's Euler equation for home bonds and foreign agent's demand for home bonds at the steady state then applying the zero net supply condition. For the home agent, $\beta(1+r) = (1+nB)$. For the foreign agent, $\beta(1+r) = (1+nB^*)$. The zero net supply condition implies $B = -B^*$. Thus, $(1+nB) = (1-nB) \rightarrow B = 0$.

The final market clearing condition ensures balance of payments. I impose mutual fund market clearing and lump-sum transfers of bond adjustment costs¹⁹:

$$\begin{aligned}
2(1+r_{t+1})B_t + 2(1+r_{t+1}^*)Q_t B_{*t} &= [C_t - Q_t C_t^*] + [N_{et}\tilde{v}_t - Q_t N_{et}^* \tilde{v}_t^*] + 2[B_{t+1} + Q_t B_{*t+1}] \\
&\quad - [w_t l_t - Q_t w_t^* l_t^*] - [N_{dt}\tilde{d}_{dt} - Q_t N_{dt}^* \tilde{d}_{dt}^*] \\
&\quad - [N_{xt}\tilde{d}_{xt} - Q_t N_{xt}^* \tilde{d}_{xt}^*]
\end{aligned} \tag{28}$$

The full equation system that characterizes the model is described in the appendix. 29 to 31 non-linear equations are generated and then log-linearized around a symmetric steady state (denoted by $\hat{\cdot}$)²⁰. In addition to these equations, the stochastic processes for the technology innovations must be specified. Productivity shocks are assumed to take the following VAR form:

$$\begin{aligned}
\begin{bmatrix} \hat{Z}_t \\ \hat{Z}_t^* \end{bmatrix} &= \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} \hat{Z}_{t-1} \\ \hat{Z}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_{Zt} \\ \xi_{Zt}^* \end{bmatrix} \\
E_t(Z_t) &= E_t^*(Z_t^*) = \bar{Z} = 1 \\
\sigma_{\xi_{Zt}} &= \sigma_{\xi_{Zt}^*} = \sigma_\xi \\
\rho_{\xi_{Zt}\xi_{Zt}^*} &= \rho_{\xi_{Zt}\xi_{Zt}^*}
\end{aligned} \tag{29}$$

Additional definitions are also required. Following Ghironi and Melitz (2005), income (GDP) is defined as:

$$Y_t = w_t l_t + N_{dt}\tilde{d}_{dt} \tag{30}$$

Total imports is constructed using the average revenue foreign exporters earn from their sales abroad:

$$IM_t = N_{xt}^* C_t \tilde{\rho}_{xt}^{*1-a} \tag{31}$$

Total exports is constructed using the average revenue domestic exporters earn from their sales abroad:

$$EX_t = Q_t N_{xt} C_t^* \tilde{\rho}_{xt}^{1-a} \tag{32}$$

Terms of trade is defines as the ratio of import prices to export prices:

$$TOT_t = \frac{\tilde{\rho}_{xt}^*}{Q_t \tilde{\rho}_{xt}} \tag{33}$$

Investment is defined as expenditures on new firm entry:

$$I_t = N_{et}\tilde{v}_t \tag{34}$$

¹⁹A description of how the balance of payments equation is generated is in the appendix.

²⁰Since the variable, B_t , has an expected value of zero in equilibrium, it is linearized with respect to steady state consumption: $\hat{B}_t = \frac{B_t - 0}{C} = \frac{B_t}{C}$.

In the model, the price of consumption (P_{Ct}) is measured as a welfare-based price index following Feenstra (1994). This price index accounts for changes in the variety of goods, while the price index in the data does not. It is thus important to transform this welfare-based index into one which closer matches the price index calculated in the data. To do so, average prices are substituted into the definition of the consumption price index:

$$1 = [N_{dt}\tilde{\rho}_{dt}^{1-a} + N_{xt}^*\tilde{\rho}_{xt}^{*1-a}]^{1/(1-a)}$$

Note that the price index is sensitive to the number of varieties. If we assume that all prices are the same on average, $\tilde{\rho} = \tilde{P}_t/P_{Ct}$, we construct:

$$\frac{P_{Ct}}{\tilde{P}_t} = [N_{dt} + N_{xt}^*]^{1/(1-a)} \quad (35)$$

Any variable measured in terms of real consumption (a "welfare-based" variable), X_t , is adjusted to this index: $\tilde{X}_t = \frac{P_{Ct}X_t}{\tilde{P}_t}$. Further, since the real exchange rate is constructed using the welfare-based price indices ($Q_t = P_{Ct}^*/P_{Ct}$), an adjusted real exchange rate is constructed with average prices:

$$\tilde{Q} = \tilde{P}_t^*/\tilde{P}_t = Q_t \left[\frac{N_{dt} + N_{xt}^*}{N_{dt}^* + N_{xt}} \right]^{1/(1-a)} \quad (36)$$

4 Numerical Experiments

The model must be calibrated before generating the set of simulated data needed to evaluate the model's fit to the real data. I start by employing a calibration that most closely matches the original Ghironi and Melitz (2005) framework. Many parameter values (reported in table 4) appear in past literature and are widely accepted. The values for the discount factor (0.99 for quarterly simulations, 0.96 for annual simulations) is standard in the literature. The values for the probability of death are calibrated to match data on U.S. job destruction (2.5% quarterly, 10 % annually). The elasticity of substitution between intermediate inputs, the shape parameter on the Pareto distribution, the lower bound on the Pareto distribution, the iceberg cost on intermediate inputs and the costs of entry and export are set to match Ghironi and Melitz (2005).

The productivity process is calibrated as²¹:

²¹Backus, Kehoe, and Kydland (1992) calibrate technology shocks as a VAR process with spillovers:

$$\begin{aligned} \begin{bmatrix} \hat{Z}_t \\ \hat{Z}_t^* \end{bmatrix} &= \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix} \begin{bmatrix} \hat{Z}_{t-1} \\ \hat{Z}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_{Zt} \\ \xi_{Zt}^* \end{bmatrix} \\ \sigma_{\xi_{Zt}} &= \sigma_{\xi_{Zt}^*} = \sigma_{\xi} = 0.00852 \\ \rho_{\xi_{Zt}\xi_{Zt}^*} &= \rho_{\xi_Z\xi_Z^*} = 0.258 \end{aligned}$$

Baxter and Crucini (1995) note that this specification is not statistically different from a near-unit-root process without

Table 4: Calibrated Parameters

Parameter		Value
β	Discount factor	0.99, 0.96
δ	Probability of death shock	0.025, 0.10
τ	Iceberg costs associated with trade	1.3
a	Elasticity of substitution between inputs	3.8
k	Shape parameter on Pareto distribution	3.4
z_{min}	Lower bound on Pareto distribution	1
\tilde{z}_d	"Special" average productivity draw	$z_{min}(\frac{k}{k+1-a})^{1/(a-1)}$
F_{Et}	Fixed cost of entry	1
F_{Xt}	Fixed exporter cost	$0.235(F_{Et}(\frac{1-\beta(1-\delta)}{\beta(1-\delta)}))$

$$\begin{aligned}
\begin{bmatrix} \hat{Z}_t \\ \hat{Z}_t^* \end{bmatrix} &= \begin{bmatrix} 0.99 & 0 \\ 0 & 0.99 \end{bmatrix} \begin{bmatrix} \hat{Z}_{t-1} \\ \hat{Z}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_{Zt} \\ \xi_{Zt}^* \end{bmatrix} \\
\sigma_{\xi_{Zt}} &= \sigma_{\xi_{Zt}^*} = \sigma_{\xi} = 0.00852 \\
\rho_{\xi_{Zt}\xi_{Zt}^*} &= \rho_{\xi_Z\xi_Z^*} = 0.258
\end{aligned} \tag{37}$$

The other model parameters, $\{n_1, n_2, \lambda, \nu, \gamma\}$ will be thought of as "free parameters" and will be calibrated under different experiments discussed shortly.

The linearized equation system is solved numerically using the brute force algorithm developed by Uhlig (1999). Simulations are performed in order to generate a set of statistics to compare to the data. In each experiment, a 200-period economy is simulated 200 times by drawing a random vector of innovations. During each simulation, I make the appropriate price adjustments as described above, then apply the HP-filter²². I calculate each summary statistic (volatility, correlation, etc.) for each simulation and then report the average statistics across simulations in the result.

4.1 The Benchmark

To match the original Ghironi and Melitz (2005) framework as closely as possible, I set $n_1 = n_2 = 0.0025$ and omit endogenous labor supply from the model. Simulation results for the adjusted variables are reported in column (1) of table 5. The fundamental shortcomings of the model are under-predicted output

spillovers

²²Since the model exhibits a high degree of persistence, we take out the low-frequency trend using the HP-filter

volatility, low correlation of output and consumption across countries, low volatilities of international market quantities (imports and exports) and prices (terms of trade and the exchange rate) and strong correlation of international prices with GDP. The model also produces strong, negative correlation of investment across countries²³.

To understand why the model poorly replicates international co-movement, I inspect the impulse response functions²⁴ for the endogenous variables of the model. In doing so, it is important to note that intuition regarding the behavior of the economy is gleaned from the welfare-based variables while the statistical performance of the model is based on the tilde-variables (which have undergone adjustment for the construction of the aggregate price index)²⁵.

When the home country experiences a positive shock to aggregate productivity (Z), output and profits for existing firms immediately rise. This drives up the discounted value of an average firm's profit stream and thus the return to mutual fund shares, spurring investment in new firm construction. With increased production by domestic firms, composite consumption rises. At the aggregate level, there is more demand for labor to pay start-up costs and to expand production, putting upward pressure on wages. Over time, new firm entry contributes to an expansion of the product variety, inducing persistent increases in consumption, output and wages. As increases to the real wage overpower the productivity shock, the marginal cost of production rises leading to increases in the domestic price level.

The economic expansion immediately increases the demand for imports, which drives up the average price for imported goods (total imports increase). The increase in productivity expands production in the export sector, contributing to an immediate fall in export prices. As exporting becomes less profitable, the size of the home country's export sector contracts.

As the return to mutual fund shares in the home country rises, the return to home bonds does as well via the Euler equation. Foreign agents adjust their asset portfolios in favor of home bonds which now yield a higher return, reducing investment in new firm construction in their own country ("investment exodus"). On one hand, less labor is demanded for new firm construction. On the other hand, more labor is demanded to expand production in the foreign export sector to satisfy increased import demand in the home country. The model predicts an overall decline in labor demand and a fall in the real

²³Note that "investment" in the model is restricted to new firm construction, while "investment" in the data is capital purchases plus inventory changes. Because capital is omitted from the framework, it is not possible to effectively compare the model's fit to the investment data.

²⁴The full set of impulse response functions for the endogenous variables of the model are available upon request. A subset of relevant impulse response functions are available in the appendix.

²⁵For sufficiently powerful changes to aggregate prices or the real exchange rate, small changes in the welfare-based variables can be statistically reversed while simulating the data. Hence, the intuition gleaned from the impulse response functions may not exactly match the simulated results used to evaluate the model's fit.

wage. Output and consumption decline in the foreign country and business cycles across countries become contemporaneously negatively correlated. Any positive relationship across countries that occurs in simulation is driven by the underlying correlation of the productivity innovations and changes in the real exchange rate used to generate comparable measures.

Although the home country's demand for imports remains high for several periods, the increase in demand drops off quickly as new varieties are produced in the home country. Also, as the variety of home products increases, a larger variety of goods are exported to the foreign country despite low export prices. The overall value of exports for the home country increases over time. With a larger variety of goods eventually coming into the country, foreign consumers experience a boost in consumption over time. The effect of an increase in labor demand in the foreign export sector overpowers the effect of a decline in labor demand for new firm construction over time, and overall wages in the foreign country eventually rise. Foreign national income and consumption increase, which induces a re-expansion in the home export sector, furthering the persistence of the positive shock in the home country. The international market in the model allows the home country business cycle to "lead"²⁶.

Since import prices rise while export prices fall in the home country in the presence of a positive productivity shock, the terms of trade rises. Import prices remain high while export prices rebound during the delayed expansionary period in the foreign country; the terms of trade declines quickly before smoothly returning to the steady state. Upon receiving a positive productivity shock, the home country begins to run a trade deficit. As the export sector expands, the trade deficit declines. When the foreign country enters their expansionary phase, the home country runs a trade surplus.

In the home country, increases in domestic prices along with increases in export prices push up the aggregate price level while increase in the foreign export sector pushes the aggregate price level down. In the foreign country, decreases in prices for domestically produced goods along with falling import prices push the price level down while a contraction of the home export sector pushes the price level up. The model predicts the impact of the changing trade pattern is dominate, leading to an increase in the real exchange rate ($Q_t = P_{Ct}^*/P_{Ct}$). Over time, increases in the domestic variety of goods and foreign firm attrition lead to further increases in the exchange rate. Since foreign variables are multiplied by the real exchange rate to make foreign measures comparable, changes in the exchange rate affect the degree of business cycle correlation. Increases in the real exchange rate induce statistical increases in foreign output and consumption in response to a positive productivity shock. Exchange rate changes are not powerful enough to statistically reverse the fall in foreign investment. Also, increases in the exchange

²⁶When one country has an expansion, the other has an immediate contraction followed by an expansion over time (in excess of recovery).

rate make exports even less profitable to the home country, contributing to the contraction of the home export sector and the decline of domestic exports.

4.2 Endogenous Labor Supply

Previous real business cycle literature suggests that endogenously determined labor supply has the potential to increase the volatility of output if employment is pro-cyclical. Since the two countries described in the model essentially trade labor services, changes in the labor market can also affect the pattern of trade, altering the international correlation of business cycles. I turn to the set of models that incorporate the consumer's labor decision and evaluate the success of models with an improved labor market.

4.2.1 Separable Consumption and Labor

In the model with separable consumption and labor ($U(C_t, l_t) = \log C_t - \frac{l_t^{1+\lambda}}{1+\lambda}$), some care must be taken when calibrating the elasticity of labor supply (λ). A variety of elasticities have been found in the empirical literature both at the micro and the macro level. I calibrate $\lambda = 1$ to match the empirical findings of Fiorito and Zanella (2008). I again consider low bond adjustment costs ($n_1 = n_2 = 0.0025$) and report the results in column (2) of table 5.

At first glance, the volatility of output (1.17) is improved. Employment in the model is pro-cyclical (0.97) and the international correlation of employment is appropriate (0.52). I inspect the impulse response functions for the endogenous variables of the model to glean intuition behind the improved results. The effect of a positive productivity shock on the domestic economy when labor is supplied inelastically has already been discussed: more labor is demanded in the home country, driving up the real wage (shown in diagram a of figure 2: labor demand shifts from LD_1 to LD_2), while in the foreign country less labor is demanded immediately after the shock due to firm attrition and ore labor is demanded in future periods due to export expansion. When the labor supply decision is endogenous, the increase in composite consumption from firm entry (more variety) and expansion (more goods produced) in the home country compels households to supply less labor at the current real wage. This is clear from the first order condition governing labor supply when $\lambda = 1$ ($l_t = w_t/C_t$) and is shown in graph a in figure 2: the labor supply curve shifts from LS_1 to LS_2 in response to a positive productivity shock. The increase in labor demand resulting from the positive productivity shock offsets the decline in labor supply, resulting in more employment and higher wages in equilibrium. The boost in employment further increases aggregate output, leading to the higher simulated output volatility seen in the results table.

The positive international correlation of employment is due directly to trade and financial linkages.

Table 5: Simulation Results

n_2 small	Data	(1)	(2)	(3)	(4a, $\gamma = 1$)	(4b, $\gamma = 2$)
Domestic Statistics						
Volatility						
$\sigma_Y(\%)$	1.54	0.92	1.17	1.00	2.07	2.11
σ_C/σ_Y	0.75	0.63	0.56	0.72	0.82	0.90
σ_I/σ_Y	3.41	3.91	4.13	3.48	3.54	3.30
σ_L/σ_Y	0.61	X	0.28	0.01	0.56	0.56
Correlation with GDP						
$\rho_{C,Y}$	0.88	0.99	0.99	0.99	0.99	0.99
$\rho_{I,Y}$	0.90	0.94	0.95	0.91	0.89	0.84
$\rho_{L,Y}$	0.81	X	0.98	0.99	0.99	0.99
International Co-movement						
ρ_{Y,Y^*}	0.56	0.16	0.18	0.16	0.04	0.04
ρ_{C,C^*}	0.45	0.05	0.08	0.16	-0.03	0.07
ρ_{I,I^*}	0.37	-0.43	-0.31	-0.65	-0.68	-0.83
ρ_{L,L^*}	0.39	X	0.52	0.19	0.09	0.07
International Market						
Volatility						
σ_{IM}/σ_Y	3.31	0.79	0.72	0.90	1.02	1.13
σ_{EX}/σ_Y	3.74	0.84	0.77	0.93	1.05	1.18
$\sigma_{NX/Y}$	0.40	0.25	0.30	0.29	0.70	0.80
σ_{TOT}/σ_Y	1.69	0.30	0.27	0.32	0.35	0.40
σ_Q/σ_Y	3.37	0.07	0.06	0.07	0.08	0.09
Correlation with GDP						
$\rho_{IM,Y}$	0.60	0.89	0.87	0.89	0.90	0.91
$\rho_{EX,Y}$	0.28	0.17	0.18	0.23	0.19	0.18
$\rho_{NX/Y,Y}$	-0.36	-0.48	-0.45	-0.47	-0.51	-0.51
$\rho_{TOT,Y}$	0.07	0.58	0.57	0.59	0.60	0.60
$\rho_{Q,Y}$	-0.18	-0.58	-0.55	-0.60	-0.60	-0.62

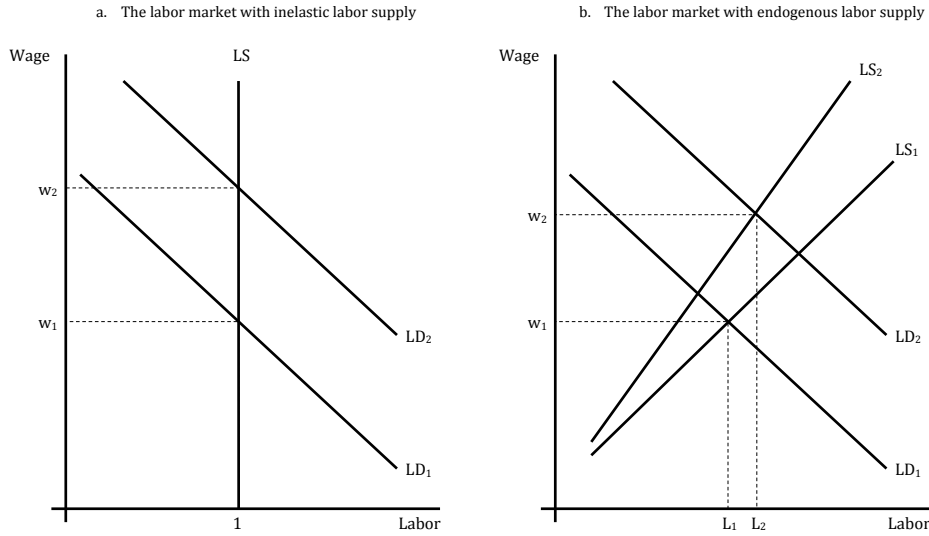


Figure 2: Labor Markets with Inelastic versus Endogenous Labor Supply: Positive Productivity Shock

As previously described, a positive productivity shock in the home country leads to firm attrition in the foreign country due to open asset markets. In foreign labor markets, there is a slight decline in the demand for labor (as seen in graph b of figure 3: labor demand shifts from LD_1^* to LD_2^*). Because the foreign economy has started to contract (C_t^* is falling), foreign households increase their supply of labor according to the first order condition governing foreign labor supply ($l_t^* = w_t^*/C_t^*$). This is shown in graph b of figure 3: the foreign labor supply curve shifts from LS_1^* to LS_2^* . The slight decline in labor demand is offset by the increase in labor supply, resulting in increased foreign employment at lower wages. There is positive international correlation of employment. Further, the model predicts an increase in overall income which induces positive output correlation across countries. While output and employment become more positively correlated across countries due to the response of the labor market to fluctuations in trade that occur during the business cycle, the correlation of consumption and investment across countries found in simulation remains due to the underlying correlation of the productivity innovations.

As the effects of the productivity shock on the home country wear off, employment smoothly returns to its steady state level. In the foreign country, however, workers shift from supplying a high level of labor during their recessionary period to supplying low levels of labor as the economy enters and expansionary

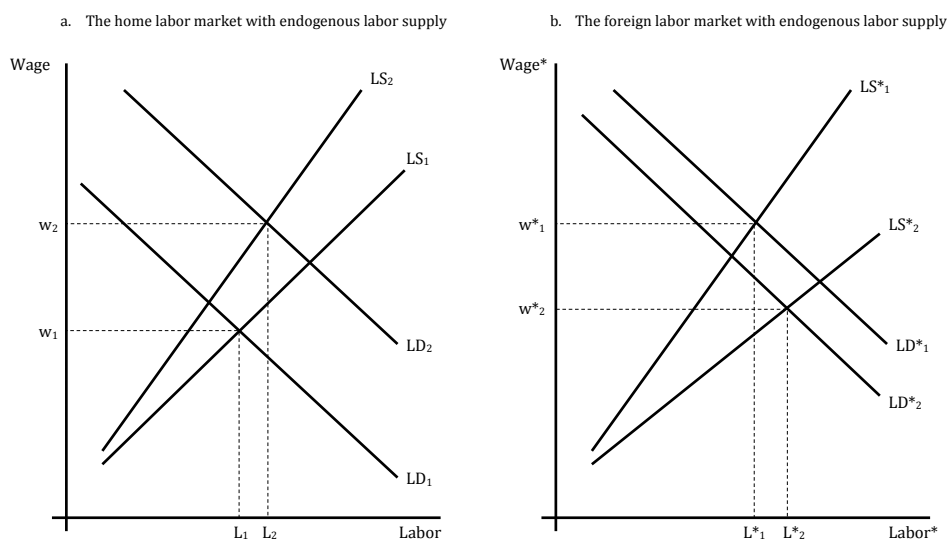


Figure 3: Home versus Foreign Labor Markets with Endogenous Labor Supply: Positive Productivity Shock

period²⁷.

Increasing the elasticity of labor supply can result in improved output volatility with little to no adverse effects on the rest of the model. Following Guo and Sturzenegger (1998), I increase the elasticity of labor supply to 4 ($\lambda = 0.25$)²⁸. Output volatility increases to 1.74. The international correlation of output and consumption are 0.22 and 0.17 respectively. The international correlation remains negative, but is less so (at -0.17). Correlation between employment and output improves (to 0.53). The more elastic labor supply curve generates a larger increase in employment in the home country when there is a positive productivity shock, raising the volatility of output and the correlation between employment and output. International correlation of output and consumption are improved statistically as a result of more mild changes in the foreign welfare-based variables.

²⁷The impulse response function for foreign employment is "humped-shaped" with an increase immediately after the shock followed by a decline below the steady state level during the expansionary phase before returning to the steady state

²⁸Full results available on demand.

4.2.2 Non-Separable Consumption and Labor

There are two reasons why we may want to consider versions of the model in which consumption and labor are "non-separable" in the consumer's problem. Although employment in the model with endogenous labor is pro-cyclical, the strength of the correlation between employment and output is low in simulation (0.28) compared with the real data (0.61). In the presence of productivity shocks, the "wealth effect" that is incorporated into the first order condition governing labor supply weakens the pro-cyclical behavior of employment. Non-separable preferences over consumption and labor of the GHH form can eliminate this wealth effect and potentially allow for stronger correlations between employment and output.

Also, with separable preferences over consumption and labor, the level of labor supplied to the market does not impact the marginal rate of inter-temporal substitution (*MRIS*). By including labor in the *MRIS*, it may be possible to offset the exodus of investment out of the foreign country. If employment is positively correlated across countries and an increase in foreign employment in response to a positive productivity shock in the home country induces a sufficiently large increase in the *MRIS*, a corresponding rise in foreign interest rates may reduce investment exodus. Correlations of output, investment and consumption across countries would then be improved.

I start with a version of the model that contains both wealth effects and labor supply in the *MRIS*. Simulation results for the model with preferences consistent with Backus, Kehoe, and Kydland (1992), a utility function used quite commonly in the literature, are reported in column (2) of table 5. In this experiment, calibrate the inverse intertemporal elasticity of substitution to 2, a common choice in the macroeconomics literature. This parameterizes $\gamma = \frac{1+\nu}{\nu}$. I differ slightly from the original Backus, Kehoe, and Kydland (1992) and set ν such the steady state level of employment in this version of the model is the same as it is in the framework with separable consumption and leisure described in the previous section ($\nu = 0.9658$). The other model parameters are calibrated as described above.

At first glance, it is clear that the model's fit worsens. Output volatility, while improved over the benchmark model, is not as high as it is when leisure and consumption are separable. Correlation of consumption across countries improves slightly, but correlation of investment across countries worsens dramatically even compared to the benchmark framework. The volatility of employment is very low relative to output. Further, correlation of employment across countries is low relative to the data.

The domestic business cycle in the presence of a positive productivity shock is qualitatively the same under the BKK specification of the utility function as it was under the model with separable consumption and leisure (as described above). The response of labor supply to a change in the consumption level, however, offsets the boost in labor demand that accompanies the productivity shock, lowering both the

correlation between employment and output as well as output volatility. The home country experiences a net increase in output, employment, consumption and import demand and investment along with a contraction in the export sector.

In the foreign country, the reduction in the number of imported varieties is relatively small. Foreign consumption changes negligibly in response to a positive shock to the home country, resulting in only a small reduction in labor supply. Firm attrition in the foreign country, however, continues to lower labor demand, leading to a fall in equilibrium employment. A fall in the exchange rate that occurs with a contracting export sector in the home country and an expanding export sector in the foreign country worsens the simulation correlations. Only the underlying correlation of the productivity innovations induces positive correlation of output, employment, and consumption.

I now remove the wealth effect all together and consider the case of GHH preferences. I calibrate the inverse of labor supply elasticity, $\lambda = 1$. I set the scale parameter A such that the steady state employment level matches that of the model with separable consumption and leisure. As in Greenwood, Hercowitz, and Huffman (1988), I consider two values of the parameter $\gamma = 1, 2$. The rest of the parameters are calibrated as described above. I report the results in column 4a and 4b of table 5.

Output volatility and correlation between employment and output are much higher when labor supply is not subjected to wealth effects. A positive productivity shock leads to a strong increase in equilibrium employment and a corresponding jump in out as shown in 4. The business cycle in the home country is qualitatively similar as in the case with separable leisure and consumption. In addition, the volatility of imports and exports are high relative to the model with separable consumption and leisure which are difficult stylized facts to improve upon. Serious drawbacks to this framework, however, are very low international correlations.

The source of low international correlation lies in the labor market. A positive productivity shock to the home country induces an increase in import demand and a contraction of the home export sector. The number of available varieties offered to the foreign country falls, leading to a fall in the level of foreign consumption. There is no reaction by labor supply since the wealth effect has been eliminated. Investment exodus leads to firm attrition which results in a reduction in labor demand. The foreign country experiences a fall in employment and therefore output in equilibrium. It becomes even less attractive in build firms in the foreign country, causing further investment exodus and reductions in consumption. International correlation between output, consumption and employment are driven by the underlying correlation of the productivity innovations. International correlation of investment worsens.

The experiments described so far bring to like a few key facts. First, endogenous labor supply is vital for matching output volatility and stylized facts related to employment. Second, models with separable

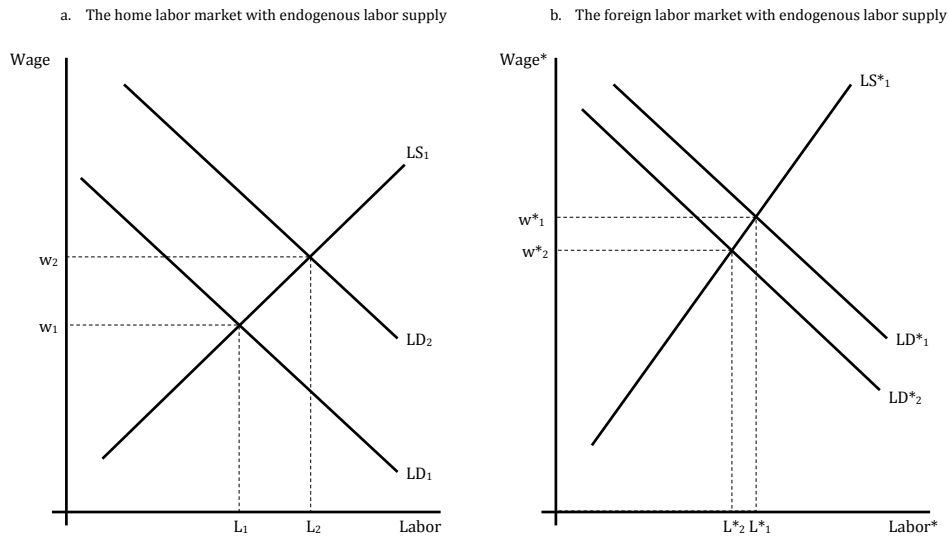


Figure 4: Home versus Foreign Labor Markets with GHH Preferences: Positive Productivity Shock

labor and consumption in the utility function generate mild but positive international correlation of output, consumption and employment while models with non-separable preferences do not. Third, GHH preferences do an improved job of capturing the stylized facts related to domestic business cycles and international market activity.

4.3 Asset Market Frictions

So far, it seems like we can't always get what we want. Preferences that omit wealth effects on labor supply (GHH) do a good job of matching domestic business cycle statistics, are worse than preferences with separable labor and consumption at matching the international data. When reviewing table 5 as a whole, it appears that the international data is never really captured adequately. The next chore I tackle is improving the international correlations when labor is endogenously supplied.

Investment exodus via open bond markets is the source for negatively correlated business cycles across countries. To reduce the desire of foreign investors to buy home bonds when the home country experiences a positive shock (and vice versa), I increase the costs associated with adjusting bond holdings. The parameter, $n_2 > 0$ is the scale parameter associated with these costs. In the exercises above, this

value was small ($n_2 = 0.0025$). I now consider the case of high bond adjustment costs. I set $n_2 = 1$ and repeat the above experiments. Results are reported in table 6.

Increasing the costs on foreign bond adjustment does not adversely affect the domestic business cycle statistics produced by the benchmark model and the three models of endogenous labor supply. In fact, the results for output, consumption, investment and employment volatility reported in table 6 are quite similar to those reported in table 5.

Large improvements are made to the international correlations when bond adjustment costs are high. When the home country experiences a positive productivity shock, there is an increase in output, investment, consumption and import demand. When foreign bond adjustment costs are high, and it is expensive to borrow from abroad to finance imports, an increase in export supply and an expansion of the home export sector attempts to keep the trade deficit low. A larger variety of goods are exported to the foreign country, increasing the value of foreign aggregate consumption. Foreign investors are less willing to purchase home bonds when the costs of doing so are high, reducing the amount of foreign firm attrition. An expansion in the foreign export sector increases the demand for labor more than the small amount of firm attrition reduces it, leading to an increase in wages and income. Output and consumption become more positively correlated across countries. Investment becomes much less negatively correlated across countries²⁹.

Some distortion, however, is introduced into the international market in the presence of restricted international borrowing. When trade is more balanced, imports and exports respond to a shock to aggregate productivity similarly. The correlation between imports and GDP is 0.60 in the data while the correlation between exports and GDP is 0.28. When bond adjustment costs are high, the correlation between imports and GDP and exports and GDP is approximately 0.82 for both statistics in simulation. The tendency to keep trade more balanced also results in low volatility of the trade balance (0.02) compared with the empirical facts (0.40). Matching some of the stylized facts of the international market must be sacrificed when attempting to improve international correlations using asset market frictions.

When labor supply is endogenous, the presence of wealth effects also results in a distortion in the labor market when foreign bond adjustment costs are high. In both the model with separable consumption and leisure and the model with BKK preferences (columns (2) and (3) of table 6 respectively), there is a reduction in the international correlation of employment across countries as business cycles become more correlated. In both countries, consumption rises when the home country experiences a positive productivity shock inducing a fall in labor supply. In the home country, however, the positive shock

²⁹The underlying correlation of the productivity innovations is responsible for the mildly positive international correlation of investment in many of the simulations.

Table 6: Simulation Results

n_2 large	Data	(1)	(2)	(3)	(4, $\gamma = 1$)	(4, $\gamma = 2$)
Domestic Statistics						
Volatility						
$\sigma_Y(\%)$	1.54	0.97	1.23	1.01	1.95	2.02
σ_C/σ_Y	0.75	0.63	0.57	0.72	0.79	0.90
σ_I/σ_Y	3.41	3.11	3.49	2.45	2.35	1.79
σ_L/σ_Y	0.61	X	0.28	0.01	0.54	0.53
Correlation with GDP						
$\rho_{C,Y}$	0.88	0.99	0.98	0.99	0.99	0.99
$\rho_{I,Y}$	0.90	0.98	0.98	0.99	0.99	0.99
$\rho_{L,Y}$	0.81	X	0.97	0.99	0.99	0.99
International Co-movement						
ρ_{Y,Y^*}	0.56	0.28	0.26	0.30	0.35	0.34
ρ_{C,C^*}	0.45	0.34	0.31	0.37	0.41	0.42
ρ_{I,I^*}	0.37	0.07	0.08	-0.04	0.03	-0.25
ρ_{L,L^*}	0.39	X	0.23	0.16	0.38	0.37
International Market						
Volatility						
σ_{IM}/σ_Y	3.31	0.58	0.51	0.67	0.78	0.90
σ_{EX}/σ_Y	3.74	0.57	0.50	0.68	0.77	0.90
σ_{NX}/σ_Y	0.40	0.01	0.02	0.02	0.07	0.09
σ_{TOT}/σ_Y	1.69	0.12	0.11	0.13	0.13	0.15
σ_Q/σ_Y	3.37	0.08	0.09	0.10	0.10	0.10
Correlation with GDP						
$\rho_{IM,Y}$	0.60	0.82	0.81	0.83	0.85	0.86
$\rho_{EX,Y}$	0.28	0.80	0.79	0.81	0.82	0.82
$\rho_{NX/Y,Y}$	-0.36	-0.22	-0.21	-0.21	-0.13	-0.20
$\rho_{TOT,Y}$	0.07	0.55	0.54	0.60	0.55	0.57
$\rho_{Q,Y}$	-0.18	0.41	0.40	0.49	0.48	0.52

induces a large increase in labor demand overall raising employment and wages. In the foreign country, the increase in labor demand is smaller and not sufficient to offset the fall in labor supply. Any positive correlation of employment is driven by the underlying correlation of the productivity innovations. When wealth effects are absent, as with GHH preferences, there is no adverse effect on labor supply and employment correlation across countries improves.

5 Conclusion

The Ghironi and Melitz (2005) model is capable of capturing many features of international business cycles. By assuming labor is supplied to the market inelastically, the model cannot account for international employment dynamics. Incorporating endogenous labor supply increases the volatility of output to a realistic level. Pro-cyclical employment and positive international correlation of employment are successfully reproduced by the model. Further, changes in labor supply improve the international correlations of output, consumption and investment.

Past researchers suggest that the correlation of business cycles produced by the model can be further improved by slowing the rate of investment exodus from one country to the other through asset market frictions. By increasing costs associated with purchasing foreign bonds, the model can in fact generate stronger correlations of output, consumption and new firm construction. However, this is achieved at the expense of the international market and, in some cases, the labor market. Increased asset market frictions boost the volatility of exports and reduce the volatility of the trade balance as countries try to maintain more balanced trade. Changes in labor supply when strong bond market frictions are present induces a reduction in employment in the foreign country, worsening the correlation of employment across countries.

Future work includes incorporating unemployment into the model. When I test whether or not the framework with the developed labor market can accommodate involuntary unemployment as in Hansen (1985) and Rogerson (1988)³⁰, I find that output volatility, the international correlation of output, investment and employment and the correlation between exports and GDP all severely worsen. Since the transmission of job loss is of concern to policy makers, it is important to include a concept of unemployment in the model without distorting the international correlations and disturbing the international market.

Other potential extensions to the model include adding multiple countries and "common shocks", which would likely result in stronger international correlations without having to sacrifice international

³⁰Results for this model are reported in the appendix.

market volatility. Including capital would provide a more accurate measure of investment, potentially resolving the problem of negatively correlated investment across countries. The inclusion of demand shocks for final goods may lead to increased volatility of imports, exports and international prices. These are left for future research.

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A Appendix

A.1 Impulse Response Functions

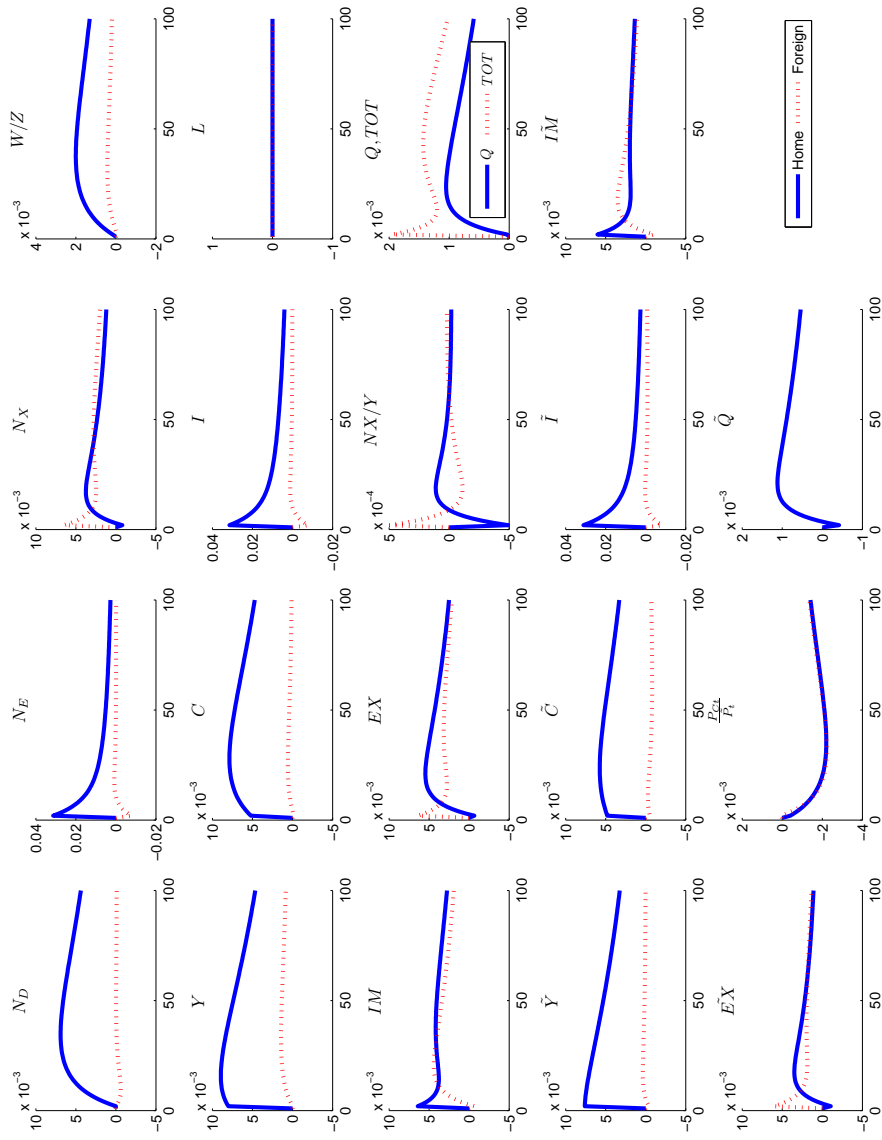


Figure 5: Impulse Response Functions - Benchmark Model - $n_2 = 0.0025$

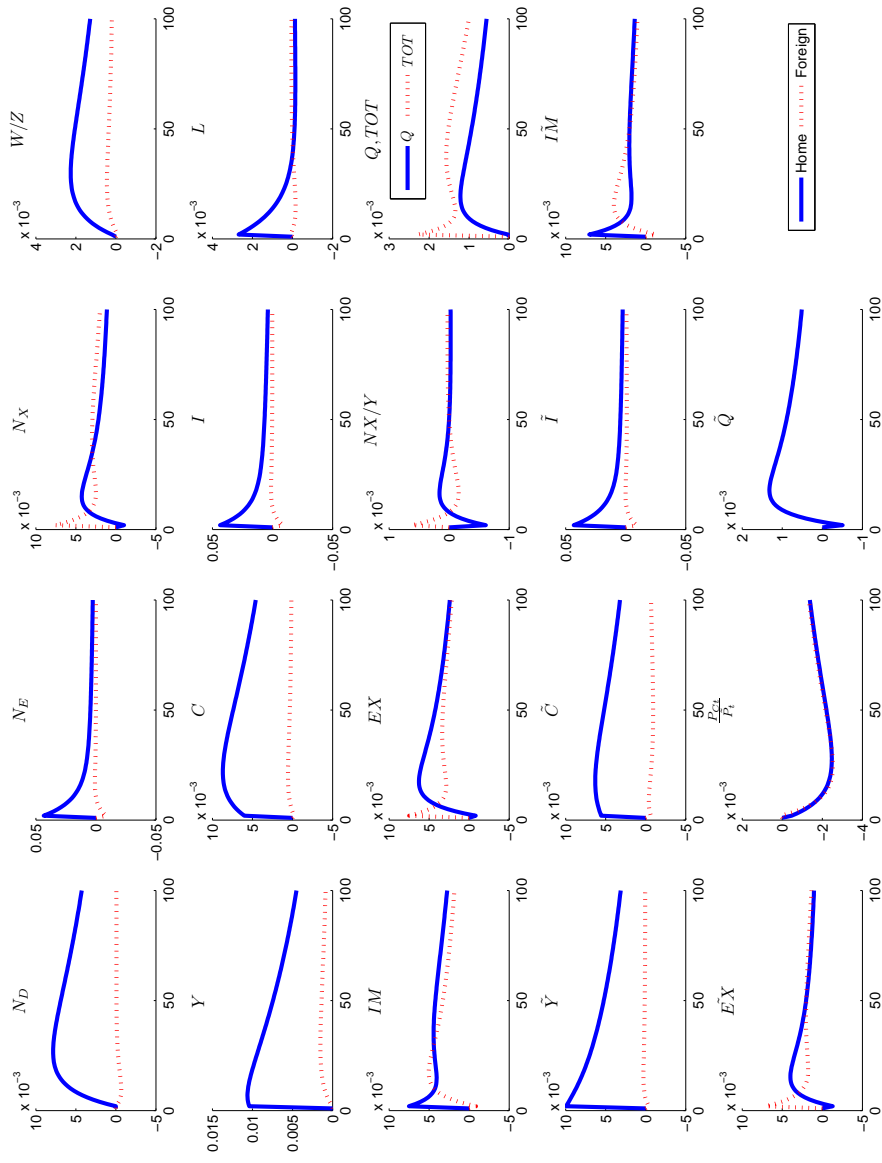


Figure 6: Impulse Response Functions - Endogenous Labor Supply with Separable Preferences for Consumption and Leisure - $n_2 = 0.0025$

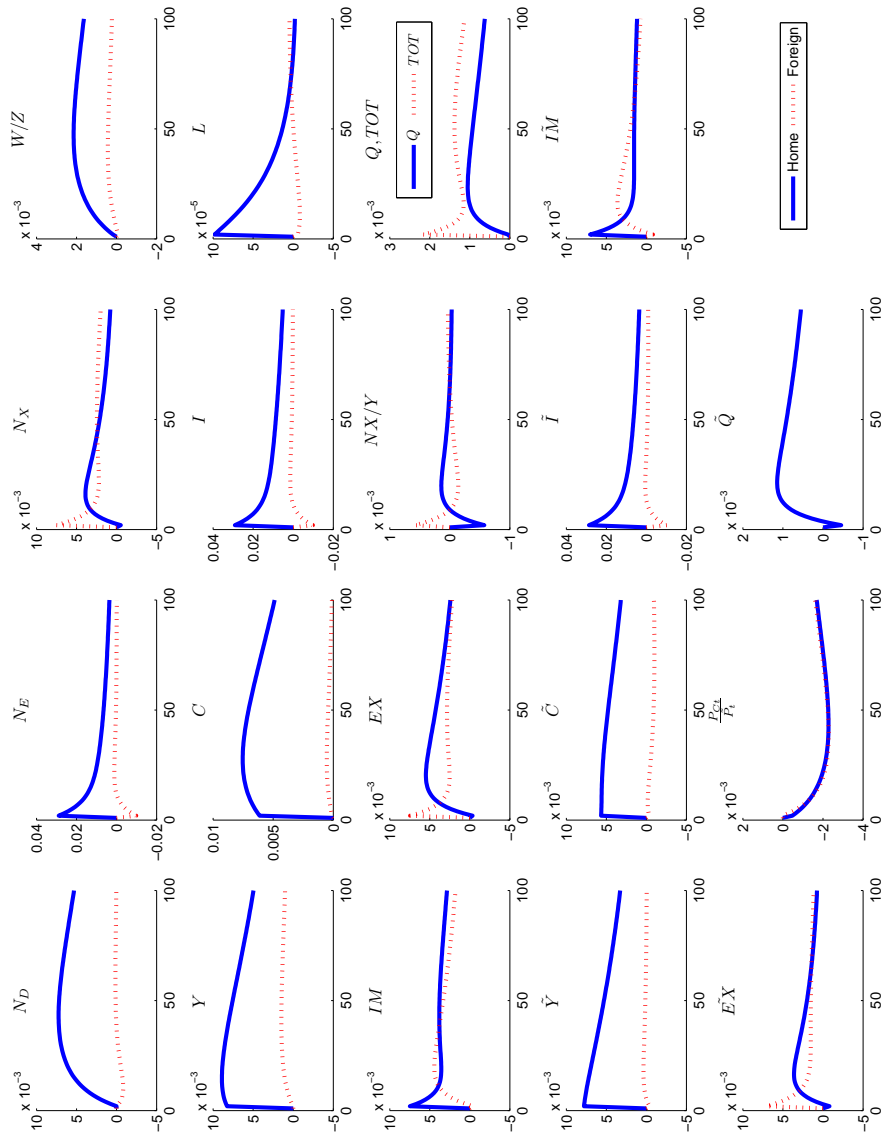


Figure 7: Impulse Response Functions - Endogenous Labor Supply with Non-Separable (BKK) Preferences for Consumption and Leisure
 - $n_2 = 0.0025$

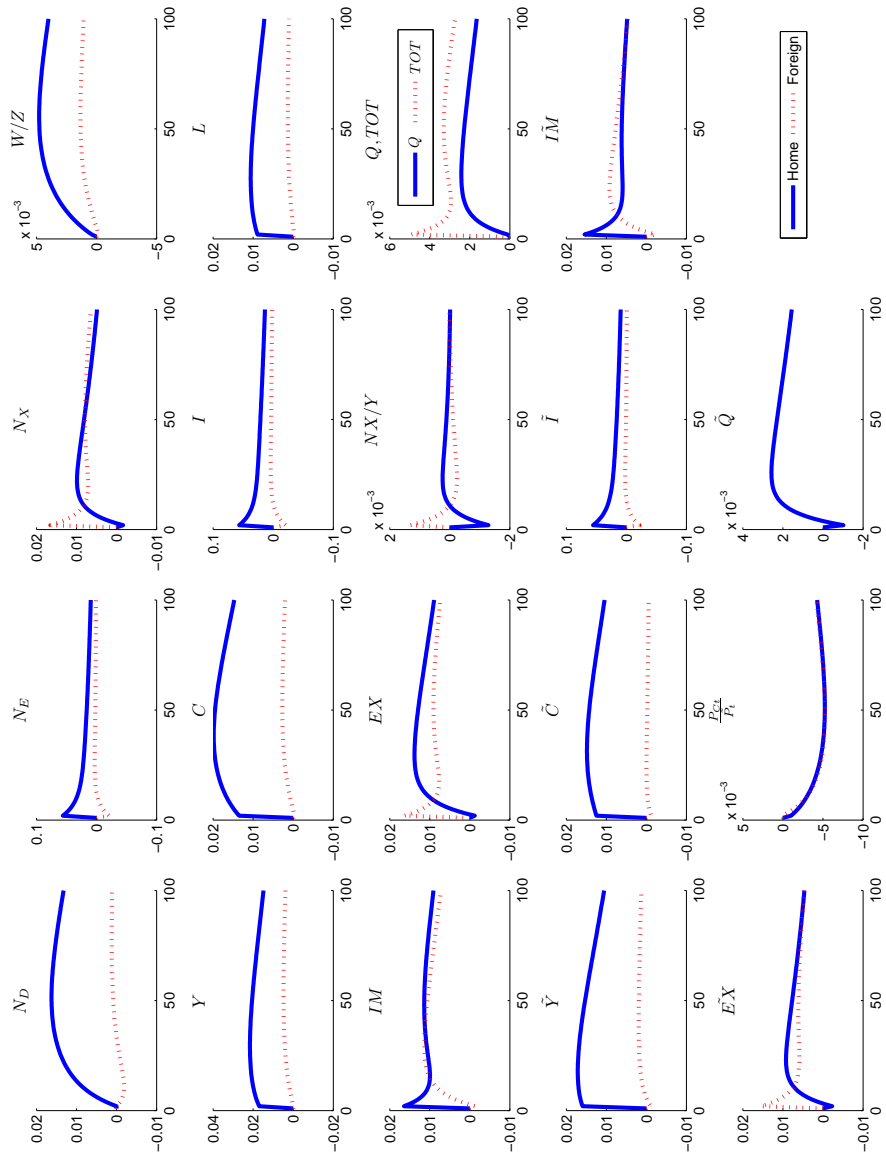


Figure 8: Impulse Response Functions - Endogenous Labor Supply with Non-Separable (GHH) Preferences for Consumption and Leisure
 - $n_2 = 0.0025$

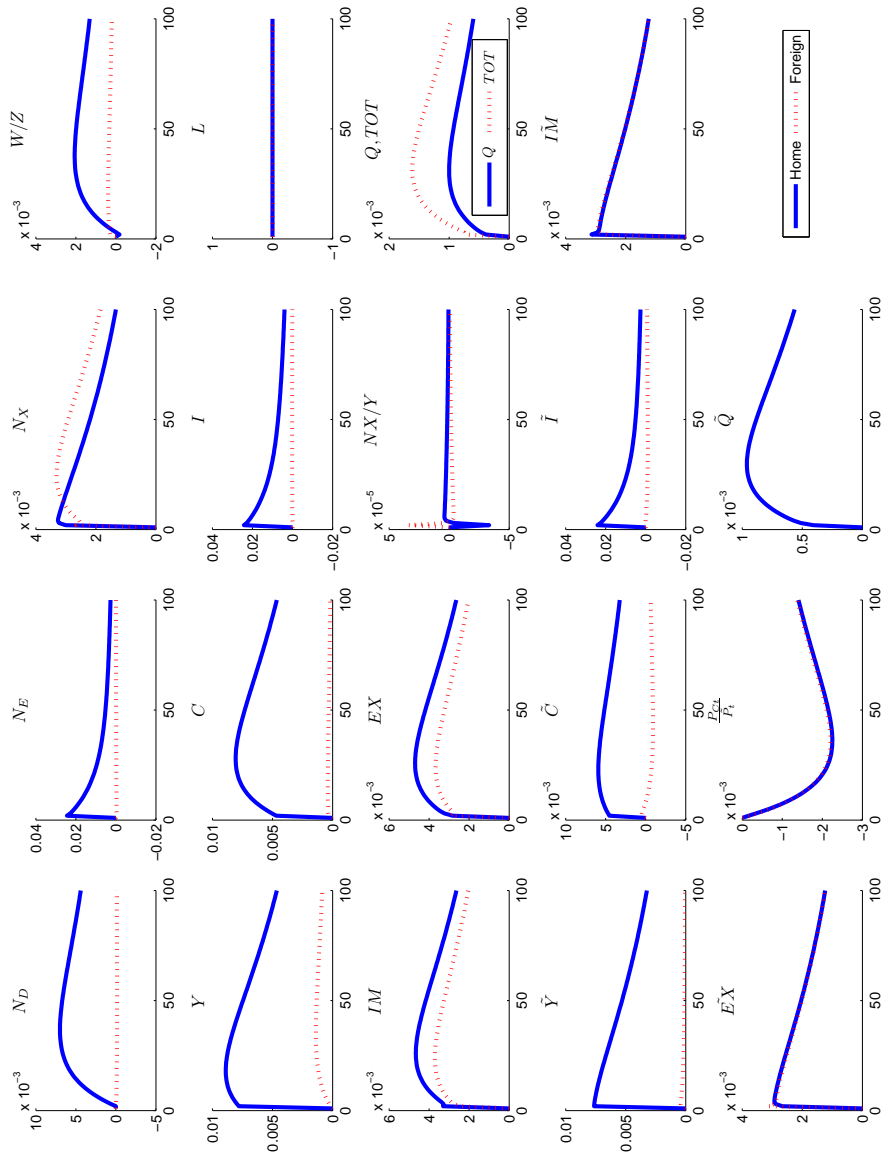


Figure 9: Impulse Response Functions - Benchmark Model - $n_2 = 1$

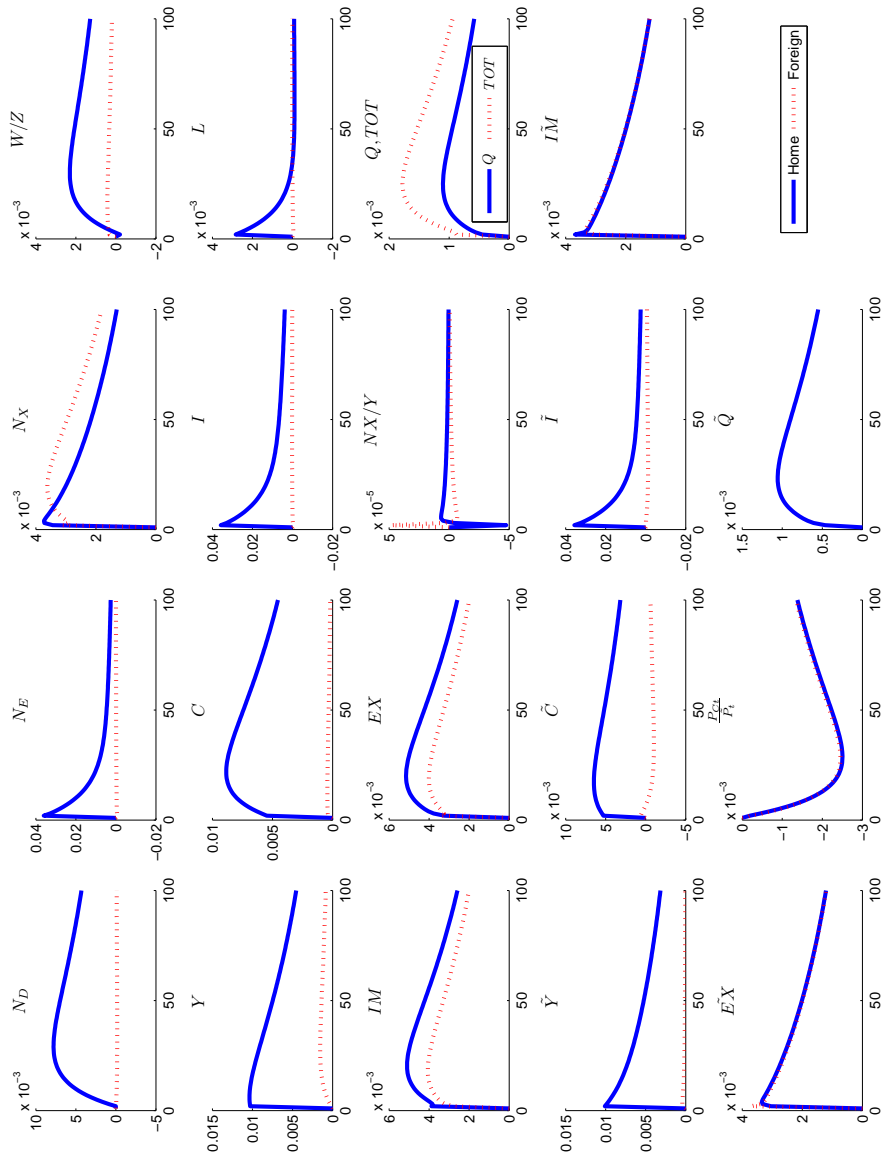


Figure 10: Impulse Response Functions - Endogenous Labor Supply with Separable Preferences for Consumption and Leisure - $m_2 = 1$

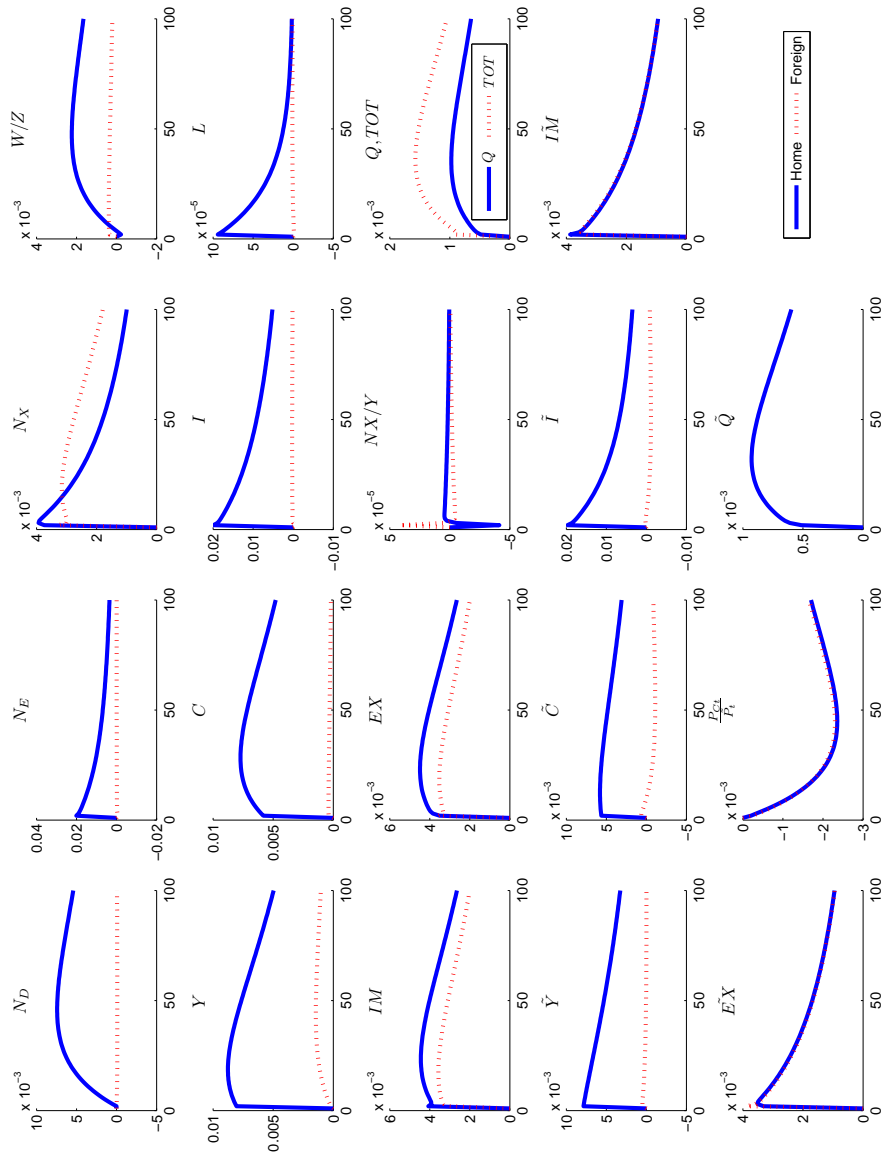


Figure 11: Impulse Response Functions - Endogenous Labor Supply with Non-Separable (BKK) Preferences for Consumption and Leisure - $n_2 = 1$

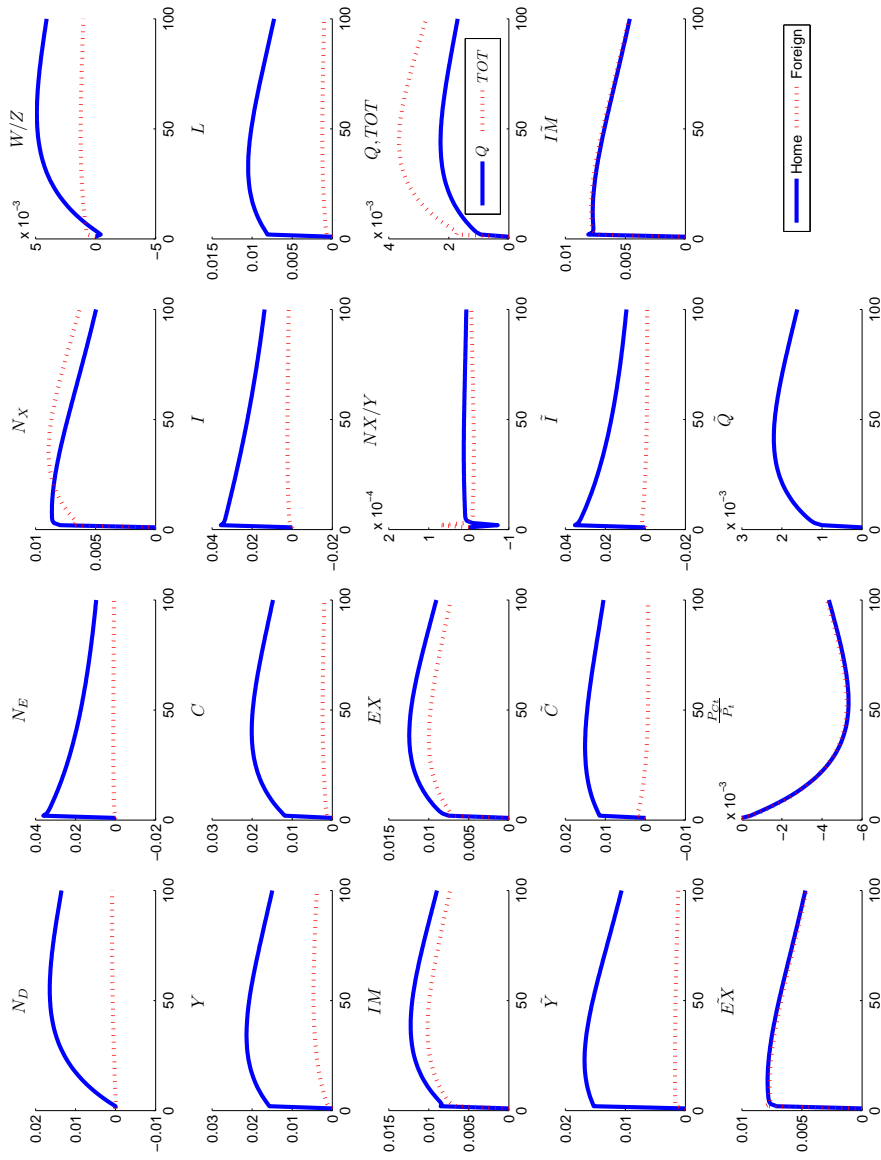


Figure 12: Impulse Response Functions - Endogenous Labor Supply with Non-Separable (GHH) Preferences for Consumption and Leisure - $n_2 = 1$

A.2 Balance of Payments

I construct the balance of payments equation by first imposing stock market clearing and bond adjustment costs transfers on the consumer's budget constraint. The budget constraint for the home country becomes:

$$C_t + B_{t+1} + Q_t B_{*t+1} + (N_{ht} \tilde{v}_t) = w_t l_t + (1 + r_t) B_t + Q_t (1 + r_t^*) B_{*t} + N_{dt} \tilde{v}_t + N_{dt} \tilde{d}_t$$

I generate the budget constraint for the foreign agent:

$$C_t^* + B_{t+1}^* + Q_t^{-1} B_{*t+1}^* + (N_{ht}^* \tilde{v}_t^*) = w_t^* l_t^* + (1 + r_t) B_t^* + Q_t^{-1} (1 + r_t^*) B_{*t}^* + N_{dt}^* \tilde{v}_t^* + N_{dt}^* \tilde{d}_t^*$$

I multiply the foreign budget constraint by Q_t to transform it into home consumption terms and subtract it from the home budget constraint. Imposing the bond market clearing conditions suggest the following equation for net foreign assets:

$$\begin{aligned} 2(1 + r_{t+1}) B_t + 2(1 + r_{t+1}^*) Q_t B_{*t} &= [C_t - Q_t C_t^*] + [N_{et} \tilde{v}_t - Q_t N_{et}^* \tilde{v}_t^*] + 2[B_{t+1} + Q_t B_{*t+1}] \\ &\quad - [w_t l_t - Q_t w_t^* l_t^*] - [N_{dt} \tilde{d}_{dt} - Q_t N_{dt}^* \tilde{d}_{dt}^*] \\ &\quad - [N_{xt} \tilde{d}_{xt} - Q_t N_{xt}^* \tilde{d}_{xt}^*] \end{aligned} \quad (38)$$

A.3 Equilibrium Conditions

The model is characterized by the following nonlinear system of equations:

1. *Price and Profit definitions:*

$$\tilde{\rho}_{dt} = \left(\frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_d} \right) \quad (39)$$

$$\tilde{\rho}_{dt}^* = \left(\frac{a}{a-1} \frac{w_t^*}{Z_t^* \tilde{z}_d} \right) \quad (40)$$

$$\tilde{\rho}_{xt} = \left(\frac{a}{a-1} \frac{w_t}{Z_t \tilde{z}_{xt}} \frac{\tau}{Q_t} \right) \quad (41)$$

$$\tilde{\rho}_{xt}^* = \left(\frac{a}{a-1} \frac{w_t^*}{Z_t^* \tilde{z}_{xt}^*} \tau Q_t \right) \quad (42)$$

$$\tilde{d}_{dt} = \frac{1}{a} C_t \tilde{\rho}_{dt}^{1-a} \quad (43)$$

$$\tilde{d}_{dt}^* = \frac{1}{a} C_t^* \tilde{\rho}_{dt}^{*1-a} \quad (44)$$

$$\tilde{d}_{xt} = \frac{1}{a} C_t^* Q_t \tilde{\rho}_{xt}^{1-a} - \frac{w_t F_{Xt}}{Z_t} \quad (45)$$

$$\tilde{d}_{xt}^* = \frac{1}{a} C_t Q_t \tilde{\rho}_{xt}^{*1-a} - \frac{w_t^* F_{Xt}}{Z_t^*} \quad (46)$$

2. *Price Index*³¹:

$$1 = (N_{dt}\tilde{\rho}_{dt}^{1-a} + N_{xt}^*\tilde{\rho}_{xt}^{*1-a}) \quad (47)$$

$$1 = (N_{dt}^*\tilde{\rho}_{dt}^{*1-a} + N_{xt}\tilde{\rho}_{xt}^{1-a}) \quad (48)$$

3. *Expected Profit*:

$$\tilde{d}_t = \tilde{d}_{dt} + \left(\frac{N_{xt}}{N_{dt}}\right)\tilde{d}_{xt} \quad (49)$$

$$\tilde{d}_t^* = \tilde{d}_{dt}^* + \left(\frac{N_{xt}^*}{N_{dt}^*}\right)\tilde{d}_{xt}^* \quad (50)$$

4. *Free Entry*:

$$\tilde{v}_t = w_t F_{Et}/Z_t \quad (51)$$

$$\tilde{v}_t^* = w_t^* F_{Et}^*/Z_t^* \quad (52)$$

5. *Zero-Profit Intermediate Exporter*:³²

$$\tilde{d}_{xt} = \left[\frac{a-1}{k+1-a}\right] \frac{w_t F_{Xt}}{Z_t} \quad (53)$$

$$\tilde{d}_{xt}^* = \left[\frac{a-1}{k+1-a}\right] \frac{w_t^* F_{Xt}^*}{Z_t^*} \quad (54)$$

6. *Share Exporting Firms*:

$$(1 - G(z_{xt})) = \frac{N_{xt}}{N_{dt}} = z_{\min}^k \left(\frac{k}{k+1-a}\right)^{k/(a-1)} \tilde{z}_{xt}^{-k} \quad (55)$$

$$(1 - G(z_{xt}^*)) = \frac{N_{xt}^*}{N_{dt}^*} = z_{\min}^k \left(\frac{k}{k+1-a}\right)^{k/(a-1)} \tilde{z}_{xt}^{*-k} \quad (56)$$

7. *Number of Firms*:

$$N_{dt} = (1 - \delta)(N_{dt-1} + N_{et-1}) \quad (57)$$

$$N_{dt}^* = (1 - \delta)(N_{dt-1}^* + N_{et-1}^*) \quad (58)$$

8. *Euler Equation for Domestic Bonds*:

$$C_t^{-1}(1 + nB_{t+1}) = (1 + r_{t+1})\beta E_t C_{t+1}^{-1} \quad (59)$$

$$C_t^{*-1}(1 - nB_{*t+1}) = (1 + r_{*t+1}^*)\beta E_t^* C_{t+1}^{*-1} \quad (60)$$

³¹Using the equation for the price index of consumption, we substitute in average prices.

³²To generate this condition, we utilize the cutoff condition, $\frac{1}{a}C_t^*Q_t\rho_{xjt}^{1-a} = \frac{w_t F_{Xt}}{Z_t}$, the useful transform, $z_{xt} = \left(\frac{k}{k+1-a}\right)^{1/(1-a)}\tilde{z}_{xt}$, and the definition for exporter profit, \tilde{d}_{xt} .

9. *Euler Equation for Foreign Bonds:*

$$Q_t \mu_t (1 + n B_{*t+1}) = (1 + r_{t+1}^*) \beta E_t Q_{t+1} \mu_{t+1} \quad (61)$$

$$Q_t^{-1} \mu_t (1 - n B_{t+1}) = (1 + r_{t+1}) \beta E_t^* Q_{t+1}^{-1} \mu_{t+1} \quad (62)$$

10. *Euler Equation for Mutual Fund Shares:*

$$\tilde{v}_t = \beta (1 - \delta) E_t \left(\frac{\mu_{t+1}}{\mu_t} \right) (\tilde{d}_{t+1} + \tilde{v}_{t+1}) \quad (63)$$

$$\tilde{v}_t^* = \beta (1 - \delta) E_t^* \left(\frac{\mu_{t+1}^*}{\mu_t^*} \right) (\tilde{d}_{t+1}^* + \tilde{v}_{t+1}^*) \quad (64)$$

11. *Labor Market Clearing:*³³

$$l_t = \frac{a-1}{w_t} (N_{Dt} \tilde{d}_{dt} + N_{xt} \tilde{d}_{xt}) + (a N_{xt} F_{Xt} + N_{Et} F_{Et}) / Z_t$$

$$l_t^* = \frac{a-1}{w_t^*} (N_{Dt}^* \tilde{d}_{dt}^* + N_{xt}^* \tilde{d}_{xt}^*) + (a N_{xt}^* F_{Xt} + N_{Et}^* F_{Et}) / Z_t^*$$

12. *Net Foreign Assets:*

$$2(1 + r_{t+1}) B_t + 2(1 + r_{t+1}^*) Q_t B_{*t} = [C_t - Q_t C_t^*] + [N_{et} \tilde{v}_t - Q_t N_{et}^* \tilde{v}_t^*] + 2[B_{t+1} + Q_t B_{*t+1}]$$

$$- [w_t l_t - Q_t w_t^* l_t^*] - [N_{dt} \tilde{d}_{dt} - Q_t N_{dt}^* \tilde{d}_{dt}^*]$$

$$- [N_{xt} \tilde{d}_{xt} - Q_t N_{xt}^* \tilde{d}_{xt}^*] \quad (65)$$

Note that the mutual fund market clearing conditions in addition to the zero net supply of bonds conditions have already been included in the above equilibrium equations. The 29-31 equations are log-linearized around the symmetric steady state to form 29-31 linear equations in terms of percentages (denoted by $\hat{\cdot}$)³⁴.

In addition to these equations, the stochastic processes for the technology innovations must be specified:

$$\begin{bmatrix} \hat{Z}_t \\ \hat{Z}_t^* \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} \hat{Z}_{t-1} \\ \hat{Z}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_{Zt} \\ \xi_{Zt}^* \end{bmatrix} \quad (66)$$

$$E_t(Z_t) = E_t^*(Z_t^*) = \bar{Z}$$

$$\sigma_{\xi_{Zt}} = \sigma_{\xi_{Zt}^*} = \sigma_\xi$$

$$\rho_{\xi_{Zt} \xi_{Zt}^*} = \rho_{\xi_Z \xi_Z^*}$$

³³When labor is supplied inelastically, we set the left-hand side of the equations to 1. When labor is supplied endogenously, we replace the left-hand side with the first-order condition governing labor supply from the consumer's problem.

³⁴Since the variable, B_t , has an expected value of zero in equilibrium, it is linearized with respect to steady state consumption: $\hat{B}_t = \frac{B_t - 0}{C} = \frac{B_t}{C}$.

A.4 Divisible versus Indivisible Labor

Although endogenizing the labor supply decision allows the model to capture employment dynamics, the formulation employed in the previous sections does not allow the model to analyze changes in unemployment. By construction, any unemployment in the model is completely voluntary. Past research by Hansen (1985) and Rogerson (1988) suggests a simple modification can be made to allow for endogenous involuntary unemployment ("indivisible labor"). In addition to improving the realism of the model, indivisible labor can also increase the volatility of labor hours relative to the real wage; a feature of the data that divisible labor models have difficulty matching. Results from incorporating indivisible labor into the framework developed so far suggest this feature can distort the pattern of trade in a way that causes excessively volatile output. In the model with indivisible labor, positive productivity shocks lead to huge gains in output which induces massive investment exodus and negatively correlated business cycles across countries.

The consumer's problem is reformulated as follows. When considering divisible labor, I will follow Rogerson (1988). Workers have 1 unit time to spend either working (l_t) or in leisure. The consumer's instantaneous utility function takes the form:

$$U_t(C_t, l_t) = \log C_t + H \log(1 - l_t)$$

where the parameter, H , is calibrated such that the steady state fraction of time working matches that of the previous models with endogenous labor supply. The agent's maximization problem is still written as:

$$\begin{aligned} & \max_{\{C_s, l_s, B_{s+1}, B_{*s+1}, x_{s+1}\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t} U_s(C_s, l_s) s.t. \\ & C_s + B_{s+1} + Q_s B_{*s+1} + \frac{n_1}{2} (B_{s+1}^2) + \frac{n_2}{2} (Q_s B_{*s+1}^2) + (N_{hs} \tilde{v}_s) x_{s+1} = \\ & w_s l_s + (1 + r_s) B_s + Q_s (1 + r_s^*) B_{*s} + \Gamma_s + x_s (N_{ds} \tilde{v}_s + N_{ds} \tilde{d}_s) \end{aligned}$$

The labor supply equation is:

$$l_t = 1 - H \frac{C_t}{w_t} \tag{67}$$

This equation is linearized and then included in the equation system that is used to solve the model.

For indivisible labor, the representative agent only has two choices of labor supply: to work a fixed number of hours or no hours at all. The agent contracts with firms to work l_0 hours with probability ϕ_t ³⁵. The expected amount of labor that the household will supply is $l_t = \phi_t l_0$.

³⁵Instead of choosing the number of labor hours to supply, workers are choosing a probability of becoming employed each period, ϕ_t .

The consumer's expected instantaneous utility function takes the form:

$$U_t(C_t, l_t) = \phi_t(\log C_{et} + H \log(1 - l_0)) + (1 - \phi_t)(\log C_{ut} + H \log(1))$$

where C_{et} is consumption when the worker is employed and C_{ut} is consumption when the worker is unemployed. It is assumed that the consumer is fully insured against becoming unemployed so that consumption in the unemployed state is the same as consumption in the employed state ($C_{et} = C_{ut} = C_t$). The expected instantaneous utility function is then:

$$U_t(C_t, l_t) = \log C_t + \phi_t H \log(1 - l_0)$$

which can be rewritten using the definition of l_t as:

$$U_t(C_t, l_t) = \log C_t + l_t \frac{H \log(1 - l_0)}{l_0} = \log C_t + G l_t$$

where $G = \frac{H \log(1 - l_0)}{l_0}$. The first order condition governing labor supply becomes:

$$w_t = -G C_t \tag{68}$$

The parameters G and l_0 are calibrated so that the steady state fraction of time spent working (l_t) matches that of the previous models with endogenous labor supply and the parameter H matches the case of divisible labor. This equation is linearized and then included in the equation system that is used to solve the model.

The simulation results for the models with divisible labor and indivisible labor that follow Hansen (1985) exercise are reported in columns (5) and (6) respectively in table 7. The model with Hansen's divisible labor generates qualitatively similar results to the model described in the previous section. The model with indivisible labor actually worsens the model's fit by exacerbating the volatility of output and investment, generating counter-cyclical exports, and inducing negative correlation of output, investment and labor hours across countries. Inspection of the impulse response functions for both the divisible labor and indivisible labor models suggest the worsened fit is due to a mixture of open capital markets and strong responses of labor demand.

Table 7: Simulation Results - Divisible versus Indivisible Labor

n_2 small	Data	(5)	(6)
Domestic Statistics			
Volatility			
$\sigma_Y(\%)$	1.54	0.99	8.65
σ_C/σ_Y	0.75	0.63	0.12
σ_I/σ_Y	3.41	3.87	11.36
σ_L/σ_Y	0.61	0.02	1.15
Correlation with GDP			
$\rho_{C,Y}$	0.88	0.99	0.47
$\rho_{I,Y}$	0.90	0.94	0.69
$\rho_{L,Y}$	0.81	0.98	0.99
International Co-movement			
ρ_{Y,Y^*}	0.56	0.17	-0.06
ρ_{C,C^*}	0.45	0.07	0.21
ρ_{I,I^*}	0.37	-0.41	-0.33
ρ_{L,L^*}	0.39	0.57	-0.11
International Market			
Volatility			
σ_{IM}/σ_Y	3.31	0.79	0.19
σ_{EX}/σ_Y	3.74	0.83	0.18
σ_{NX}/σ_Y	0.40	0.27	0.65
σ_{TOT}/σ_Y	1.69	0.29	0.05
σ_Q/σ_Y	3.37	0.06	0.03
Correlation with GDP			
$\rho_{IM,Y}$	0.60	0.87	0.64
$\rho_{EX,Y}$	0.28	0.21	-0.27
$\rho_{NX/Y,Y}$	-0.36	-0.45	-0.56
$\rho_{TOT,Y}$	0.07	0.57	0.66
$\rho_{Q,Y}$	-0.18	-0.57	-0.08

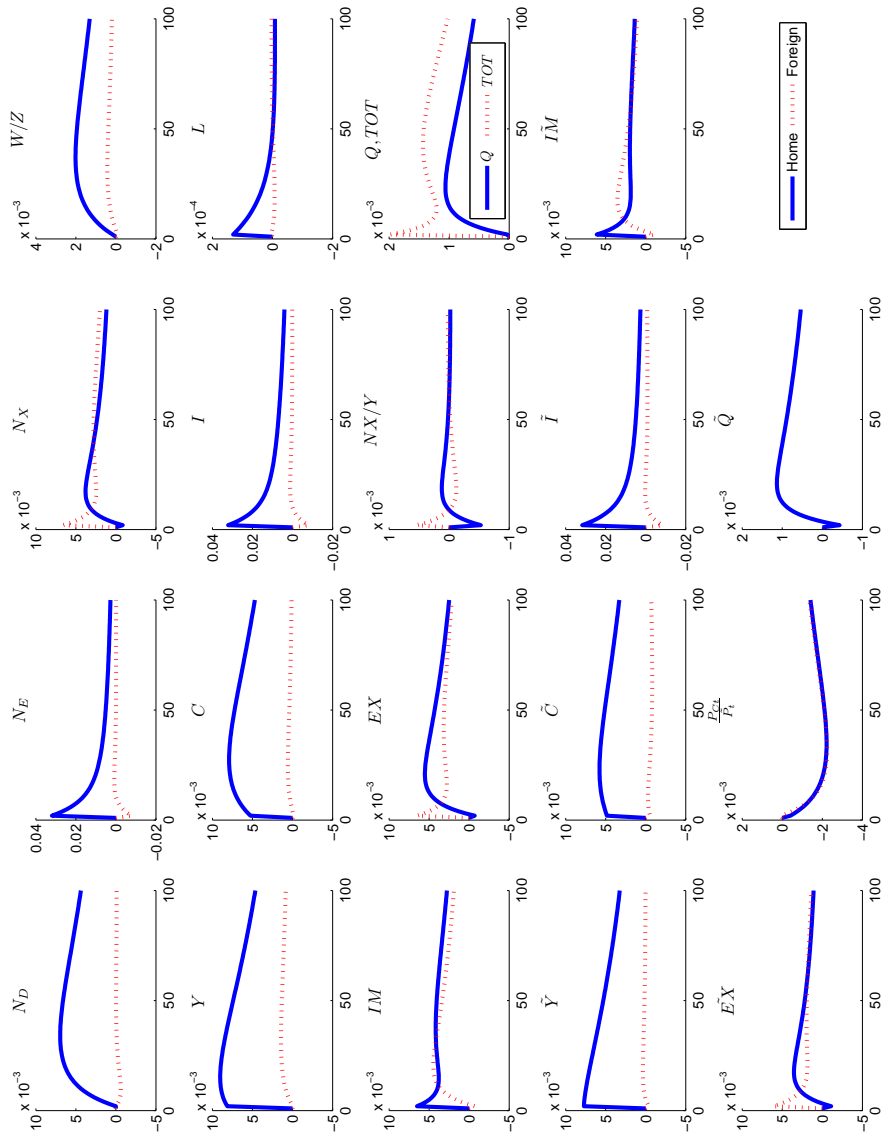


Figure 13: Impulse Response Functions - Divisible Labor - $n_2 = 0.0025$

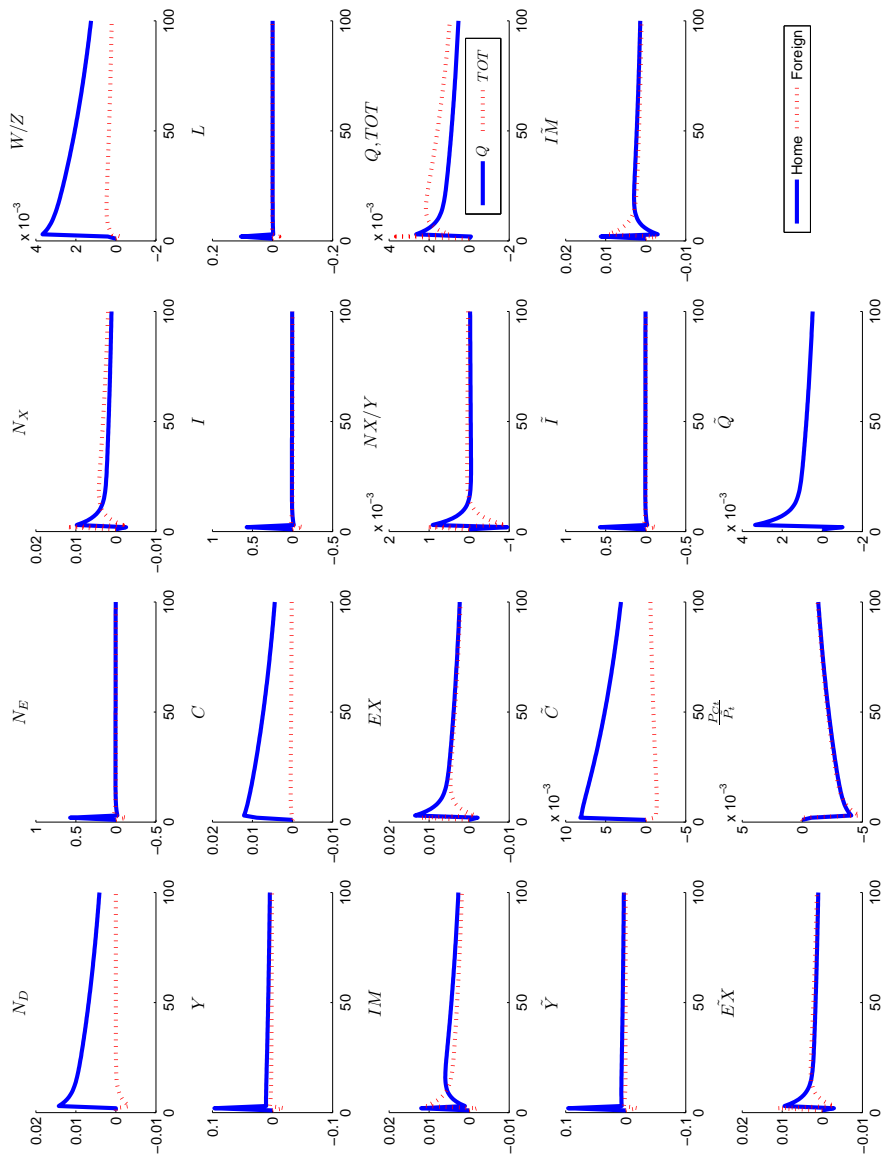


Figure 14: Impulse Response Functions - Indivisible Labor - $n_2 = 0.0025$

In the model with divisible labor, a positive productivity shock in the home country leads to an overall expansion at home. Consumption increases, the profitability of firms rises, there's increased firm construction and higher demand for imported goods as aforementioned. In the labor market, there is increased demand for labor to spend on production and new firm construction but reduced supply of labor due to the expansion of consumption. The model predicts an increase in the real wage and in increase in employment, which boosts output. In the foreign country, less is spent on firm construction and investment dollars are redirected abroad. There is downward pressure on labor demand due to the fall in firm construction but upward pressure on labor demand due to the expansion of the export sector. Labor supply increases due to a reduction in the variety of goods which resulted from firm attrition. The model predicts a fall in the real wage, but an increase in employment and a boost in output.

When labor is indivisible, inspection of the impulse response functions for the endogenous variables of the model suggest an strong increase in employment in the home country in response to a positive productivity shock. Holding labor supply and new firm construction constant, a positive productivity shock leads to an increase in labor demand and higher employment in equilibrium. In the model with indivisible labor, the increase in labor demand leads to an larger increase in employment than in the model with divisible labor. Firms in the home country become more profitable under the indivisible labor specification than they do under divisible labor. Since firms are more profitable under the indivisible labor specification, there is a larger boost in new firm construction than in the divisible labor framework. Finally, allowing for changes in labor supply, the labor supply curve in the indivisible labor model rises quite dramatically since the positive productivity shock boost consumption relatively more than in the model with divisible labor. Employment is more volatile as is the real wage, leading to a dramatic increase in income.

In the foreign country, there is a large exodus of investment in the indivisible labor framework due to the dramatic increase in the profitability of home firms. Firm attrition leads to a large decrease in labor demand. Although labor supply increases in the foreign country due to a fall in consumption, the model predicts a lower real wage as well as lower employment in equilibrium. National income falls as does the demand for home's exports. The model predicts negatively correlated output and employment across countries and counter-cyclical exports, none of which match the empirical facts. The inclusion of indivisible labor, in the presence of firm entry and open asset markets.