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Money, Output and Inflation in the Longer Term: Major Industrial Countries, 1880-2001[†]

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[†] We thank the co-editor, Dean Corbae, two anonymous referees, and seminar participants at the ECB, IGIER, and at Calgary, Otago, Simon Fraser, Singapore, Victoria (Wellington) and York universities for many useful comments. The usual caveat applies. This is a substantially revised version of an earlier paper (Haug and Dewald, 2004), however, the results for the role of money growth are unchanged.

Abstract: We study how fluctuations in money growth correlate with fluctuations in real output growth and inflation. Using band-pass filters, we extract cycles from each time series that last 2 to 8 (business cycles) and 8 to 40 (longer-term cycles) years. We employ annual data, 1880-2001 without gaps, for eleven industrial countries. Fluctuations in money growth do not play a systematic role at business cycle frequencies. However, money growth leads or affects contemporaneously inflation, but not real output growth, in the longer run. Also, formal break tests indicate no structural changes for the longer-term money growth and inflation relationship, despite changes in policy regimes.

JEL code: E3

Key words: Band-pass filters; role of monetary aggregates; longer-term cycles.

1. Introduction

The empirical role of money growth in the macroeconomy has been studied by many researchers using different data and methods. The most prominent studies focusing on the quantity theory of money are Friedman and Schwartz (1963, 1982) using data averaged over the phases of the business cycle in order to capture longer-run movements with United States and United Kingdom data back to 1867. McCandless and Weber (2001) is a representative example for a different approach for 110 countries in the post-WW II period: longer-run growth rates are approximated by growth of a variable over a 30 year period (average growth rates).¹ The problem with phase-averaging is that it does not deal with possible non-stationarity of the data.² The problem with growth averages over long periods, while informative to a degree, is that they have limitations, too. It is impossible to assess the timing of movements in these variables.³

In the frequency domain, the “long run” is associated with frequency zero, which in our terminology is the long-run trend. Previous studies in the frequency domain on the long-run neutrality of money at zero frequency include Lucas (1980) and Geweke (1986). The theory of spectral analysis provides a rigorous framework for extracting specific frequency bands from the data, instead of considering only frequency zero. Stock and Watson (1998) applied the band-pass filter of Baxter and King (1999) to extract cycles (frequency bands) for United States data and focused on the business cycle and on mostly the post-WWII period. Christiano and Fitzgerald (2003) proposed an improved band-pass filter and provided an application to the longer-run relation of money and inflation, for bands of 8 to 20 years and 20 to 40 years, in the United States from 1900 to 1997.⁴

The usefulness of monetary aggregates for conducting monetary policy has been seriously called into question due to the instability of money demand. A prominent empirical study is Estrella and Mishkin (1997). Recent mainstream New Keynesian models have no explicit role for money (Woodford, 2008). A contrasting view is that the longer-term role of money is still of relevance for monetary policy. The European Central Bank explicitly emphasizes a supporting role that monetary aggregates should play in order to provide an anchor for inflation expectations (Issing, 2006; see also King, 2002; Nelson, 2003; and Hafer

¹ Similarly, Dewald (2003) used 10-year averages.

² Friedman and Schwartz completed their studies before the impact of Nelson and Plosser (1982) showing that many macroeconomic variables are well approximated by a unit root process.

³ For further criticism of data averaging, see Ericsson, Irons and Tyron (2001), and DeGrauwe and Polan (2005).

⁴ See also Fitzgerald (1999).

et al., 2007)). Christiano et al. (2007) provided a theoretical New Keynesian model where inflation expectations lose their anchor, unless money is monitored and reacted to. Also, Beck and Wieland (2008) presented a New Keynesian model, where cross-checking with money growth movements improves inflation control. Similarly, Assenmacher-Wesche and Gerlach (2006, 2007) found a significant role for a low frequency (low-pass filtered) component of money in the empirical New Keynesian Phillips-curve equation.⁵

In the spirit of Friedman (1961), we use long time spans so that our results are unlikely to be due to a specific policy rule in place in a given period. We have 122 years of annual data, from 1880 to 2001, before the introduction of the euro in 2002. We have real GDP, money and consumer price indices, with no gaps, for 11 industrialized countries.⁶ The countries included in our study differ in such terms as their institutions, fiscal and monetary policies, and economic growth rates. Nonetheless, at least in peace time they were similar in generally having comparatively free markets in which prices and interest rates were determined. Our aim is to apply analytic methods that are free from economic and statistical models as much as possible in order to explore empirical regularities, or stylized facts, in the data. Also, the focus of our research is the longer run.

We employ band-pass filters to pick out cycles from each time series that last 8 to 40 years, i.e., the longer-term component.⁷ We also filter out cycles that last 2 to 8 years, i.e., the short-run business-cycle component.⁸ Our data span allows us to extract longer cycles. This is in contrast to most previous research with band-pass filters that concentrated on the business cycle instead. We apply the band-pass filter of Christiano and Fitzgerald (2003) and calculate the dynamic cross-correlations among filtered components of money growth with, in turn, real GDP growth and inflation, following the approach in Stock and Watson (1998), among others.⁹ We also provide critical values for the filtered correlations based on bootstrap methods.

In our analysis, the longer-run components of 8 to 40 years reveal a systematic pattern consistent with the quantity theory of money. We formally test for its structural stability by

⁵ However, they applied the low-pass filter of Engle (1974) that requires covariance stationary variables. Inflation and other variables that they used are likely not stationary so that their results could be spurious.

⁶ We are indebted to Michael Bordo and Lars Jonung who provided most of the data for the period before the late 1940s (see, e.g., Bordo and Jonung, 1987). Data sources are listed in the Appendix of Dewald (2003).

⁷ Comin and Gertler (2006) and Müller and Watson (2008) found pronounced dynamics below the business cycle frequency. However, these authors did not study the role of money.

⁸ In addition, there is a long-run cycle or trend at frequencies below 40 years that is shaped by slow moving factors like changes in demographics.

applying tests proposed by Bai and Perron (1998, 2003). These tests are for breaks at unknown dates.¹⁰ In contrast to Friedman (1961), Sargent and Surico (2008) and Rolnick and Weber (1997) argued that long-run relations implied by the quantity theory of money depend on prevailing monetary policy rules and are, therefore, subject to structural breaks. However, we find, consistent with Christiano and Fitzgerald (2003) for the U.S. for 20 to 40 year cycles, that the money-inflation relationship is positive, significant and very stable in the longer run.

In Section 2, we briefly describe band-pass filters and outline the basic features of the Christiano and Fitzgerald (2003) filter. In Section 3, we present examples of typical spectra and coherences for our data, along with examples of graphs for the raw and filtered data.¹¹ Next, we discuss the empirical results from the analysis of the dynamic cross-correlations and the structural change tests. The conclusion summarizes results and suggests directions for future research.

2. Brief Description and Comparison of Frequency Domain Filters

One of the most often used methods to extract business cycles is the filter of Hodrick and Prescott (1997). The extracted business cycle component is covariance stationary (i.e., integrated of order zero) even though the original unfiltered series may be integrated of order one to four. However, the trend or longer-run component of the Hodrick-Prescott filter inherits the non-stationarity of the original series and is therefore not suitable for our purposes.¹² We apply instead band-pass filters that are designed for data with unit roots and avoid this problem.

We found evidence for unit roots in several of our variables.¹³ Weber (1994) also found that the logarithms of nominal monetary aggregates and inflation follow a process with either one or two unit roots for the post-WWII period. Similarly, Rapach (2003) found inflation to be “convincingly” I(1) for 14 OECD countries.

We need to specify the relevant frequency bands for the band-pass filters. Stock and Watson (1998) pointed out that 90% of the business cycles in the United States lasted from 1 ½ or 2

⁹ We used GAUSS for the calculations of the spectra and coherences, for all filters applied in this paper, and for the bootstrapping. The basic GAUSS code for the band-pass filter of Christiano and Fitzgerald (2003) was downloaded from: <http://www.clevelandfed.org/research/tjfbandpass.html>.

¹⁰ See Hansen (2001) on the importance of treating break dates as unknown when testing for structural change.

¹¹ The coherence measures, at a given frequency, the proportion of the variance of one series that is explained by the variance of the other series.

¹² Granger and Newbold (1974) demonstrated that unit root processes can lead to spurious inferences.

¹³ Results are available from the authors on request.

years to 8 years over the period 1854 to 1990.¹⁴ Researchers have used different definitions of the business cycle and therefore different methods to extract the business cycle from the data. We use the Christiano-Fitzgerald band-pass filter, and for robustness checking of our results also the filter of Baxter and King (1999).¹⁵ These two filters produce covariance stationary components for extracted cycles when the unfiltered data have unit roots. The filtering for both is carried out in the time domain with a moving average. However, the weights for the moving average are chosen in the frequency domain. The Baxter-King filter minimizes a quadratic loss function for the difference between the ideal but unfeasible filter and the proposed feasible filter. The Baxter-King filter imposes symmetry of the weights for K observations on either side of t . K observations at the beginning and at the end of the sample are hence lost for further analysis. But, symmetry assures that the filtered series have no phase shift. On the other hand, the Christiano-Fitzgerald filter is based on a somewhat different optimising rule. The squared deviation of the proposed filter from the ideal filter is weighted with the spectrum of the raw series. In order to avoid estimating the spectrum of each series, Christiano and Fitzgerald suggested using the spectrum of a random walk as an approximation to the true spectrum. They carried out an extensive sensitivity analysis for deviations from the random walk assumption and found that such a “random walk filter” works well for United States macroeconomic time series even when the underlying time series process is quite different. Also, the Christiano-Fitzgerald filter uses for filtering a variable at time t all observation available in the sample, forwards and backwards, and is therefore not symmetric. Because of the non-symmetry, the Christiano-Fitzgerald filter may induce phase shift, i.e., the timing of peaks and troughs may not be consistent with the behaviour of the unfiltered series. We used the random walk (with drift) assumption for filtering throughout this paper. However, we also checked whether our results are sensitive to this assumption by not removing a drift and by assuming instead of a random walk that the series are stationary. We found that the filtered components were almost identical across these assumptions. Further, Christiano and Fitzgerald’s sensitivity analysis showed that phase shift and remaining non-stationarity in the filtered series are likely to be negligible in magnitude in applications.

Several authors criticised the type of filters discussed above. For example, Cogley and Nason (1995) demonstrated that the Hodrick-Prescott filter could introduce what they

¹⁴ The recent cycle in the 1990s and early 2000s lasted somewhat longer.

¹⁵ We also applied a new leakage-corrected frequency-domain filter of Corbae and Ouliaris (2006) to extract 8 to 40 year bands and found very similar results to the other two filters.

labelled “spurious cycles”, and Murray (2003) extended this analysis to the Baxter-King filter. Pedersen (2001) clarified the issue of “spurious cycles” by showing that only the deviations from the ideal filter can cause spurious cycles by re-weighting spectra at relevant frequencies with weights other than the ideal filter weights of 0 and 1 (the so-called “Slutsky effect”). He demonstrated that the Hodrick-Prescott filter does not produce spurious cycles. Cogley and Nason (1995) defined the business cycle in terms of the Beveridge and Nelson (1981) metric and then judged the Hodrick-Prescott generated cycle by this metric, as Cogley (2001) acknowledged. The same criticism applies to the analysis in Murray. The Hodrick-Prescott, Baxter-King, and Christiano-Fitzgerald filters are each designed to extract components with periodic fluctuations of specific durations, whereas the Beveridge-Nelson definition of the cycle is fundamentally different. The frequency domain filters extract cyclical fluctuations of chosen durations, whereas the Beveridge-Nelson method separates a time series into a stochastic trend and a stationary deviation from this trend that is not periodic by construction.

Figure 1 presents gain functions of the ideal filter, the Hodrick-Prescott filter, the Baxter-King filter, and the Christiano-Fitzgerald random-walk filter. The gain function for a specific filter determines the weight that it assigns to a given frequency when the raw series is filtered. In our figures, we use cycles per period on the horizontal axis instead of the frequency ω . Cycles per period, denoted by $1/p$, relate to frequency as $1/p = \omega/(2\pi)$. The ideal filter picks cycles per period between .025 and .125, which corresponds to cycles with periods between 40 and 8 years, respectively. The Christiano-Fitzgerald random walk filter provides the closest approximation to the ideal but unfeasible filter. The Hodrick-Prescott filter (for the trend component) is a low-pass filter and the other two filters are band-pass filters. The Hodrick-Prescott filter does not eliminate frequencies below .125 cycles per period, which corresponds to more than 40 years per cycle, whereas the Baxter-King and Christiano-Fitzgerald filters do eliminate those frequencies and pass through the band from 8 to 40 years. Figure 1 was constructed for large samples with $T=1000$ observations, however, the gain functions look very similar when $T=121$, which corresponds to our long sample.

3. Empirical Results

3.1 The Data

The countries for which we could find continuous data from the 1880s onwards are: Canada, Denmark, France, Italy, Japan, the Netherlands, Norway, Sweden, Switzerland, the United Kingdom, and the United States. The variables considered in our study are a measure

of the money stock, which is mostly a broad measure based on the IMF definition of money plus quasi money, real national income, which is mostly represented by GDP, and CPI-based inflation. The data were constructed so as to assure consistency of definitions across countries as much as possible.¹⁶ The data cover the period from 1880 to 2001, with a few exceptions.¹⁷ The endpoint of our data is 2001 because the euro was introduced in 2002. We also include Belgium and Germany in our analysis of the post-WWII period.¹⁸ Other euro area countries lack sufficiently long annual data spans for our analysis. Data are the same as in Dewald (2003), who provided a data appendix with details. We construct annual growth rates for our measure of money, real GDP and a consumer price index as the first difference of the natural logarithms of each.¹⁹ The growth rates closely correspond with each other in amplitude and frequency but not with real GDP growth.²⁰ We will carry out a systematic analysis in the next section to assess these relationships and the timing patterns.

3.2 The Empirical Spectra

The spectrum of a series shows the contribution of each frequency to the overall variance. For the entire period 1880-2001, Figures 2.1 to 2.3 show the typical spectra for a few representative countries: France, Italy, and the US.²¹ The low frequencies are particularly important in explaining the variances of money growth and inflation.

Figures 3.1 and 3.2 illustrate coherences for the same set of countries. Coherences in our figures are very large and not far from one at the lower frequencies for money growth and inflation. The coherences indicate that money growth may play an important role for variations in inflation for cycles that last longer than 8 or 10 years, and less of a role for fluctuations in real GDP growth at any frequency.

¹⁶ Our measure of money is a broader measure of money, mostly M2 or a definition as close as possible to it. While one would want to explore the sensitivity of results to using different definitions of money, like base money, M1, and broader aggregates than M2, we were limited by historical data available to us.

¹⁷ The last observation for the money measure for Denmark is 1999 and for Sweden it is 2000.

¹⁸ Data for Belgium cover 1947 to 2001 and for Germany 1949 to 2001.

¹⁹ An anonymous referee pointed out that first differencing data is a particular filter that amplifies high frequency fluctuations and that an alternative specification would be using log-levels instead of growth rates. However, we prefer to think of quantity theoretic relations in terms of growth rates.

²⁰ Friedman and Schwartz (1963, 1982) explored this issue in detail. See, e.g., Lucas (1996) for evidence that money growth is closely related to inflation but not to real output

²¹ These time series are not inconsistent with covariance stationary behavior, so that the spectra do exist.

3.3 Analysis of Dynamic Cross-Correlations

We start with analyzing the correlation pattern for 2 to 8 year cycles, the typical business cycle frequency, for the full sample period from 1880 to 2001. We employ our preferred filter, the Christiano-Fitzgerald random walk band-pass filter. In order to assess whether the reported correlations are significantly different from zero, we calculate critical values with the bootstrap technique. We follow Christiano and Fitzgerald (2003) and fit to each unfiltered time series a moving average (MA) process. The best fitting MAs take on orders from 1 to 7, with an MA(3) and MA(4) being the most common cases. Next, we parametrically bootstrap this MA process for the actual sample size (after deleting 200 observations to mitigate start-up effects) and 20,000 replications under the null hypothesis of no correlation in the data generating process with Gaussian errors. We start each data generating process at the historical level. In each replication, we apply the band-pass filter to the artificially generated sample and calculate the correlations in the filtered components. Then, from 20,000 replications for each correlation, we calculate the 1%, 5% and 10% critical values.

Figure 4 illustrates for Italy the component that the filter extracts for money growth for 8 to 40 year cycles, in relation to the raw series. We will consider dynamic cross-correlations for the filtered series from two lags to five leads: between money growth and real GDP growth, and between money growth and inflation. We look for the peak effect for each country in terms of these cross-correlations, i.e., the maximum absolute value of the cross-correlations. Next, we separate out the post-WWII period in order to find out whether results differ when the two World Wars are not included in the sample.

Our particular interest is in the correlations of money growth with real output and with inflation in the longer term, beyond the business cycle frequency. Therefore, we filter out cycles of 8 to 40 year length and calculate cross-correlations of the components to study the effects of longer-term fluctuations in money growth. Figure 5 graphs for Italy the pairs of components for money growth and inflation that we study for the 8 to 40 year cycles. We also carry out tests for unit roots on the filtered series. Furthermore, we assess how sensitive our results are to different bands. We consider 2 to 10 year, 8 to 20 year, 10 to 50 year, and 2 to 40 year cycles. Finally, we contrast our results for 8 to 40 cycles with the Christiano-Fitzgerald filter to those with the Baxter-King filter to check sensitivity to a specific filter. Many of the results not reported in the Tables are available in the working paper version of this article (Haug and Dewald, 2004), or otherwise on request from the authors.

3.3.1 Money Growth and Real GDP Growth

Table 1a reports cross-correlations for 2 to 8 year components of money growth and real GDP growth using data from 1880 to 2001. We look for the peak effect for each country, going from 2 lags to 5 leads (i.e., from $i = -2$ to $i = 5$). However, there is large variability over leads and lags for every country and the cross-correlations show no common pattern across the eleven countries. Most correlations have relatively low values in absolute terms. Also, of the 88 reported correlations, only 26 are significant at the 5% level.

Next, we explore whether our results are due to the WWI and WWII years and therefore, we consider the post-WWII period from 1946 to 2001 separately for 2 to 8 year cycles (not reported). The standard deviation of the money growth component relative to the real GDP growth component is generally lower in the postwar period than in the full sample. The cross-correlations do not systematically differ from those in Table 1a and a clear pattern for the role of money growth is not evident either. Overall, the number of significant correlations falls considerably, however, this may be due to small sample problems.

Our next step is to study the cross-correlations for the longer run. For this purpose, we extract cycles of 8 to 40 years. The correlations of the money growth and real GDP growth components in the longer run are given in Table 1b. When compared to Table 1a, the relative standard deviation of the real GDP to money growth component decreased for all countries, except for the Netherlands. Also, correlations increased noticeably, in absolute terms, as we move from business cycles to longer cycles. The peak correlations are all significant at the 5% level, however, the timing of peak correlations in Table 1b differs substantially across countries, as does the sign. Seven of the eleven countries have negative peak correlations and no obvious common pattern emerges. These differences across countries reflect differences in real growth patterns. Countries with high real growth rates can accommodate higher monetary growth as illustrated by post-WWII Germany and Japan or pre-WWI United States and Canada.

Our next step is to omit the years 1880 to 1945 and to study the period 1946 to 2001 for 8 to 40 year cycles. The timing for the peak effects changes for nine countries when compared to the full sample in Table 1b, and there are sign changes for two countries. However, more than half of the peak correlations become insignificant at the 5% level. Also, the mixed pattern of signs remains, and the range of magnitudes is quite similar to those in the full sample of 8 to 40 year cycles.

3.3.3 Money Growth and Inflation

The peak correlations between the components of money growth and inflation in Table 2a show no common pattern across countries at the business cycle frequency and are relatively small and mostly insignificant. For the post-WWII period, results lead to the same conclusion (not reported). Next, we consider the correlations among money growth and inflation components for 8 to 40 year cycles. Table 2b shows pronounced increases in correlation magnitudes when moving from business cycles to longer-term cycles. The peak correlations are now all in the range from 0.64 to 0.90 and significant at the 1% level, except for 0.40 for Switzerland, which is significant at the 5% level. Also, 56 of the 88 correlations in the table are now significant at the 5% level. Money growth leads inflation by 1 to 3 years for Canada, Denmark, Japan, the Netherlands, Sweden, and the US. The peak effects occur contemporaneously for France, Italy, Norway, and the UK. Only Switzerland has a one period lag for the peak correlation, which, though, is very close to the effect at $i = 0$ (0.39). These results suggest that, in the longer run, money growth and inflation are closely linked. The peak effects from money to inflation occur either contemporaneously or with a 1 to 3 year lag for the majority of the countries in our sample.

This obvious pattern for correlations for 8 to 40 year cycles is also present once we delete the pre-1946 observations from our sample (Table 2c). For money growth and inflation components, we find seven changes in timing for the peak correlations compared to Table 2b. All peak correlations are positive and mostly significant at the 5% level, with the exception of Sweden (-0.27). Furthermore, money growth leads inflation or the peak effects occur contemporaneously. The general pattern is the same as for the full sample period, though considerably fewer cases are significant possibly due to low power of the test in small samples. Again, we find that money growth and inflation are closely linked in the longer run.

3.3.4 Sensitivity Analysis

So far, we have considered 2 to 8 year and 8 to 40 year cycles in our study. Our next step is to analyze the sensitivity of our results to changes in the cycle length. We extract cycles that last 2 to 10 years, 8 to 20 years, 10 to 50 years, and 2 to 40 years. For 2 to 10 year cycle components, we generally find increases in the absolute values of the cross-correlations, compared to 2 to 8 year cycles. However, the correlations remain relatively low for this cycle length. The overall mixed pattern of the correlations is preserved for the full period and for the postwar period.

A comparison of correlations for 8 to 20 year cycle components to those for 8 to 40 year cycles reveals generally smaller correlations in absolute terms for the shorter band. Some changes take place in the timing and sign of the peak correlations. Correlations for components extracted for 10 to 50 year cycles show almost the very same pattern in terms of magnitude and timing as those for 8 to 40 year cycles. The peak correlations for money growth and inflation components show larger, all positive, values for 10 to 50 year cycles.

Next, we extract 2 to 40 year cycles. One may argue that the cycles of 2 to 8 years are not orthogonal to the cycles of 8 to 40 years. The cross-correlations for 2 to 40 year cycles are typically lower than the ones for the 8 to 40 year cycles. However, the pattern is mostly preserved. Money growth shows no common pattern across countries for correlations with real GDP growth, whereas the pattern for correlations of money growth with inflation is evident and similar to the pattern for 8 to 40 year cycles.

In addition, we separate out periods of large changes in money growth. The average across the eleven countries shows a money growth rate of 3.2% in 1913 that increases to 24.2% in 1917 and falls to -3.3% in 1921. It moved up to 3.3% by 1925 and fell to -2.8% by 1932. It peaked again by 1944 with 23.2% growth and eventually came down to 6% by 1950. Another episode of dramatic money growth changes occurred in the 1970s and 1980s. In 1970, money growth was at 9.5% and climbed to 17% in 1972 followed by a slow decline to below 8% not until 1988. Hence, we analyze correlations for the combined period 1913-1950 and 1970-1988. For 2 to 8 year and 8 to 40 year cycles, the correlations of money growth and real GDP growth typically increase with some negative correlations becoming positive. However, the mixed pattern across countries persists and no common features are apparent. On the other hand, correlations for both frequency bands get mostly amplified during these extreme periods for inflation. Some cases also show a sharper spike around the peak correlation. These results are essentially what one would expect if money growth is a driving force behind the fluctuations of inflation.²²

Lastly, we extend our analysis to extracting cyclical components with the Baxter-King instead of the Christiano-Fitzgerald band-pass filter. A comparison will allow us to assess the extent of phase shift and remaining non-stationarity in the filtered components of the Christiano-Fitzgerald method. The Baxter-King filter imposes stationarity and is symmetric,

²² We also studied the period of low money growth rates, namely 1880-1912, 1951-1969, and 1989-2001. The qualitative results remain unchanged. However, the majority of correlations decreased somewhat in absolute terms and there are numerous sign changes. The ranges of magnitude of the correlations are similar to the ones obtained for the postwar period.

so that phases are not shifted by this filter. We find that some phase shift is present, but it is generally not very extensive. Also, standard unit root tests on the Christiano-Fitzgerald filtered components show little evidence of unit roots.

Over the full sample, the cross-correlation results for the Baxter-King filtered components for 8 to 40 year cycles are not much different from the ones obtained with the Christiano-Fitzgerald filter, with only a few changes in timing for the peak correlations. We conclude from these results that the Christiano-Fitzgerald filter does introduce small phase shifts for some series, but it has no effect on the overall qualitative results in our study.²³

3.3.5 *Structural Change*

An important question is whether the long run relationships, that show a systematic pattern for the calculated correlations, are stable over time or whether they show structural changes over time due to changes in monetary and fiscal policy regimes, or other factors like changes in institutions. Sargent and Surico (2008) argued that low-frequency quantity theoretic relationships are not policy invariant. They fitted time-invariant and time-varying vector autoregressions to US data from 1900 to 2000 and calculated a spectral measure at frequency zero that approximates the time-domain equivalent of correlation. Their approximation is valid at or near frequency zero and requires covariance stationary variables. They related the estimated vector autoregressions to DSGE models and derived how changes in monetary policy affect the zero-frequency relationships between money growth and inflation. They found that their zero-frequency spectral measure of the relation between money and nominal variables changes over time. Based on graphs of filtered series, they pick 1929, 1955 and 1976 as break dates for the time-invariant VAR case but do not formally test for breaks.

Rolnick and Weber (1997) argued instead that the relationship of money with other variables depends on whether there is a commodity or fiat monetary standard. They calculated average growth rates and inflation rates over an entire sample period for each of 15 countries. They found that under fiat standards correlations of money growth with output and inflation are higher than under commodity standards.²⁴ They did not apply formal tests for breaks either

²³ Using these filters for real time analysis would require a careful assessment of the trade-offs between phase shifts and sharpness of the filter, which is beyond the scope of this paper.

²⁴ They considered a commodity standard for the US from 1820-1932 and the fiat standard from 1933-1991.

In our analysis, we apply formal tests for multiple structural changes at unknown dates, proposed by Bai and Perron (1998, 2003).²⁵ It is important not to impose candidate break dates. Hansen (2001) illustrated for the Chow test how imposing a break date can lead to false inference about breaks, e.g., finding breaks when there are none. We apply the tests of Bai and Perron to the dynamic correlations that we calculated for the longer-run filtered components of 8 to 40 years in order to check whether the uncovered regularities in the data are structural features of the economy that are independent of monetary and fiscal policies which have changed over time. The methodology of Bai and Perron allows us to estimate the number of breaks and the break dates consistently and also to test for the significance of the breaks. Bai and Perron (2003) developed an efficient algorithm to obtain global minima for the sum of squared residuals. The method also deals with possible heteroskedasticity and autocorrelation in the residuals.

We first apply the double maximum test UDmax that is based on sequential F-type tests of the null hypothesis of no break against several alternative hypothesis of 1 and up to 5 breaks at all possible (unknown) break dates. This test allows us to establish whether there are any breaks at all in the sample. If there are breaks, we will apply Bai and Perron's sequential sup-F test of the null hypothesis of l breaks against the alternative hypothesis of $l+1$ breaks in order to determine the number of breaks and the break dates. We set the trimming parameter $\varepsilon = 0.15$ so that the minimal segment has at least εT observation, where T is the full sample size. We consider a partial structural change model where only the constant term in the model changes and also a full structural change model where the constant and the slope coefficient change both. We find much less evidence of structural change in the full model and therefore present in Table 3 only the results for the partial structural change model.

We apply the structural change tests to a linear model with the filtered money component as the left hand side variable and the filtered component of real GDP and inflation in turn as the left hand side variable in bivariate regressions.²⁶ We are interested only in the stability of the longer-run relations for the 8 to 40 year cycles for which we found systematic patterns. Table 3 reports results for the UDmax test for the peak correlations.

For the UDmax test and the sequential sup-F tests, we calculate bootstrap critical values by fitting again MA processes to the unfiltered data. However, this time we impose the variance-covariance structure found in the real data onto the data generating process used

²⁵ We downloaded the GAUSS code from Pierre Perron's website at: <http://people.bu.edu/perron/code.html>

²⁶ Reversing the order does not affect the test results.

for the simulations. We again filter each artificial sample and apply Bai and Perron's UDmax and sequential sup-F tests to the filtered data. We use 5000 replications and calculate the 1%, 5% and 10% critical values for the tests.

The results in Table 3 for the UDmax test show some instability for the relation of money with real GDP for the 8 to 40 year components. Canada has significant breaks at the 5% level in 1928, 1946, 1964 and 1982. Denmark shows even more breaks but only at the 10% level of significance: in 1902, 1919, 1936, 1949, 1953 and 1971. The UK also has breaks that are significant at the 5% level in 1932 and 1949. On the other hand, the longer-run relationships of money growth and inflation components are stable across all countries. This result is consistent with the findings in Christiano and Fitzgerald (2003) for the US that there are no structural breaks in longer cycles of 20 to 40 years. However, they found that cycles of 8 to 20 years show instability when a break is imposed in 1961. This implies that results are driven by the longer end of the cycle or are due to imposing a break date instead of determining it endogenously (Hansen, 2001). Our results show that the longer-run money and inflation relationship is a structural feature or stylized fact of the economies that is not affected by changes in monetary policies or monetary standards.

4. Conclusion

We studied the behavior of money growth in relation to real GDP growth and inflation with a data set spanning 122 years without gaps. For this purpose, we extracted the business cycle and longer-term components from each time series for eleven countries over the period 1880 to 2001 and for thirteen countries over the period 1946 to 2001. As we have a harmonized data set, we were able to compare results across countries in a consistent framework. We looked for effects of money growth that are valid despite different policies and institutions across countries and over time. The filtered components allowed us to assess the role of money growth at the business cycle frequency and for longer-term cycles.

We applied the recently developed band-pass filter of Christiano and Fitzgerald (2003) to extract from the data the periodic fluctuations for various frequency bands. This filter produces covariance stationary longer-term components when data have unit roots, which low-pass filters for example do not. It also provides a better approximation to the ideal but unfeasible filter than other commonly used filters. Previous research with filters has mostly focused on the business cycle only. Furthermore, we applied tests for multiple structural changes at unknown dates (Bai and Perron, 1998 and 2003).

At the business cycle frequency, we generally find relatively low dynamic cross-correlations when money growth is involved, consistent with other studies. Across countries, real GDP growth and inflation components show no obvious pattern in relation to the money growth component. Separating out the postwar period does not improve matters either. Country and time-specific features seem to matter at this frequency, like idiosyncratic shocks and differences in monetary policies.

Beyond the business cycle frequency, results are quite different for the longer-term effects in the frequency band from 8 to 40 years. Compared to the business cycle frequency, correlations are in general much larger in absolute terms. There is no obvious pattern for the correlations of the components of money growth and real GDP growth and structural change tests reveal some instability. However, a clear pattern emerges for components of money growth in relation to those of inflation. Correlations are almost all positive and highly significant for the long spans. We found the longer-run relationships for money growth and inflation to be highly stable over time for all countries. Qualitative results are quite similar for the full and postwar periods and are not sensitive to excluding periods of high money growth from the sample. The results for money growth and inflation components show for six countries that money growth leads inflation by one to three years and the other five countries show that the effects occur contemporaneously for the full sample. These correlations are in the range from 0.64 to 0.90, with only one exception of 0.40 for Switzerland.

Our findings are consistent with those of Friedman and Schwartz (1963, 1982) and therefore they are also consistent with Friedman's interpretation of the quantity theory of money. We contributed to the literature by applying modern econometric techniques that account for possible non-stationarity in the data instead of using phase-averages. Furthermore, we contributed to the literature by extending the historic analysis in Friedman and Schwartz to nine other countries besides the US and the UK. The band-pass filter that we applied produces covariance-stationary longer-run components of money. These components should prove useful in empirical versions of theoretical models for anchoring inflation expectations, such as Christiano et al. (2007) and Beck and Wieland (2008).

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Table 1a. Correlations of Band-Pass Filtered Components for Cycles of 2 to 8 Years (Christiano-Fitzgerald Filter): Money Growth and Real GDP Growth Components, 1880-2001

Country	Relative st. dev.	Cross-correlations of money in period (t) with real GDP in period (t+i):							
		i= -2	i= -1	i=0	i=1	i=2	i=3	i=4	i=5
Canada	1.19	-.05	-.008	.20**	.08	-.14	.03	-.19*	.02
Denmark	.54	.001	-.15*	-.18**	.74***	-.33***	-.23**	-.05	.20**
France	1.18	-.12	-.07	.12	-.09	-.21**	.26***	-.11	.05
Italy	1.63	.20**	.17*	.05	-.28***	.04	.14	-.11	-.005
Japan	.72	-.07	.20**	-.20**	-.08	.18*	.08	-.05	.05
Netherlands	.62	-.36***	.44***	.20**	-.39***	.08	.02	-.11	.18**
Norway	.86	.16*	.14*	-.16*	-.22**	.05	.21**	.01	-.13
Sweden	.41	-.18**	.31***	.06	-.24**	-.06	.17**	-.02	-.16*
Switzerland	.71	-.12	-.10	-.04	.12	.05	.05	-.12	-.03
UK	.86	-.04	.12	.13	-.01	-.15	-.19*	.06	.18*
US	1.29	-.04	.12	.50***	-.06	-.22**	-.22**	-.12	.12

Note: Real GDP for Japan covers the period 1885 - 2001. Data for money were available up to 1999 for Denmark, and up to 2000 for Sweden. The relative standard deviation is the standard deviation of the real GDP component divided by that of the money component. Significance, based on 20,000 bootstrap replications for each correlation reported, is denoted by *, ** and *** for the 10%, 5% and 1% level, respectively.

Table 1b. Correlations of Band-Pass Filtered Components for Cycles of 8 to 40 Years (Christiano-Fitzgerald Filter): Money Growth and Real GDP Growth Components, 1880-2001.

Country	Relative st. dev.	Cross-correlations of money in period (t) with real GDP in period (t+i):							
		i= -2	i= -1	i=0	i=1	i=2	i=3	i=4	i=5
Canada	.88	.56***	.57***	.47**	.26	.03	-.17	-.26	-.23
Denmark	.32	.37**	.28	.08	-.20	-.45**	-.59***	-.59***	-.45**
France	.78	-.23	-.27	-.27	-.21	-.11	.04	.20	.37**
Italy	.41	-.51***	-.46**	-.34*	-.15	.06	.26	.39**	.45**
Japan	.37	-.57***	-.59***	-.52***	-.38**	-.21	-.05	.05	.07
Netherlands	.68	-.16	-.34*	-.47**	-.52***	-.44**	-.26	-.003	.25
Norway	.28	-.35*	-.47**	-.48**	-.37**	-.20	-.02	.12	.20
Sweden	.30	-.21	-.32*	-.41**	-.46**	-.46**	-.38**	-.26	-.11
Switzerland	.63	-.02	.27	.45**	.47**	.30*	.04	-.18	-.26
UK	.57	.43**	.19	-.13	-.44**	-.65***	-.71***	-.62***	-.43**
US	.62	.65***	.69***	.56***	.31*	.02	-.20	-.29*	-.26

Note: See Table 1a.

Table 2a. Correlations of Band-Pass Filtered Components for Cycles of 2 to 8 Years (Christiano-Fitzgerald Filter): Money Growth and Inflation Components, 1880-2001

Country	Relative st. dev.	Cross-correlations of money in period (t) with inflation in period (t+i):							
		i= -2	i= -1	i=0	i=1	i=2	i=3	i=4	i=5
Canada	.94	-.15	-.09	.28**	.23**	.02	-.16*	-.19*	-.07
Denmark	.36	-.05	.006	-.37***	-.02	.18*	.14	.09	-.11
France	1.09	-.0007	.02	.05	.33***	-.20*	-.16	-.19*	.10
Italy	2.89	.02	-.22**	-.06	.22**	-.14	-.01	.07	-.006
Japan	1.35	-.09	.12	.12	.46***	.009	-.30**	-.30**	-.19*
Netherlands	.30	.005	.14*	-.15*	.002	-.09	.07	.17*	.02
Norway	1.03	-.26**	.01	.04	.11	-.15	.09	.24**	-.09
Sweden	.59	.08	.26**	.08	-.16	-.22*	-.002	.10	.08
Switzerland	.78	-.08	.004	.009	.05	.03	-.001	-.04	-.03
UK	.96	-.04	-.009	-.03	.14	.10	-.008	-.12	-.07
US	.87	-.12	.02	.008	.26**	.09	-.20*	-.10	-.15

Note: See Table 1a. The relative standard deviation is the standard deviation of the inflation component divided by that of the money component.

Table 2b. Correlations of Band-Pass Filtered Components for Cycles of 8 to 40 Years (Christiano-Fitzgerald Filter): Money Growth and Inflation Components, 1880-2001

Country	Relative st. dev.	Cross-correlations of money in period (t) with inflation in period (t+i):							
		i= -2	i= -1	i=0	i=1	i=2	i=3	i=4	i=5
Canada	.82	.20	.46**	.66***	.74***	.68***	.54***	.36**	.22
Denmark	.66	-.24	-.04	.22	.46**	.62***	.64***	.53***	.33*
France	1.26	.60***	.76***	.82***	.77***	.64***	.48**	.32*	.21
Italy	1.59	.73***	.85***	.89***	.83***	.68***	.46**	.22	.01
Japan	1.37	.42**	.68***	.84***	.87***	.75***	.51***	.23	-.01
Netherlands	.57	.34**	.55***	.70***	.73***	.66***	.51***	.31*	.14
Norway	.72	.68***	.86***	.90***	.80***	.58***	.31*	.07	-.10
Sweden	.91	-.06	.03	.20	.41**	.58***	.66***	.60***	.42**
Switzerland	1.42	.30*	.40**	.39**	.27	.08	-.12	-.28*	-.36**
UK	1.06	.59***	.73***	.76***	.69***	.53***	.33*	.14	.002
US	.66	.32*	.54***	.68***	.71***	.61***	.43**	.24	.10

Note: See Table 2a.

Table 2c. Correlations of Band-Pass Filtered Components for Cycles of 8 to 40 Years (Christiano-Fitzgerald Filter): Money Growth and Inflation Components, 1946-2001

Country	Relative st. dev.	Cross-correlations of money in period (t) with inflation in period (t+i):								
		i=-2	i=-1	i=0	i=1	i=2	i=3	i=4	i=5	
Belgium	.66	-.04	.13	.32	.47*	.51*	.45	.32	.19	
Canada	.92	.20	.35	.54**	.64**	.69**	.68**	.60**	.48*	
Denmark	.68	-.28	-.21	-.007	.25	.45*	.52*	.45*	.30	
France	1.97	.67**	.75***	.78***	.64**	.46*	.29	.15	.05	
Germany	.70	-.18	.04	.29	.49*	.57**	.53*	.40	.24	
Italy	1.51	.58**	.68**	.76***	.43	.18	.03	-.01	-.0003	
Japan	1.70	.31	.63**	.93***	.83***	.61**	.33	.08	-.08	
Netherlands	.78	-.03	.04	.11	.21	.30	.37	.42*	.44*	
Norway	.74	.46*	.64**	.69**	.66**	.58**	.50*	.45*	.43	
Sweden	.67	-.01	-.09	-.20	-.27	-.27	-.20	-.09	.04	
Switzerland	.64	-.26	-.42*	-.42*	-.23	.07	.35	.49*	.43*	
UK	1.17	.09	.19	.33	.47*	.58**	.62**	.58**	.47*	
US	.70	.20	.28	.42	.40	.40	.41	.44*	.46*	

Note: See Table 2a. Data for Belgium cover the period 1947 to 2001. Data for Germany cover 1949 to 2001.

Table 3. Structural Change Test of Bai and Perron (2003) for Band-Pass Filtered Components for Cycles of 8 to 40 Years (Christiano-Fitzgerald Filter), 1880-2001 and the Postwar Period for Belgium and Germany

Country	Relation of money component in period (t) with real GDP component in period (t+i): UDmax-test	Relation of money component in period (t) with inflation component in period (t+i): UDmax-test
Belgium	120.75, i=-1	94.35, i=2
Canada	384.70**, i=-1	2.53, i=1
Denmark	91.58*, i=3	16.42, i=3
France	7.09, i=5	3.25, i=0
Germany	73.76, i=5	8.55, i=2
Italy	11.53, i=-2	5.90, i=0
Japan	26.63, i=-1	3.71, i=1
Netherlands	8.89, i=1	3.79, i=1
Norway	15.76, i=0	7.99, i=0
Sweden	14.26, i=1	35.19, i=3
Switzerland	8.39, i=1	28.03, i=-1
UK	311.16**, i=3	25.01, i=0
US	10.92, i=-1	3.43, i=1

Note: We allow for up to 5 breaks at unknown points in time. Critical values for the UDmax test are calculated from 5000 bootstrap replications of the test for every correlation in the table. The values chosen for "i" correspond to the peak correlations of the filtered components. Significance is denoted by *, ** and *** for the 10%, 5% and 1% level, respectively.

Figure 1. Filter gains for cycles of 8 to 40 years

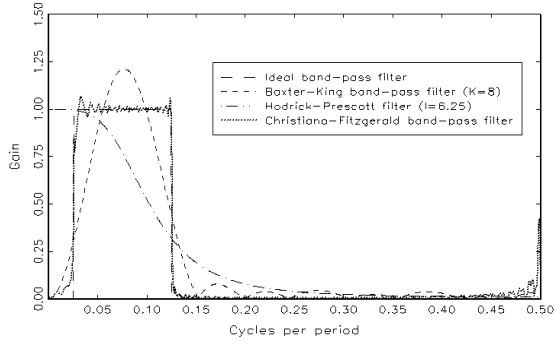


Figure 2.1 Spectrum of money growth

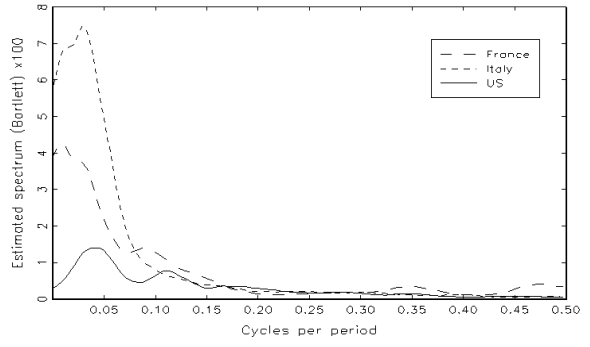


Figure 2.2 Spectrum of real GDP growth

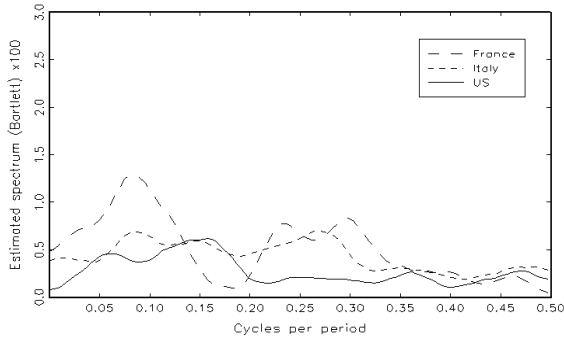


Figure 2.3 Spectrum of inflation

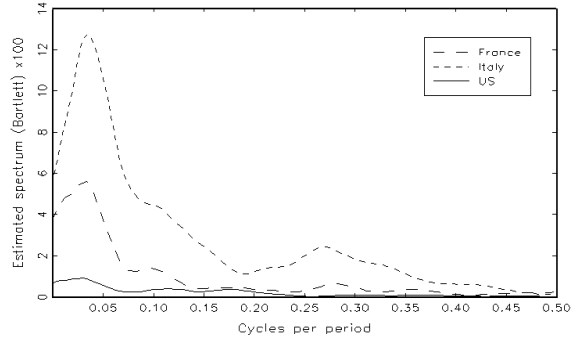


Figure 3.1 Coherence of money growth and real GDP

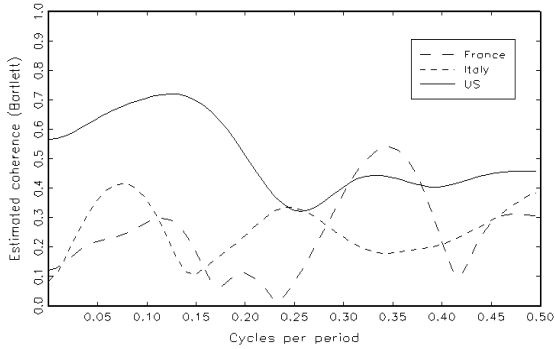


Figure 3.2 Coherence of money growth and inflation

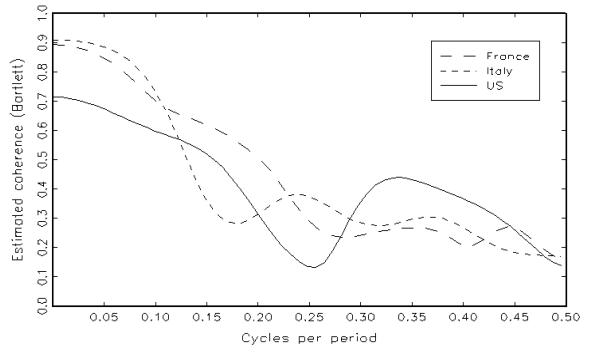


Figure 4. Money growth for Italy for 8-40 year cycles

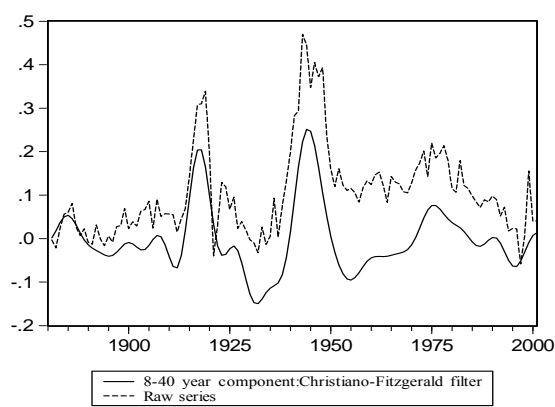


Figure 5. Money growth and inflation for 8-40 year cycles: Italy

