



BUSINESS SCHOOL
Te Kura Pakihi

ISSN 1178-2293 (Online)

University of Otago
Economics Discussion Papers
No. 1715

DECEMBER 2017

Simulation Evidence on Herfindahl-Hirschman Indices as Measures of Competitive Balance

P. Dorian Owen¹ · Caitlin A. Owen²

Address for correspondence:

Dorian Owen
Department of Economics
University of Otago
PO Box 56
Dunedin
NEW ZEALAND
Email: dorian.owen@otago.ac.nz
Telephone: 64 3 479 8655

Simulation Evidence on Herfindahl-Hirschman Indices as Measures of Competitive Balance

P. Dorian Owen¹ · Caitlin A. Owen²

Abstract Measurement of the degree of competitive balance, how evenly teams are matched, is central to the economic analysis of professional sports leagues. A common problem with competitive balance measures, however, is their sensitivity to the number of teams and the number of matches played by each team, i.e., season length. This paper uses simulation methods to examine the effects of changes in season length on the distributions of several widely used variants of the Herfindahl-Hirschman index applied to wins in a season. Of the measures considered, a normalized measure, accounting for lower and upper bounds, and an adjusted measure perform best, although neither completely removes biases associated with different season lengths.

Keywords Herfindahl-Hirschman · Competitive balance · Simulation

JEL Classification D63 · C63 · L83 · Z20

Contact:
Dorian Owen
e-mail: Dorian.Owen@otago.ac.nz

¹ Department of Economics, University of Otago, PO Box 56, Dunedin 9054, New Zealand

² Department of Information Science, University of Otago, PO Box 56, Dunedin 9054, New Zealand

1 Introduction

Measurement of the degree of competitive balance, how evenly teams are matched, continues to attract attention in the economic analysis of professional sports leagues. In any single match, it takes two teams, each attempting to beat their opponent, to jointly produce a sporting contest (Neale 1964). Similarly, the overall league competition reflects the aggregation of all the outcomes of the individual pairwise matches; this output is the joint product of all the teams in the league. The extent to which playing strengths vary across teams therefore has important implications for the degree of uncertainty surrounding the outcomes of individual matches and of overall championships. According to the uncertainty of outcome hypothesis (Rottenberg 1956), the more predictable the outcome of a sporting contest, the less interest there will be from consumers, reflected in lower match attendances and lower television audience ratings.

Measurement of competitive balance is therefore important, whether in tracking its movements over seasons and evaluating the effects of regulatory and institutional changes, or in examining the effects of changes in competitive balance on consumer demand for the sporting product (Fort and Maxcy 2003). Because competitive balance is concerned with the degree of inequality of match and/or championship outcomes arising from differences in the strengths of teams, it is natural that summary measures of dispersion, inequality and concentration are commonly used (Humphreys and Watanabe 2012; Owen 2014).

A common problem with such measures, however, is their sensitivity to the number of teams and the number of matches played by each team, i.e., season length. This makes comparisons of levels of competitive balance difficult, especially when these commonly involve different leagues that exhibit widely differing numbers of teams or games played. Major League Baseball, for example, has 30 teams playing 162 games each in a regular season, whereas the English Premier League has 20 teams playing 38 games each. A drawback with the use of standard ‘off-the-shelf’ measures of dispersion, inequality and concentration is that they do not

take into account the design characteristics of sports leagues. Leagues' playing schedules (the list of fixtures) impose limits on the dispersion of the distribution of wins or points and consequently limit the range of feasible values of these measures (Depken 1999; Utt and Fort 2002; Owen et al. 2007; Owen 2010; Gayant and Le Pape 2015); moreover, these limits depend on the number of teams and games.

A desirable property of any measure of competitive balance used for cross-league comparisons (or for comparison of balance in a single league with changing numbers of games per season over time) is independence with respect to the numbers of teams or games played per season. Recent simulation analyses show that the location of the distribution of the popular ratio of standard deviations measure (Noll 1988; Scully 1989), which is commonly advocated for comparisons involving scenarios with different numbers of teams and/or games played, is in fact highly sensitive to season length due to an inappropriate normalization (Owen and King 2015; Lee et al. 2016). In this paper, we examine simulation evidence on the distributional properties of an alternative family of CB measures based on the Herfindahl-Hirschman index applied to wins.

In Section 2 we describe the different variants of the Herfindahl-Hirschman index commonly used in the sports economics literature; these vary in the extent to which they incorporate information on the limits imposed by the league's playing schedules. In Section 3 we outline the details of the simulation design used to examine the effects of different distributions of team strength, number of teams and number of games played on the distributions of these different variants. The results of the simulation analysis are reported and interpreted in Section 4. We find that accounting for both the lower and upper bounds of the concentration measure improves its performance across the degrees of imbalance considered. However, all the variants tend to provide values that are biased upwards if the number of

matches is small, so we investigate further some adjustments that could improve this aspect of their performance. Conclusions are summarized in Section 5.

2 Herfindahl-Hirschman Indices of Competitive Balance

Drawing on the industrial organization literature on firm concentration, a common measure of competitive balance is the Herfindahl-Hirschman index (*HHI*), which is based on the sum of squares of market shares. When applied to the distribution of wins across teams in a season, ‘market share’ is interpreted as the number of wins by a team as a proportion of total wins by all the teams in the league in that season (Depken 1999):

$$HHI = \sum_{i=1}^N (w_i / \sum_{i=1}^N w_i)^2, \quad (1)$$

where w_i is the number of wins for team i and N is the number of teams in the league. Equal shares of wins for each team minimize the value of *HHI* at $1/N$ (corresponding to a situation of perfect balance); increases in the value of *HHI* reflect decreases in competitive balance as wins become less equally distributed and more concentrated among the stronger teams in the league.

This definition is appropriate for sports for which the result of each match is a win for one team and a loss for the other (i.e., there are no draws or ties). In some sports, drawn (tied) matches are feasible or common (as in the case of association football), so that the points assigned to each outcome (win, draw, loss) need to be taken into account. In such cases, *HHI* can be defined in terms of points instead of wins, and total points can represent the total of points actually accumulated by all teams or the feasible maximum of available points.

Because the lower-bound value of *HHI*, $HHI^{lb} = 1/N$, corresponding to perfect balance in terms of the shares of wins or points, depends on the number of teams in the league, Depken (1999) suggests controlling for variation in N when interpreting movements in *HHI* over time or comparing balance in different leagues. He proposes an adjusted measure:

$$dHHI = HHI - 1/N, \quad (2)$$

i.e., the deviation of HHI from its lower-bound perfect-balance value. Equal shares of wins (perfect balance) imply $dHHI$ has a minimum value of zero and, as with HHI , increases in $dHHI$ away from zero represent decreases in competitive balance (increases in competitive imbalance).

Rather than subtracting the lower bound of HHI , Michie and Oughton (2004) adopt a multiplicative adjustment, defining their ‘ H Index’, here denoted $mHHI$, as a ratio form:

$$mHHI = HHI/(1/N) = N.HHI. \quad (3)$$

As the degree of competitive imbalance increases, $mHHI$ also increases, but $mHHI \geq 1$, i.e., the lower bound of $mHHI$ is unity.¹

A distinctive feature of market share in the context of teams’ wins in a sports league is that (if $N > 2$) no team can be the equivalent of a monopolist, because teams cannot win games in which they do not play.² As a result, the league’s playing schedules imply an upper limit on the degree of imbalance in the distribution of wins, and consequently impose an upper bound on HHI . The upper bound is determined by the ‘most unequal distribution’ of match outcomes (Horowitz, 1997; Fort and Quirk, 1997; Utt and Fort, 2002). This involves one team winning all of its games, the second team winning all except its game(s) against the first team, and so on down to the last team, which wins none of its games. If playing schedules are balanced, each team in the league plays every other team the same number of times, K . Each team plays $G = K(N - 1)$ games and, overall, there are $KN(N - 1)/2$ games in the season. Assuming balanced

¹ Often, this form of the index is multiplied by 100 to give a perfect parity score of 100.

² HHI or $dHHI$ can also be applied to shares of championships over several seasons (e.g., Eckard 1998; Kringstad and Gerrard 2007; Dittmore and Crow 2010; Addesa 2011; Leeds and von Allmen 2014, p.164; York and Miree 2015). In that context, it is feasible, in principle, for one team to be a monopolist and win the championship in every season in the time span considered.

scheduling, Owen et al. (2007) derive an expression for the upper bound for the HHI for wins (or points), denoted HHI^{ub} , given by:

$$HHI^{ub} = 2(2N - 1)/[3N(N - 1)], \quad (4)$$

with $HHI^{ub} < 1$ if $N > 2$. They propose a normalized version of HHI , HHI^* , which adjusts for the lower and upper bounds:

$$HHI^* = (HHI - HHI^{lb})/(HHI^{ub} - HHI^{lb}). \quad (5)$$

As with all the previously discussed variants of HHI , decreases in competitive balance (increases in competitive imbalance) are associated with increases in HHI^* . An advantage of this normalization is that, for any set of match outcomes, HHI^* is bounded in the interval [0, 1], with zero indicating perfect balance and one representing maximum imbalance. Because of its ease of use and interpretation, Hall and Tideman (1967) consider having a [0, 1] range to be a desirable property for any concentration measure.³ However, Van Scyoc and McGee (2016, p.1040) ask: “[d]oes an [HHI^*] of 0.43 in Major League Baseball mean exactly the same thing as a 0.43 in the National Football League? ... it is not clear that [the] arithmetic transformation actually leaves us with a useful measure.” They suggest that neither $dHHI$ nor HHI^* is fully purged of the influence of N and G . For the case of perfect balance (i.e., all teams of equal strength) and a balanced playing schedule, they show that $E(dHHI) = 1/NG = 1/[KN(N - 1)]$ and $E(HHI^*) = 3(N - 1)/[(N + 1)G] = 3/[K(N + 1)]$ (Van Scyoc and McGee, 2016, p.1040, fn. 10, and substituting $G = K(N - 1)$). At least for the case of perfect competitive balance, this implies both $dHHI$ and HHI^* have expected values very close to zero only for large N and/or K .

³ Gayant and Le Pape (2015) show that HHI^* (which they refer to as the ‘Herfindahl Ratio of Competitive Balance’) is equivalent to a normalized measure defined in terms of the variance of teams’ shares of total points earned. This “strengthens the validity of the normalization process” and “shows clearly that there is intrinsically no difference between calculating a variance or a Hirschman-Herfindahl index when measuring the level of competitive imbalance in a league” (Gayant and Le Pape 2015, p.115).

All these variants of HHI , applied to shares of wins or points in a season, are widely used in recent empirical analyses of competitive balance. The unadjusted HHI , as in equation (1), continues to be applied to wins despite arguments for the desirability of adjusting for its lower and upper bounds. For example, Jane (2014, 2016), analysing the determinants of game-day attendance for the National Basketball Association (NBA), uses the unadjusted HHI applied to the shares of wins.⁴ Del Corral et al. (2016) also calculate unadjusted HHI indices (applied to the end-of-season *expected* number of victories in the NBA).

Following Depken's (1999) suggestion, adjustment for HHI 's lower bound is commonly implemented. For example, Larsen et al. (2006) calculate $dHHI$ applied to the shares of wins in the National Football League (NFL) to allow for league expansions (increases in N) over time. They use $dHHI$ as their dependent variable in modelling the effects of different determinants of competitive balance (e.g., the introduction of free agency, the salary cap, player strikes and the distribution of playing talent).⁵ Fenn et al. (2005), in a study of the National Hockey League (NHL), and Totty and Owens (2011), for the NBA, NHL and NFL, adopt a similar approach.

In addition to Michie and Oughton (2004, 2005), a multiplicative normalization taking into account HHI^{lb} (equivalent to $mHHI$) is also widely used, including by Brandes and Franck (2007), Lenten (2008, 2015, 2017), Pawlowski et al. (2010), Mills and Fort (2014), Gasparetto and Barajas (2016), Eckard (2017) and Tainsky et al. (2017).

Normalized versions of HHI that take into account both lower and upper bounds are also becoming more widely adopted. In addition to Owen et al. (2007), Manasis et al. (2015) use HHI^* , along with six other seasonal balance measures in a panel data analysis of attendance

⁴ HHI is applied cumulatively to take into account the timing of each game; for a game at time t , the share of wins for team i is calculated as team i 's cumulative wins divided by the total of games played in the league prior to the game at time t .

⁵ They also proxy the upper bound of HHI "by consulting actual playing schedules and by assuming that wins are distributed in alphabetical order" (Larsen et al., 2006, p.380); they plot the value for this proxy graphically but $dHHI$ is used in their regression analysis.

demand functions for eight European football leagues. Martinez and Willner (2017) apply HHI^* (along with Gini and standard deviation measures) to data for the top division of English football. Ramchandani (2012) uses a normalized version of HHI , defined as $(HHI - 1/N)/(1 - 1/N)$, applied to points in 10 European football leagues; this accounts for the lower bound of HHI , but sets the upper bound at unity, which is not feasible in a sports league with $N > 2$, as previously discussed.

In addition to these widely used variants of HHI , we also examine a version of the ‘record-based’ balance measure proposed by McGee (2016), which can be viewed as an adjusted version of the other HHI measures. For the case of a balanced playing schedule with each team playing $G = K(N - 1)$ matches, and no draws, his ϕ_r measure is defined (in our notation) as:

$$\phi_r = \left[\frac{3(\theta_r - N)}{N(G + 2K - 3)} \right], \quad (6)$$

where $\theta_r = \sum_{i=1}^N (2w_i - G)^2 / G$ (McGee, 2016, eq. (6)). McGee makes the simplifying assumptions that the degree of imbalance is transitive (team A is always favoured over all other teams, B is favoured over all others apart from A, and so on) and uniform, such that for each of the $N(N - 1)/2$ pairings of teams, the stronger team always has a common probability, p , of winning. Under these conditions, McGee shows that $E(\phi_r) = (2p - 1)^2$ and is, therefore, independent of N or K . If $p = 0.5$ (perfect balance), $E(\phi_r) = 0$, and if $p = 1$, $E(\phi_r) = 1$ (perfect imbalance). McGee’s measure can be interpreted as equivalent to an adjusted HHI measure as $dHHI = HHI - (1/N) = (\theta_r/N^2G)$ (Van Scyoc and McGee 2016, eq. (7)). Substituting for $dHHI$ and its upper bound, $(N + 1)/[3N(N - 1)]$ (Owen et al., 2007, p.301), in equation (5) yields:

$$HHI^* = \frac{dHHI}{(N + 1)/[3N(N - 1)]} = \frac{3\theta_r}{NK(N + 1)}.$$

Substituting for θ_r in terms of ϕ_r , from equation (6), and solving for ϕ_r yields:

$$AdjHHI^* = \phi_r = \left[\frac{K(N+1)HHI^* - 3}{K(N+1) - 3} \right]. \quad (7)$$

Under McGee's assumptions, although the expected value of $AdjHHI^*$ is zero if there is perfect balance ($p = 0.5$), this measure will produce sample values that are negative. If it is considered important to maintain zero as the lower bound of the calculated balance measure, one possibility, following the approach adopted by Lee et al. (2016) with standard deviation measures, is to define a truncated version of this measure as:⁶

$$TruncAdjHHI^* = \max(0, AdjHHI^*) \quad (8)$$

To examine whether any of these variants of HHI serves as a useful measure for comparing competitive balance in situations with differing values of K and N , we conduct a simulation analysis. This allows us to examine how the distributions of the different balance measures behave as different aspects of league design (such as N or K) are varied, for known distributions of the strengths of teams in the league.

3 Simulation Design

The effects of varying season length on the distributional properties of the different HHI -based measures of within-season competitive balance are studied by simulating results for different scenarios corresponding to different values of N (the number of teams), K (the number of rounds of matches), and different distributions of team strengths. The simulation design is similar to that used by Owen and King (2015).

⁶ Interpretation of $TruncAdjHHI^*$ compared to $AdjHHI^*$ is analogous to comparing adjusted R^2 values with the conventional R^2 . Negative measures of the truncated measure will usually occur only for relatively low levels of competitive imbalance. Note also that ϕ_r can be expressed as an adjusted version of each of the different variants of HHI previously considered; we focus on the relationship between ϕ_r and HHI^* because it is the simplest.

In the simulations, the playing schedules are balanced, in that each team plays every other team in the league the same number of times (a common format in, for example, association football leagues). The number of games played by each team, $G = K(N - 1)$, is therefore the same for every team.

Teams' playing strengths (S_i , $i = 1, 2, \dots, N$) determine the outcomes of matches. Strength ratings are normalized, so that the S_i ratings have a mean of zero. A team of average strength therefore has a strength rating of zero. Better/stronger teams have positive strength ratings; poorer/weaker teams have negative ratings. We use the Bradley-Terry (1952) model for paired comparisons to generate probabilities of each match outcome (home win, home loss) based on the relative strength ratings of the two opposing teams. If there are no draws (ties), the probability that home team i beats away team j , $p_{win,i,j}$, is given by:

$$p_{win,i,j} = \frac{\exp(S_i)}{\exp(S_i) + \exp(S_j)},$$

and the probability that home team i loses to away team j , $p_{lose,i,j} = 1 - p_{win,i,j}$. Match outcomes are simulated using the `rbinom()` function in R version 3.0.2 (R Core Team 2014) to produce a sequence of 1s (home wins) and 0s (home losses) for each match.⁷

The Bradley-Terry model design is flexible and can, in principle, incorporate a generic home advantage, team-specific home advantages, drawn (tied) matches (with different possible ratios of points allocated for wins and draws), or combinations of these (Rao and Kupper 1967; King 2011; Agresti 2013). However, the simulation results in Owen and King (2015) suggest that these variations have only minor effects on the key distributional properties of standard-deviation-based measures of competitive balance as N and K are varied. We therefore focus attention on the simplest model design with no home advantage and no draws. Team strengths

⁷ The R code for the simulations draws on and extends code in Marchi and Albert (2014, sections 9.3.2-9.3.4).

are also assumed to remain constant throughout the season, although the design can easily be generalized to allow updating of team strengths in response to simulated results as the season evolves (Clarke 1993; King et al. 2012).

The simulated outcomes for all $KN(N - 1)/2$ matches in the playing schedule for a season are combined to produce end-of-season shares of wins for each of the N teams and hence values of the different variants of HHI described in section 2. This process is repeated for 1000 seasons, giving a distribution of values for each end-of-season HHI measure, for a given distribution of team strengths and given values of league parameters N and K . Finally, all the stages of the simulation exercise are repeated for different assumptions about the distribution of teams' strengths and different values of N and K .

Match outcomes are simulated for five different distributions of strength ratings, ranging from perfect balance (with all teams of equal strength, i.e., $S_i = 0$ for all i) to a relatively high degree of imbalance. In principle, deviations of strength distributions from perfect parity can be specified in an infinite number of different ways. In the simulations, we follow Owen and King (2015) and characterize the different distributions by increasing the range of team strengths, $R = (\text{maximum strength} - \text{minimum strength})$, from 0 through to 5 with, in each distribution, teams equally spaced, from the strongest to the weakest team. Specifically, R takes the values 0, 1.25, 2.5, 3.75 and 5. Because the strength ratings are normalized to have zero means, each distribution also has a zero mean. Figure 1 illustrates the strength ratings for $N = 20$.

When constructing distributions of strength ratings for different values of N , but with the same level of 'strength inequality', a constant *range* of strength ratings is maintained but the slope of the plot of strength ratings against team number decreases as N increases. Details of the five strength rating distributions considered, for different values of N , are reported in Owen and King (2015, Supporting Information, Appendix A, Tables A1 to A4). While this is clearly

not the only possible pattern of departures from perfect balance, it has some desirable features. As N changes, the probability of the strongest team beating the weakest team remains constant, and an average-strength team has unchanged probabilities of beating the strongest and weakest teams. In addition, as N varies the standard deviation of strength ratings is approximately preserved for each of the values of R considered.

Simulations for 1000 seasons are repeated for combinations of different numbers of teams ($N = 10, 15, 20, 25$) and different numbers of rounds per season ($K = 2, 4, 6, 8, 10$). Although the number of games each team plays, G , can change as a result of varying N or K or both, we consider changes in N and K separately because both the lower and upper bounds of HHI are explicit functions of N but not K . We therefore expect variations in N and K to have different effects on the distributions of the HHI measures.

4 Simulation Results

For ease of interpretation, distributions of the various HHI -based measures of competitive balance, for different distributions of strength ratings, numbers of teams and rounds, are presented graphically by kernel density estimates (using the Epanechnikov kernel function in R).

Kernel densities for the unadjusted HHI (equation (1)), $dHHI$ (equation (2)), $mHHI$ (equation (3)) and HHI^* (equation (5)), for $N = 20$, $K = 2$ and different values of R , are presented in Figure 2. For all the measures, increasing competitive imbalance, i.e., increasing R from 0 through to 5, is reflected in the densities shifting to the right. Although the densities overlap, increasing degrees of competitive imbalance are associated with higher mean values of each of the variants of HHI , as would be expected for any credible balance measure. In this comparison, the main differences are the ranges and scales on the horizontal axes, reflecting the different adjustments to HHI . If we increase the number of rounds of matches to $K = 8$, as

in Figure 3, this increases the number of matches played overall; this reduces the variance of the density functions and, consequently, the separation between the densities for different values of R . Figures 2 and 3 demonstrate that, with fixed N and K , all of the HHI measures appropriately track the increased imbalance as R increases.

To examine the effects of varying the number of rounds of matches played by each team, we fix N and vary K , for a specific value of R . Figure 4 shows the results for $N = 20$ if there is perfect competitive balance, i.e., $S_i = 0$ for all i , so that $R = 0$. All the HHI measures display a similar pattern. As K increases and more matches are played between the same number of teams, the density functions shift leftwards towards each measure's minimum value and the variances of the densities decrease. With perfect balance, all the measures on average overestimate the degree of imbalance, but this upward bias decreases with more matches played. A similar pattern is observed for other values of R . For example, in Figure 5, with $R = 5$, $N = 20$ and K varying, the densities shift left as more matches are played. The main difference compared to the case of $R = 0$ is the positioning of the densities at higher values of their respective scales (reflecting a relatively severe case of imbalance between team strengths). Despite a high degree of imbalance, all the measures display similar responses to varying K , regardless of whether they adjust for the upper bound on HHI or not. This is not surprising given that HHI 's upper bound, HHI^{ub} in equation (4), is not a function of K .

However, both the lower and upper bounds of HHI do depend on the number of teams, so we would expect more obvious differences if N is allowed to vary for a given value of R . Therefore, we next compare the densities for a specific value of R and with K fixed, but varying N . Figure 6 shows the densities for $R = 0$ (perfect balance), $K = 2$ and varying N . In this experiment, the effects on the locations of the densities of the different HHI measures are much more dramatic. Even though, all the teams are equal in terms of strength, the unadjusted HHI measure shifts markedly towards zero as the number of teams increases, with no overlap

between the densities, reflecting the property that the lower bound of HHI , i.e., $1/N$, decreases as the number of teams increases. The other three measures are not subject to this problem because they take into account the lower bound of HHI . Otherwise, the density functions of the other HHI measures reflect the pattern observed with variation in K . i.e., a decrease in their variances and a reduction in their means as N , and hence the number of matches overall, increases. These patterns are all accentuated if K is set at a larger value.

Similar patterns are observed if we consider higher degrees of imbalance, as shown in Figures 7 and 8. For smaller values of R , the densities for Depken's $dHHI$ measure exhibit similar properties to those of $mHHI$ and HHI^* . However, for larger values of R , the effects on the densities for $dHHI$ are more marked as N varies, with the overlap between the densities decreasing as R increases (again, a feature that is accentuated for larger values of K). This is not unexpected, because as the degree of imbalance increases, the location of HHI 's upper bound becomes more relevant, and the calculation of $dHHI$ does not take this into account. What is perhaps more surprising is that the densities of $mHHI$, which adjusts multiplicatively for HHI 's lower bound, display less separation as N increases compared to $dHHI$. Apart from the scales, the densities for $mHHI$ in Figure 7 (and to a lesser extent Figure 8) exhibit similar behaviour to those of HHI^* , which does take into account HHI 's upper bound. However, the lack of an adjustment for the upper bound using $mHHI$ shows up more clearly as the number of matches increases due to higher values of K , as in Figure 9 for which $R = 5$ and $K = 10$.

Of the four measures considered, HHI^* , which accounts for both the lower and upper bounds of HHI , performs best across the various different combinations of values of R , N and K . However, as with the other three HHI -based measures, HHI^* tends to overestimate the degree of imbalance if season length is short, with fewer matches. As the number of matches played increases, the density of HHI^* shifts leftwards and converges with a decreasing variance. A similar property is observed with the standard deviation of win (or points) ratios, which also

overestimates imbalance for shorter seasons (Owen and King 2015; Lee et al. 2016). It is therefore relevant to examine whether an adjustment to HHI^* , based on McGee's measure in equation (6), can reduce or eliminate this 'short-season' overestimation.

Kernel density plots for $AdjHHI^*$, equal to McGee's ϕ_r measure (equation (7)), and its truncated version (equation (8)) are presented in Figure 10 for a league with $N = 20$ and varying K . With $R = 0$ (in the upper panel), the mean of $AdjHHI^*$ is approximately zero, even for a relatively small number of rounds ($K = 2$); this is confirmed by examining the numerical values of the quantiles of the simulated values. Consistent with this, negative values are common, with median values (for any K) being slightly negative. Not surprisingly, truncation leads to a piling up of the relative frequency at zero and upward bias in the measure with a mean value that is slightly positive (e.g., the mean value of $TruncAdjHHI^*$ is 0.010 for $N = 20$ and $K = 2$).⁸ As with all the other variants, increasing the number of games reduces the variance of the distribution. As the degree of imbalance increases, so does HHI^* , and the truncation has increasingly less effect, as can be seen for $R = 1.25$ in the lower panel of Figure 10.

As R increases further, $AdjHHI^*$ (and its truncated variant) begin to exhibit upward bias for low values of K . $AdjHHI^*$'s tendency to be biased upwards when R is larger (a higher degree of imbalance) is more obvious when we fix K at 10 and vary N , as in Figure 11. Indeed, for larger values of R , the adjustment implied by McGee's measure makes relatively little difference; for example, the distributions of $AdjHHI^*$ (in Figure 11) and HHI^* (in Figure 9) are very similar for $R = 5$, $K = 10$. The distributions of the two measures are also similar for $R = 2.5$ and $R = 3.75$. This suggests that McGee's assumption of a common probability of the stronger team winning, which underpins his ϕ_r measure and determines its mean value, improves

⁸ The apparent negative values in the kernel density for $TruncAdjHHI^*$ in the case of $K = 2$ is an artefact of the smoothing process; inspection of the quantiles of the simulated values confirms that all values up to and including the median are zero for all values of K .

on HHI^* when R is low ($R = 0$ or 1.25). However, it does not provide uniformly better performance compared to HHI^* as the degree of imbalance increases.

5 Conclusions

Several variants of the Herfindahl-Hirschman index of concentration applied to the distribution of wins across teams in a season are widely used as measures of competitive balance in professional sports leagues. Some of these measures take into account, to varying degrees, the constraints on the range of feasible values of HHI imposed by the league's playing schedules. Given the HHI 's emphasis on teams' shares of wins, a key feature is that teams cannot win games in which they do not play, which is reflected in the existence of upper bounds for HHI -related measures. Both the upper bounds and lower bounds of HHI depend on the number of teams in the league, which has implications for comparing such balance measures for leagues made up of different numbers of teams or for the same league over time if the number of teams changes.

To examine the properties of four variants of HHI -based measures of within-season competitive balance for leagues with different season lengths, we conduct a simulation analysis in which the degree of competitive imbalance can be specified. The unadjusted HHI is highly sensitive to variation in N , the number of teams, and is therefore not recommended for comparisons where N varies. Of the measures that adjust only for the lower bound of HHI , the ratio form, $mHHI$, is less sensitive to N . Of the four main measures considered, HHI^* , which takes into account the lower and upper bounds of HHI performs best across the various combinations of degrees of imbalance, number of teams and number of rounds of games. However, HHI^* , as with the other measures, tends to overstate the extent of imbalance when the number of matches is relatively small. McGee's (2016) suggested measure, which can be interpreted as an adjusted version of HHI^* , produces approximately zero bias when the league

is perfectly balanced, even when the number of matches is relatively small. However, as the number of teams and hence matches increases it also tends to overestimate the degree of imbalance when the degree of competitive imbalance is higher. Overall, the normalized *HHI** and McGee's adjusted balance measure are therefore recommended as the most useful of the measures considered, although neither completely removes biases associated with shorter season lengths, especially for higher degrees of imbalance.

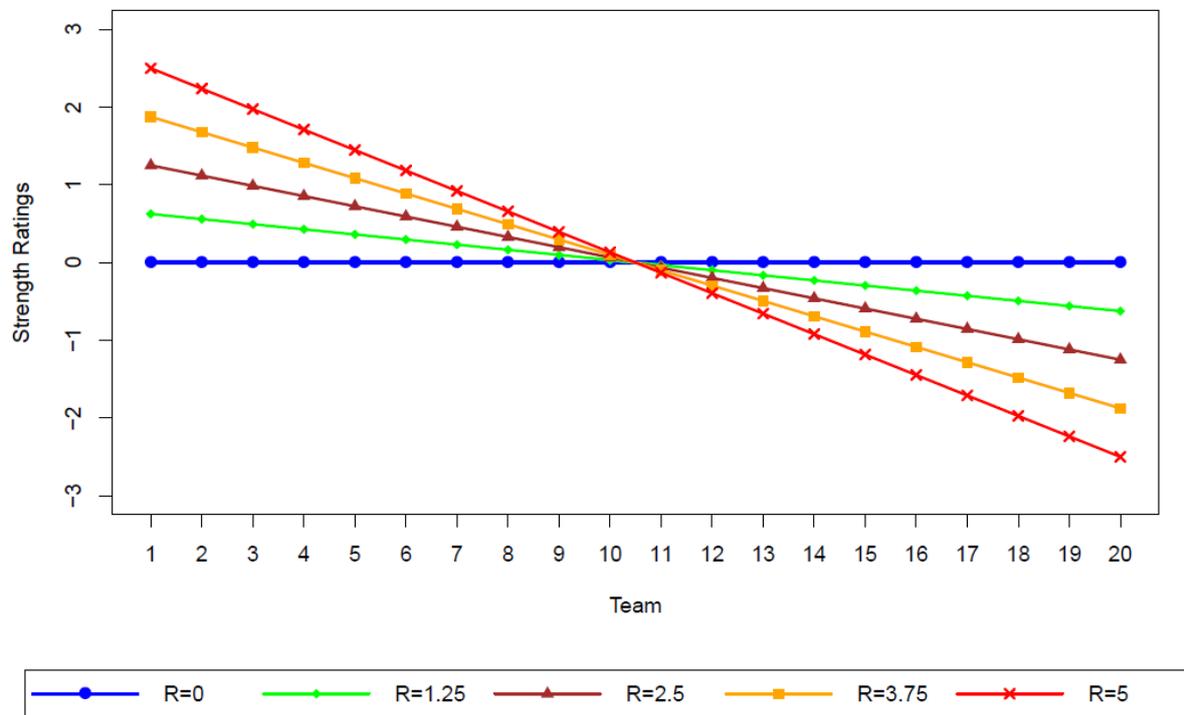


Fig. 1 Strength rating distributions used for simulations, $N = 20$

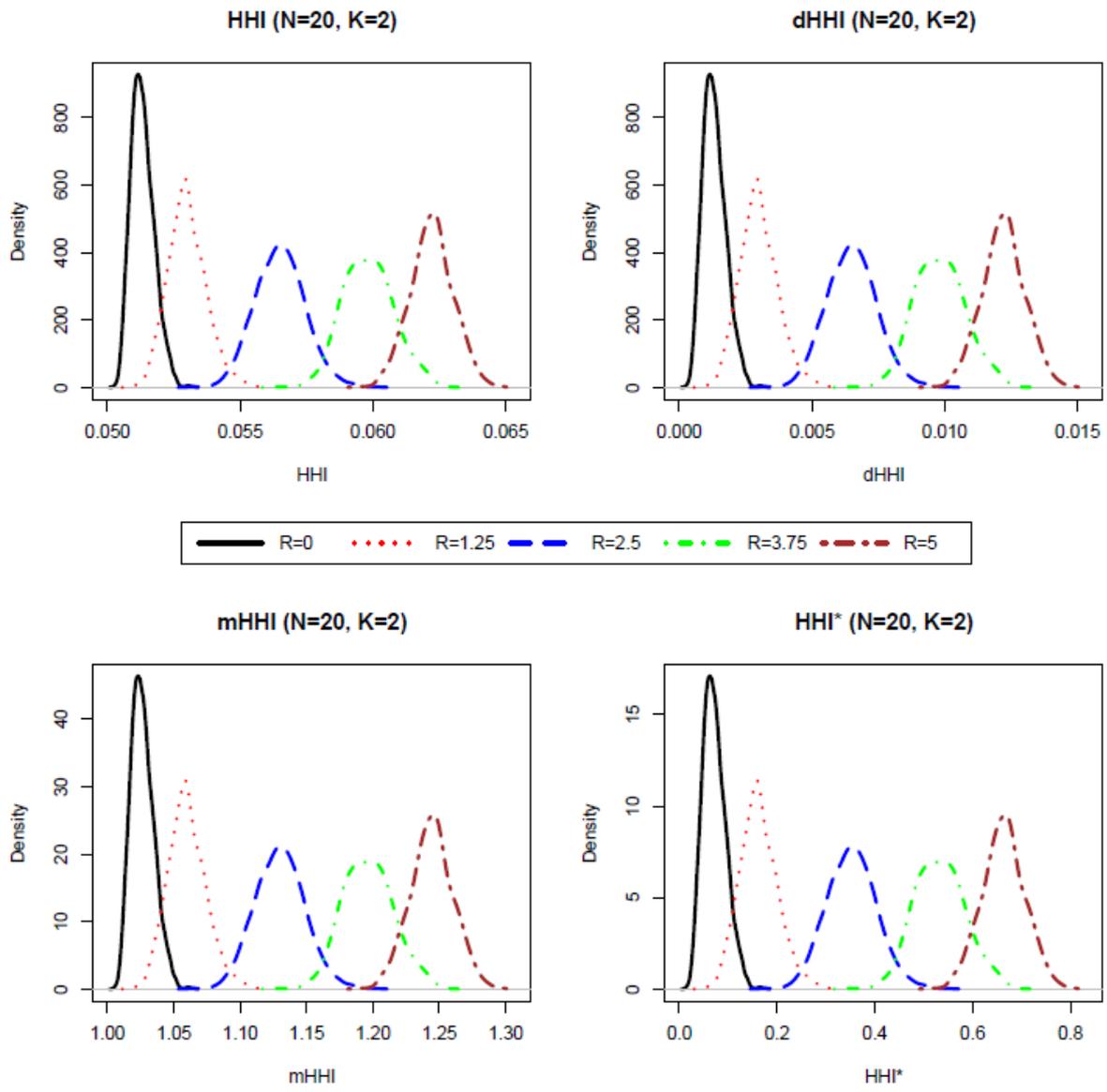


Fig. 2 Density functions of HHI measures for different degrees of competitive imbalance ($N = 20, K = 2$)

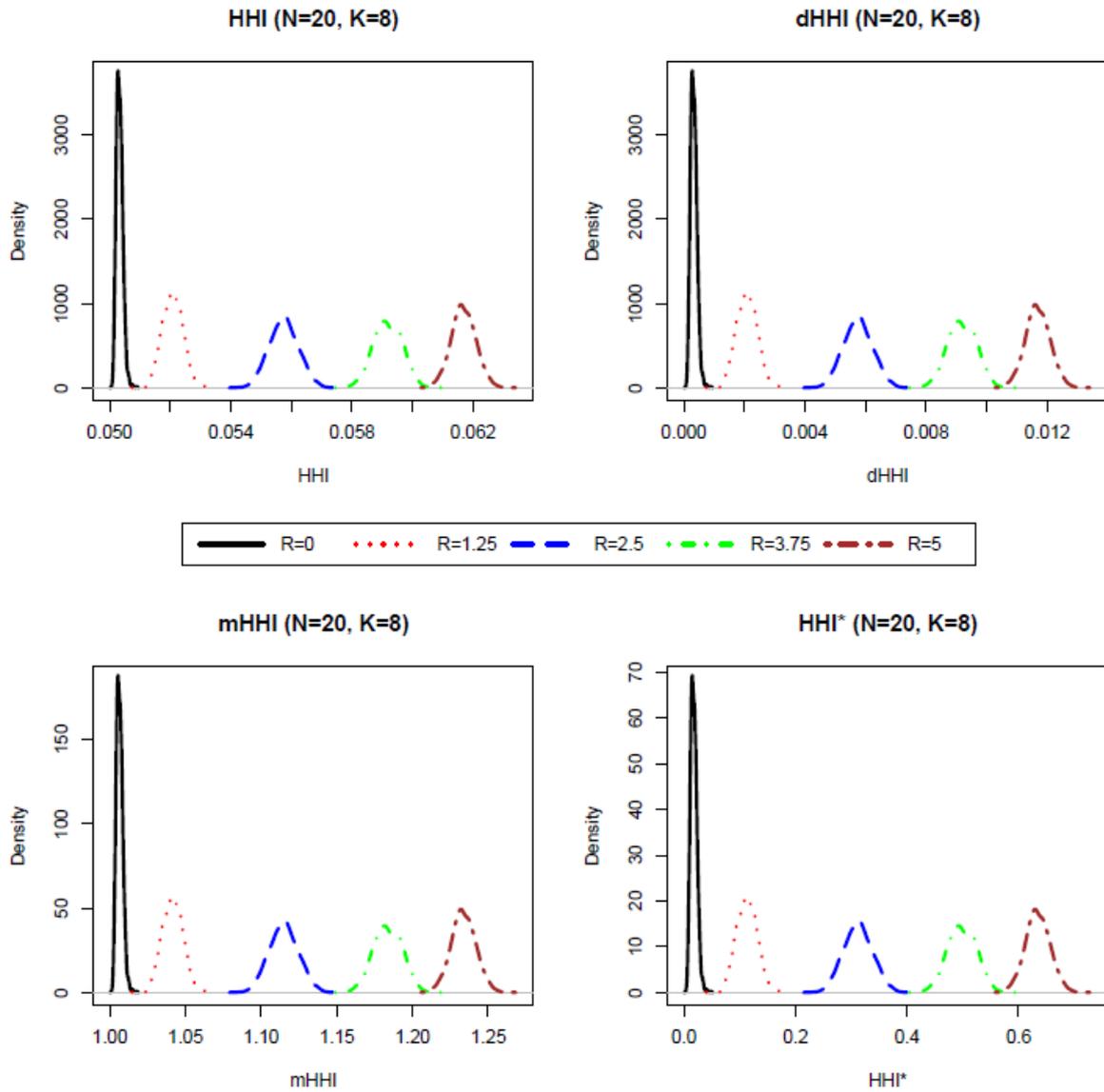


Fig. 3 Density functions of HHI measures for different degrees of competitive imbalance ($N = 20, K = 8$)

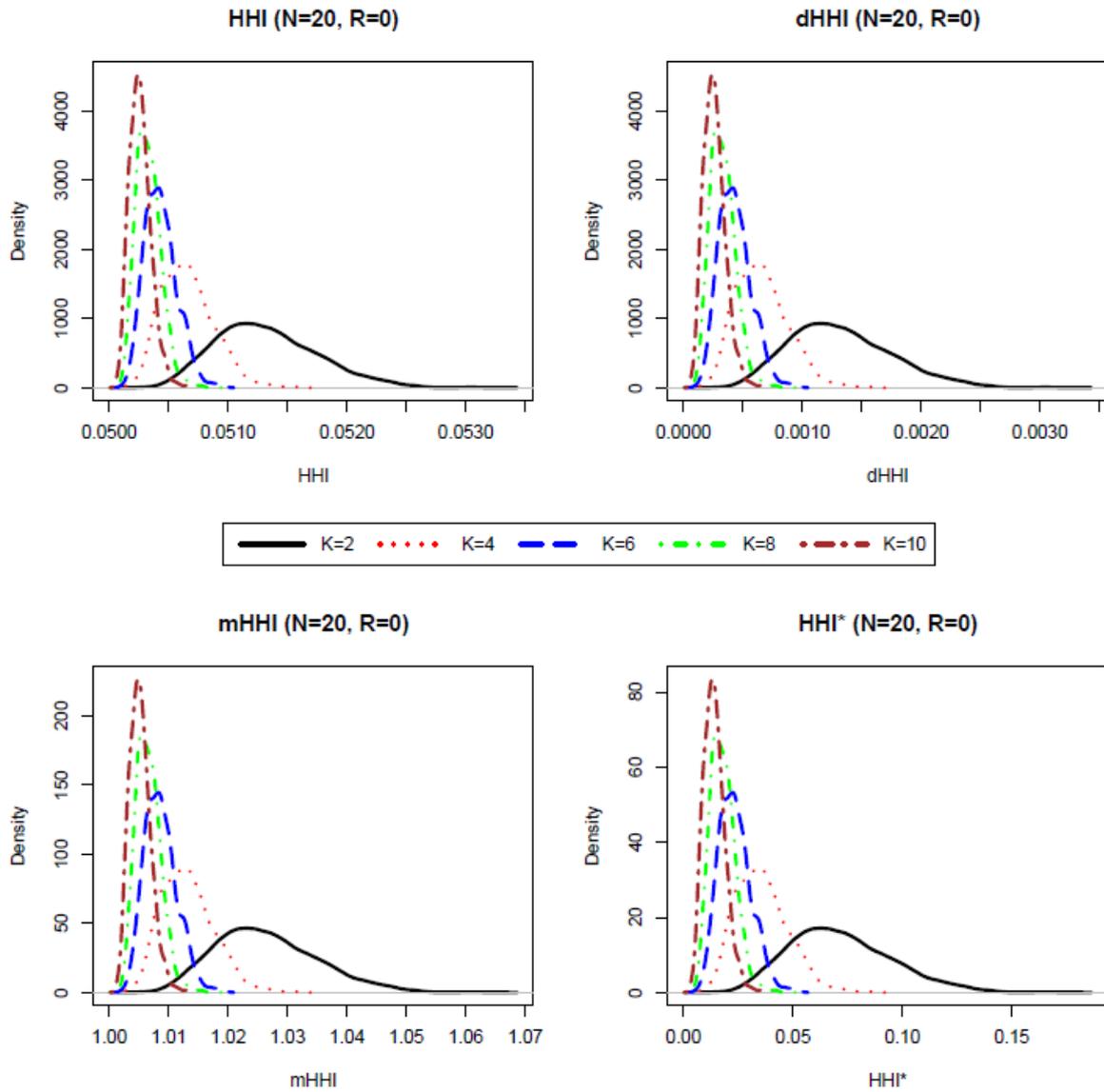


Fig. 4 Density functions of HHI balance measures for $R = 0$ (perfect balance), $N = 20$, varying K

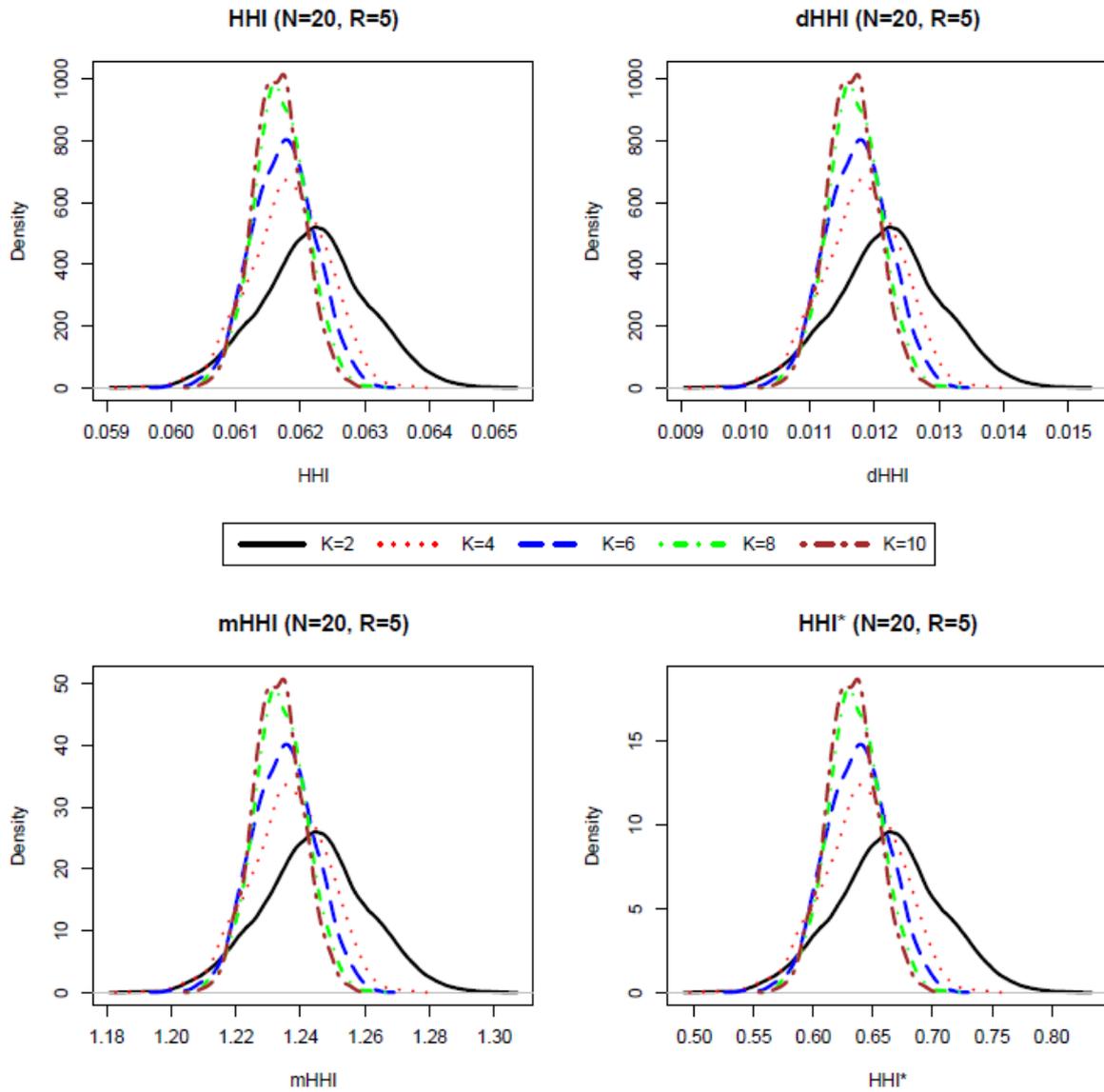


Fig. 5 Density functions of HHI balance measures for $R = 5$ (severe imbalance), $N = 20$, varying K

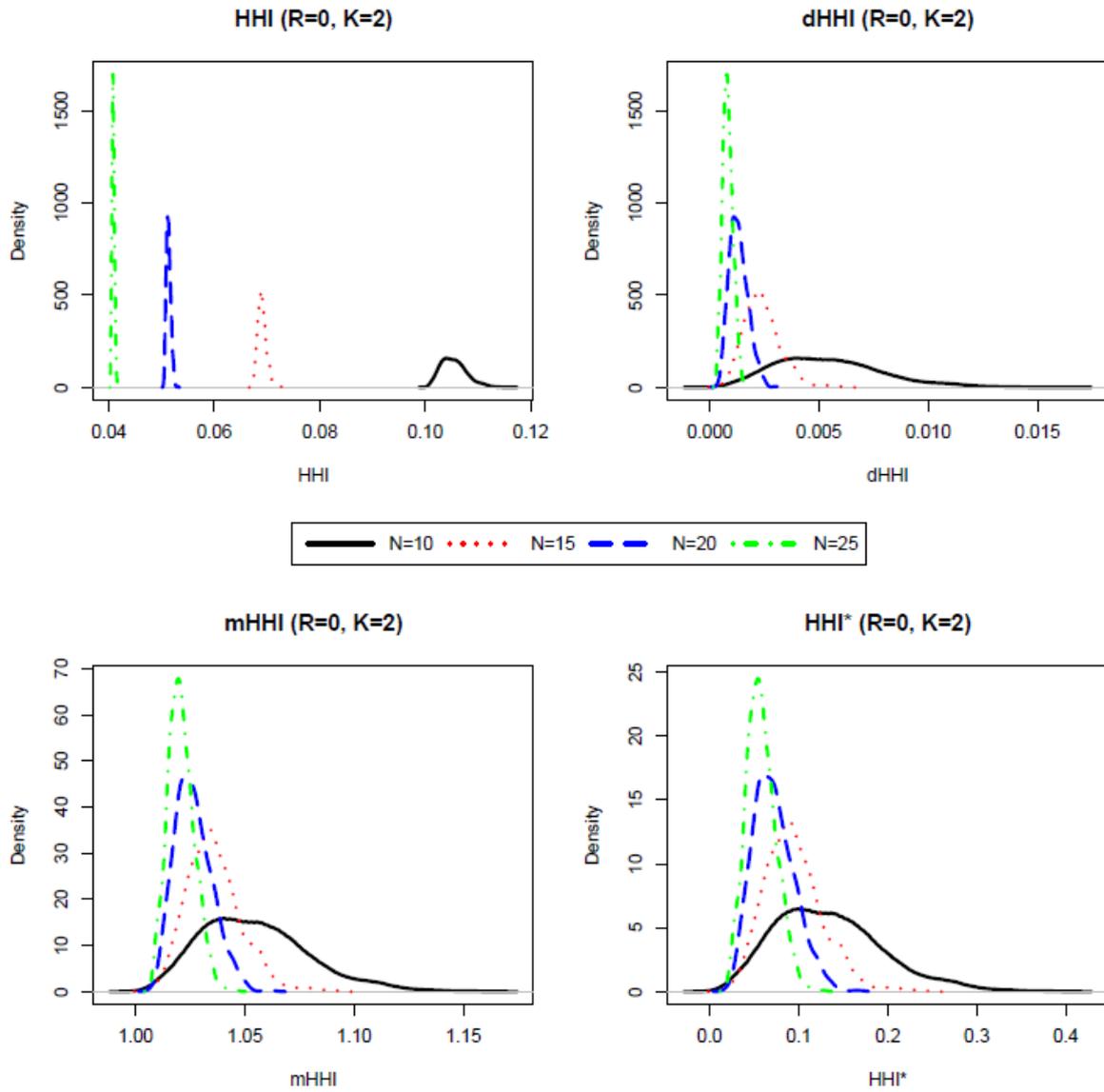


Fig. 6 Density functions of HHI balance measures for $R = 0$ (perfect balance), $K = 2$, varying N .

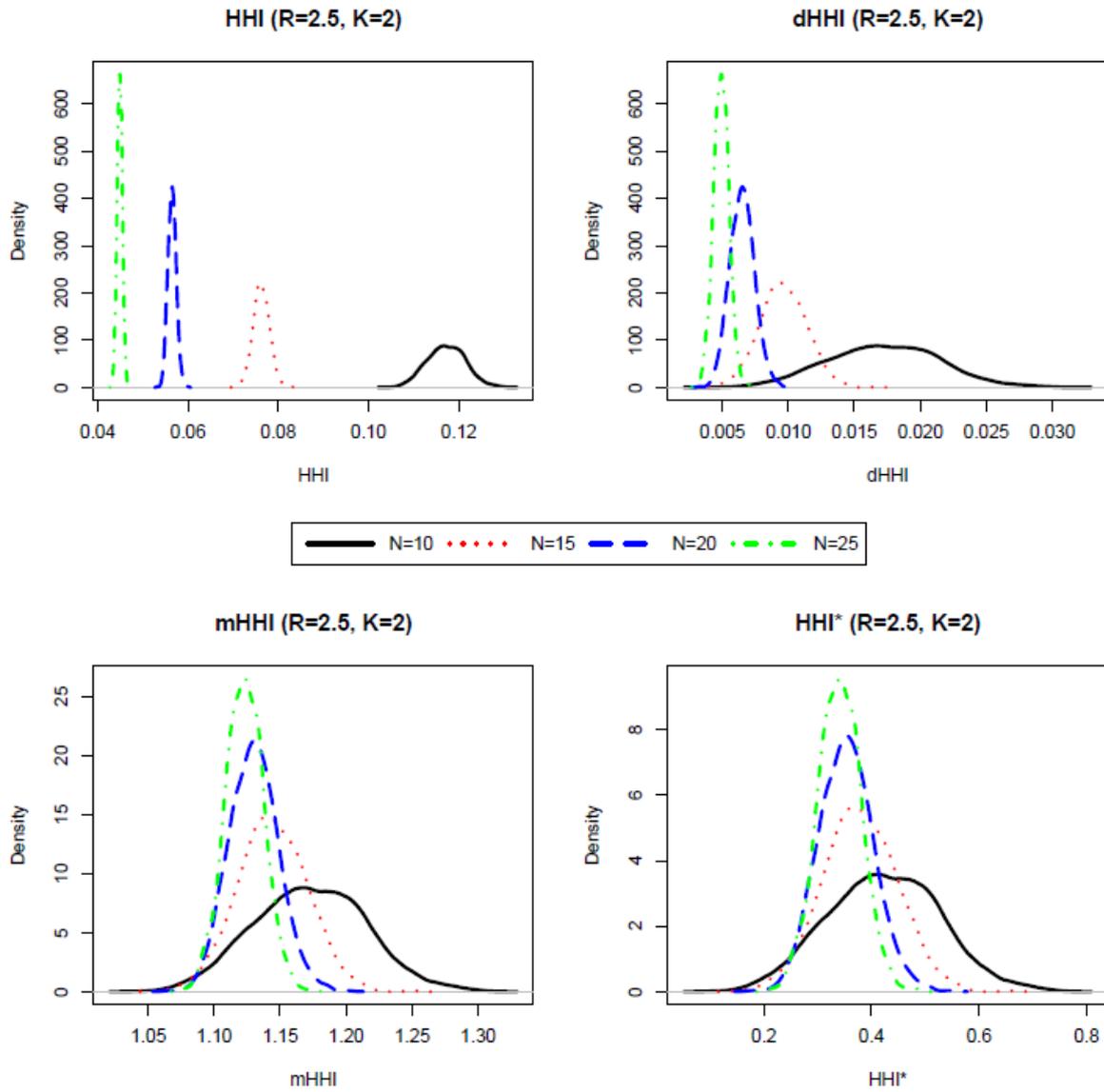


Fig. 7 Density functions of HHI balance measures for $R = 2.5$ (moderate imbalance), $K = 2$, varying N .

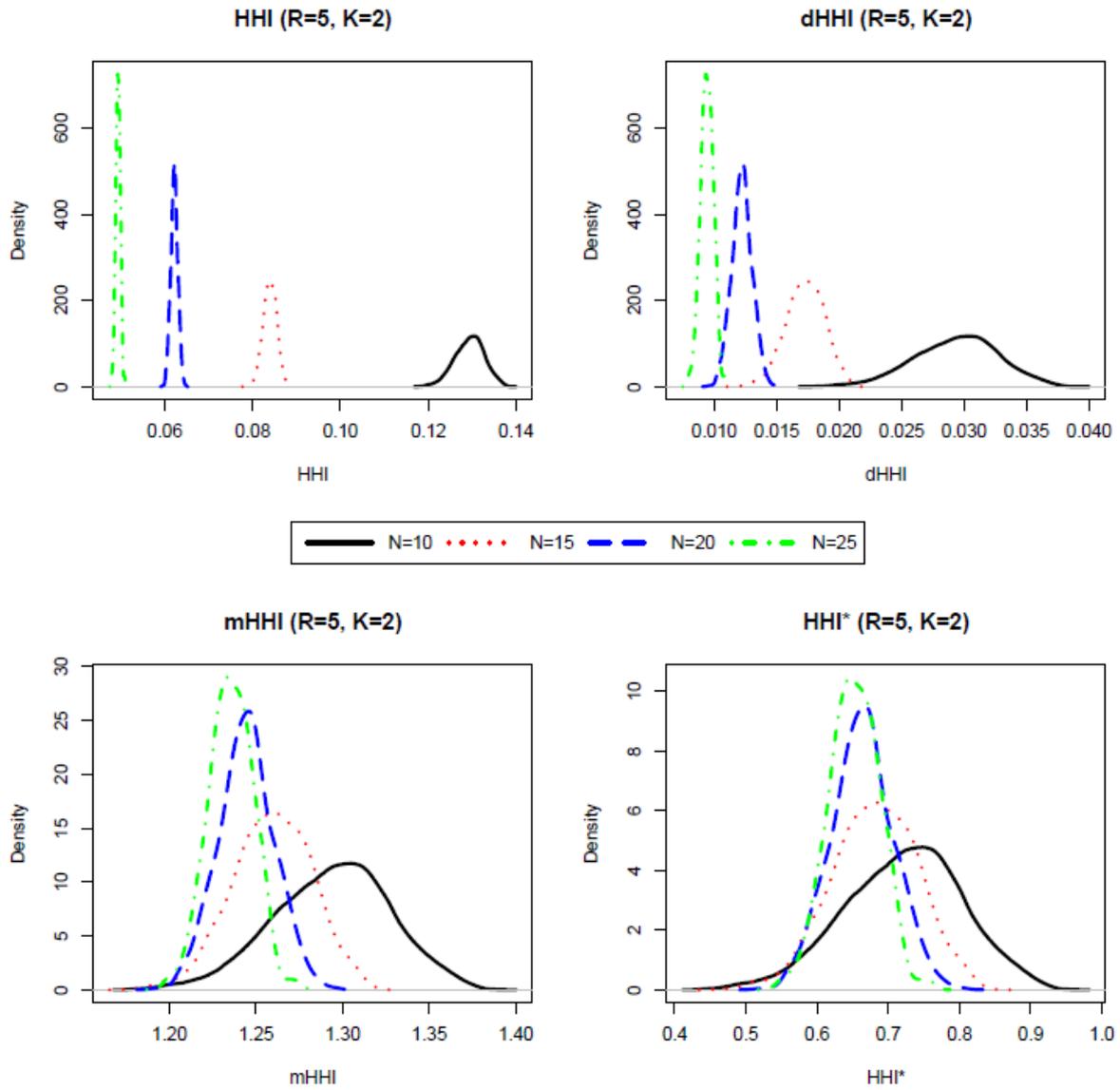


Fig. 8 Density functions of HHI balance measures for $R = 5$ (severe imbalance), $K = 2$, varying N .

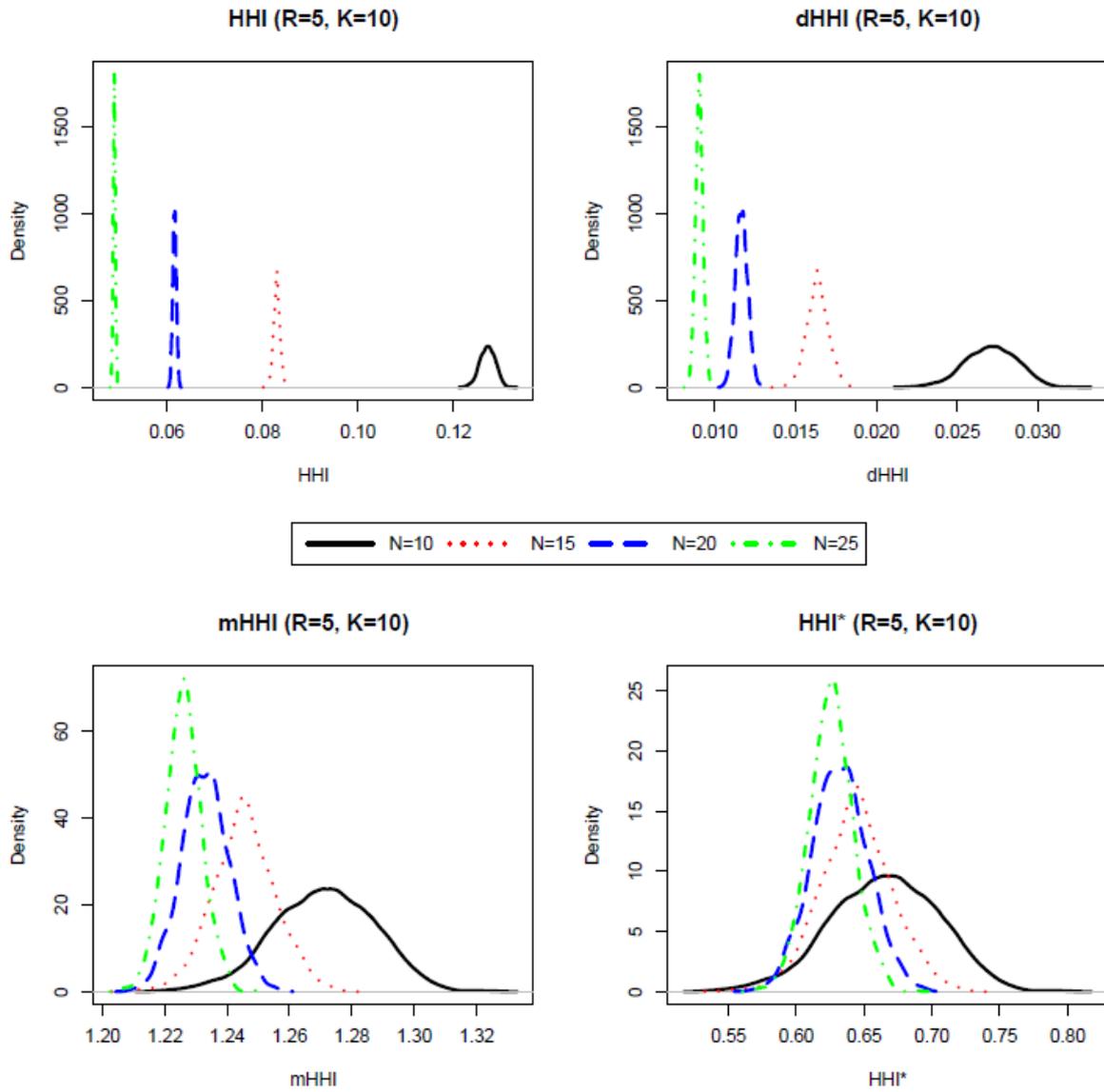


Fig. 9 Density functions of HHI balance measures for $R = 5$ (severe imbalance), $K = 10$, varying N .

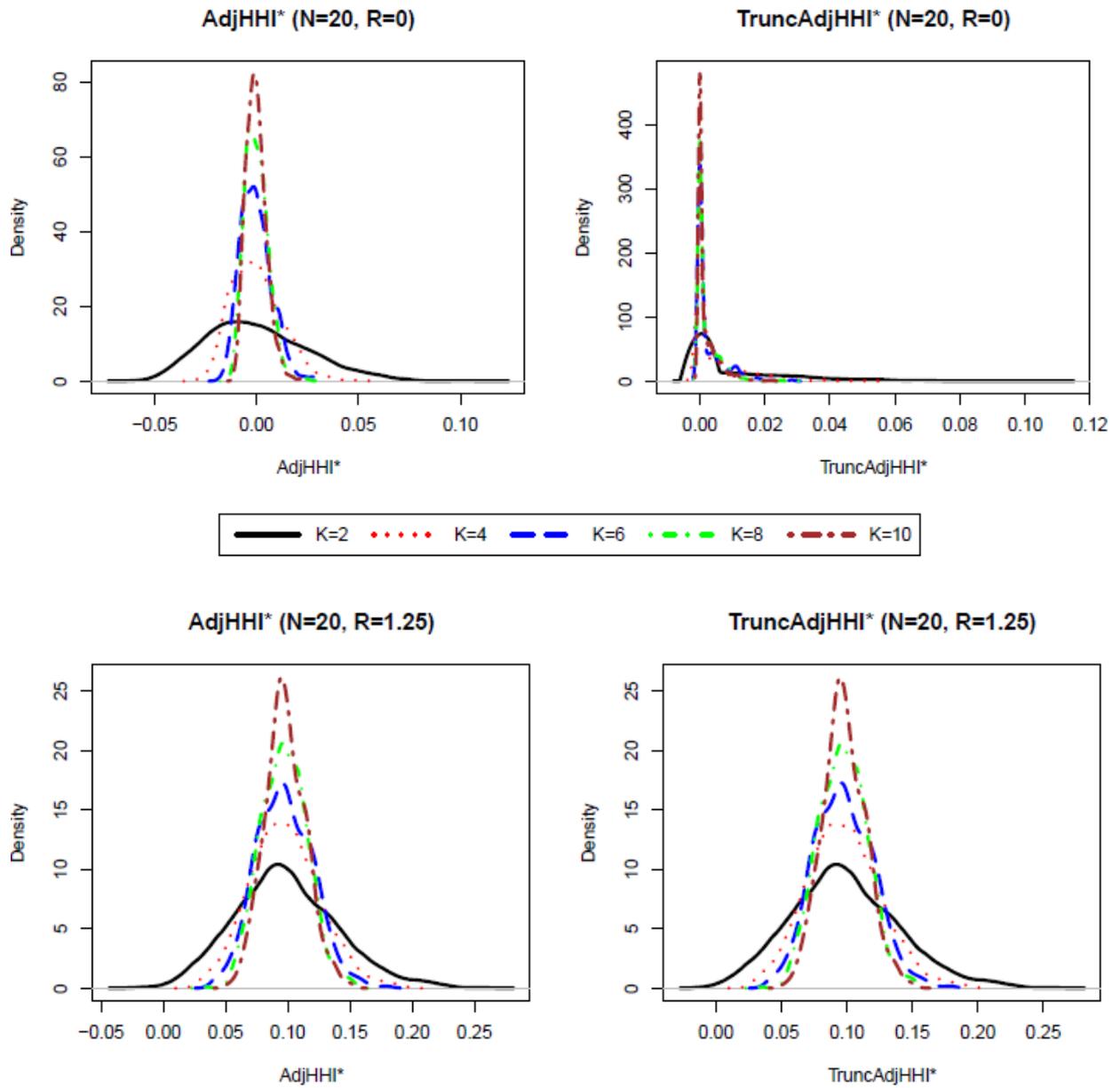


Fig. 10 Density functions of adjusted HHI balance measures for $R = 0$ and $R = 1.25$, $N = 20$, varying K .

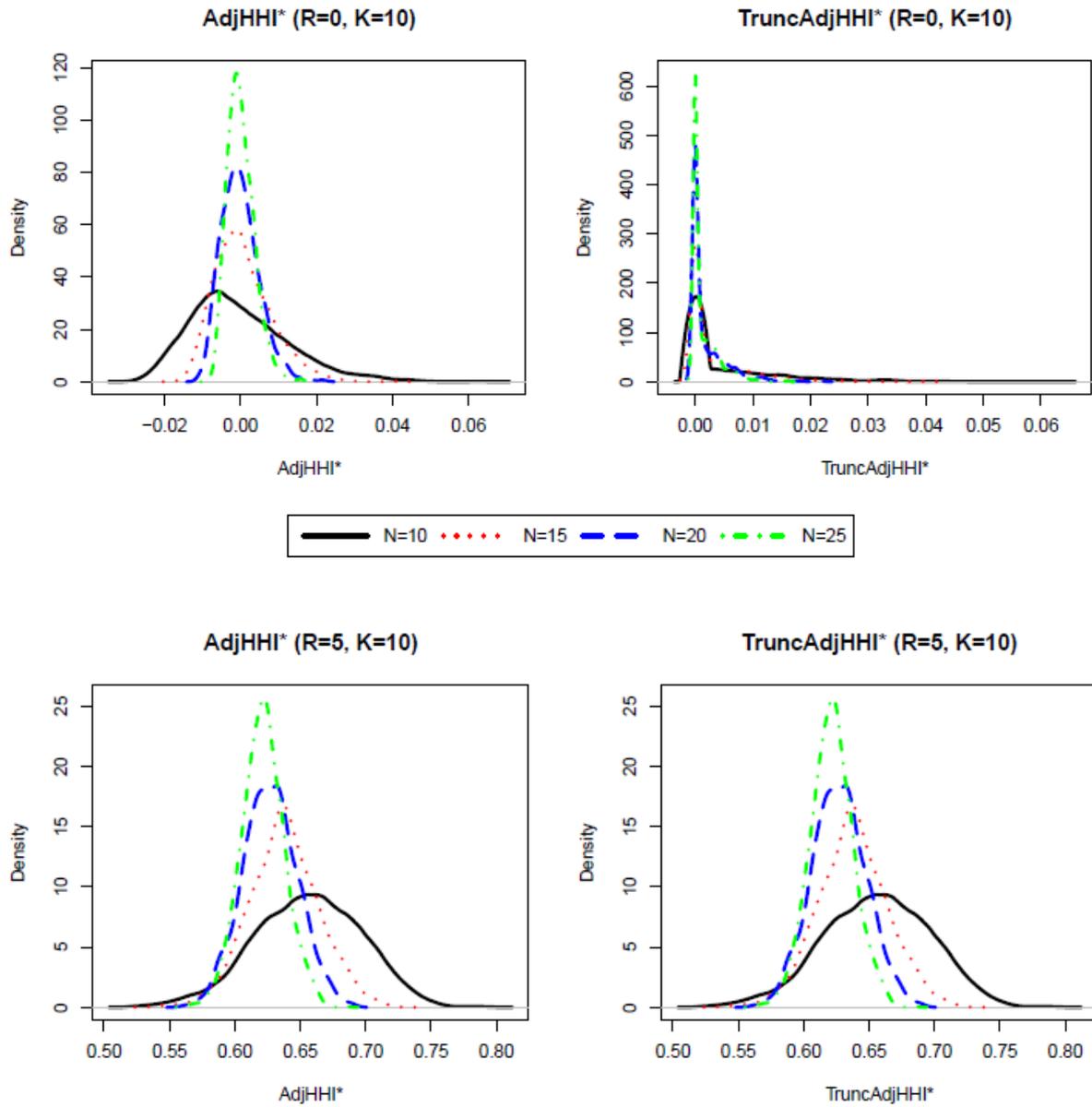


Fig. 11 Density functions of adjusted HHI balance measures for $R = 0$ and $R = 5$, $K = 10$, varying N .

References

- Addesa, F. (2011). Competitive balance in the Italian Basketball Championship. *Rivista di Diritto ed Economia dello Sport*, 7(1), 107-125.
- Agresti, A. (2013). *Categorical data analysis, 3rd edition*. Hoboken, NJ: Wiley.
- Bradley, R. A., & Terry, M. E. (1952). Rank analysis of incomplete block designs: I. The method of paired comparisons. *Biometrika*, 39(3/4), 324-345.
- Brandes, L., & Franck, E. (2007). Who made who? An empirical analysis of competitive balance in European soccer leagues. *Eastern Economic Journal*, 33(3), 379-403.
- Clarke, S. R. (1993). Computer forecasting of Australian Rules football for a daily newspaper. *Journal of the Operational Research Society*, 44(8), 753-759.
- del Corral, J., García-Unanue, J., & Herencia-Quintanar, F. (2016). Are NBA policies that promote long-term competitive balance effective? What is the price? *Open Sports Sciences Journal*, 9(Suppl-1, M10), 81-93.
- Depken, C. A., II (1999). Free-agency and the competitiveness of Major League Baseball. *Review of Industrial Organization*, 14(3), 205-217.
- Dittmore, S. W., & Crow, C. M. (2010). The influence of the Bowl Championship Series on competitive balance in college football. *Journal of Sport Administration and Supervision*, 2(1), 7-19.
- Eckard, E. W. (1998). The NCAA cartel and competitive balance in college football. *Review of Industrial Organization*, 13(3), 347-369.
- Eckard, E. W. (2017). The uncertainty-of-outcome hypothesis and the industrial organization of sports leagues: Evidence from U.S. college football. *Journal of Sports Economics*, 18(3), 298-317.
- Fenn, A. J., von Allmen, P., Brook, S., & Preissing, T. J. (2005). The influence of structural changes and international players on competitive balance in the NHL. *Atlantic Economic Journal*, 33(2), 215-224.

- Fort, R., & Maxcy, J. (2003). Comment on “Competitive Balance in Sports Leagues: An Introduction”. *Journal of Sports Economics*, 4(2), 154-160.
- Fort, R., & Quirk, J. (1997). Introducing a competitive economic environment into professional sports. In W. Hendricks (Ed.), *Advances in the economics of sports (Volume 2)* (pp. 3-26). Greenwich, CT: JAI Press.
- Gasparetto, T., & Barajas, A. (2016). Playoffs or just league: A debate in Brazilian football. *Open Sports Sciences Journal*, 9(Suppl-1, M11), 94-103.
- Gayant, J.-P., & Le Pape, N. (2015). The metrics of competitive imbalance. In W. Andreff (Ed.), *Disequilibrium sports economics: Competitive imbalance and budget constraints* (pp. 104-130). Cheltenham: Edward Elgar.
- Hall, M., & Tideman, N. (1967). Measures of concentration. *Journal of the American Statistical Association*, 62(317), 162-168.
- Horowitz, I. (1997). The increasing competitive balance in Major League Baseball. *Review of Industrial Organization*, 12(3), 373-387.
- Humphreys, B. R., & Watanabe, N. M. (2012). Competitive balance. In L. H. Kahane and S. Shmanske (Eds), *The Oxford handbook of sports economics, Volume 1: The economics of sports* (pp.18-37). Oxford: Oxford University Press.
- Jane, W.-J. (2014). The relationship between outcome uncertainties and match attendance: New evidence in the National Basketball Association. *Review of Industrial Organization*, 45(2), 177–200.
- Jane, W.-J. (2016). The effect of star quality on attendance demand: The case of the National Basketball Association. *Journal of Sports Economics*, 17(4), 396-417.
- King, N. (2011). The use of win percentages for competitive balance measures: An investigation of how well win percentages measure team ability and the implications for competitive balance analysis. MCom thesis, University of Otago.

- King, N., Owen, P. D., & Audas, R. (2012). Playoff uncertainty, match uncertainty and attendance at Australian National Rugby League matches. *Economic Record*, 88(281), 262-277.
- Kringstad, M., & Gerrard, B. (2007). Beyond competitive balance. In M. Parent & T. Slack (Eds), *International perspectives on the management of sport* (pp. 149-172). Burlington, MA: Butterworth-Heinemann.
- Larsen, A., Fenn, A. J., & Spenner, E. L. (2006). The impact of free agency and the salary cap on competitive balance in the National Football League. *Journal of Sports Economics*, 7(4), 374-390.
- Lee, Y. H., Kim, Y., & Kim, S. (2016). A bias-corrected estimator of competitive balance in sports leagues. Sogang University, Research Institute for Market Economy, Working Paper No. 2016-11
- Leeds, M. A., & von Allmen, P. (2014). *The economics of sports*, Fifth edition. Boston, MA: Pearson.
- Lenten, L. J. A. (2008). Unbalanced schedules and the estimation of competitive balance in the Scottish Premier League. *Scottish Journal of Political Economy*, 55(4), 488-508.
- Lenten, L. J. A. (2015). Measurement of competitive balance in conference and divisional tournament design. *Journal of Sports Economics*, 16(1), 3-25.
- Lenten, L. J. A. (2017). A formal test for asymmetry in the uncertainty of outcome hypothesis. *Journal of Sports Economics*, 18(3), 253-270.
- Manasis, V., Ntzoufras, I., & Reade, J. (2015). Measuring competitive balance and uncertainty of outcome hypothesis in European football. arXiv preprint arXiv:1507.00634.
- Marchi, M. & Albert, J. (2014). *Analyzing baseball data with R*. Boca Raton, FL: CRC Press.
- Martinez, M., & Willner, J. (2017). Competitive balance and consumer demand in the English Football League. *Applied Finance and Accounting*, 3(2), 49-60.
- McGee, M. K. (2016). Two universal, probabilistic measures of competitive imbalance. *Applied Economics*, 48(31), 2883-2894.

- Michie, J., & Oughton, C. (2004). *Competitive balance in football: Trends and effects*. London: The Sports Nexus.
- Michie, J., & Oughton, C. (2005). *Competitive balance in football: An update*. London: The Sports Nexus.
- Mills, B., & Fort, R. (2014). League-level attendance and outcome uncertainty in U.S. pro sports leagues. *Economic Inquiry*, 52(1), 205–218.
- Neale, W. C. (1964). The peculiar economics of professional sports. *Quarterly Journal of Economics*, 78(1), 1-14.
- Noll, R. G. (1988). Professional basketball. Stanford University, Studies in Industrial Economics Paper No. 144.
- Owen, P. D. (2010). Limitations of the relative standard deviation of win percentages for measuring competitive balance in sports leagues. *Economics Letters*, 109(1), 38–41.
- Owen, D. (2014). Measurement of competitive balance and uncertainty of outcome. In J. Goddard and P. Sloane (Eds), *Handbook on the economics of professional football* (pp. 41-59). Cheltenham: Edward Elgar.
- Owen, P. D., & King, N. (2015). Competitive balance measures in sports leagues: The effects of variation in season length. *Economic Inquiry*, 53(1), 731–744.
- Owen, P. D. et al. (2007). Measuring competitive balance in professional sports using the Herfindahl-Hirschman index. *Review of Industrial Organization*, 31(4), 289-302.
- Pawlowski, T., Breuer, C., & Hovemann, A. (2010). Top clubs' performance and the competitive situation in European domestic football competitions. *Journal of Sports Economics*, 11(2), 186-202.
- R Core Team (2014). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL <http://www.R-project.org/>.

- Ramchandani, G. (2012). Competitiveness of the English Premier League (1992-2010) and ten European football leagues (2010). *International Journal of Performance Analysis in Sport*, 12(2), 346-360.
- Rao, P. V., & Kupper, L. L. (1967). Ties in paired-comparison experiments: A generalization of the Bradley-Terry model. *Journal of the American Statistical Association*, 62(317), 194-204.
- Rottenberg, S. (1956). The baseball players' labor market. *Journal of Political Economy*, 64(3), 242-258.
- Scully, G. W. (1989). *The business of Major League Baseball*. Chicago, IL: University of Chicago Press.
- Tainsky, S., Xu, J., & Yang, Q. (2017). Competitive balance and the participation–spectatorship gap in Chinese table tennis. *Applied Economics*, 49(3), 263-272.
- Totty E. S., & Owens M. F. (2011). Salary caps and competitive balance in professional sports leagues. *Journal for Economic Educators*, 11(2), 46-56.
- Utt, J., & Fort, R. (2002). Pitfalls to measuring competitive balance with Gini coefficients. *Journal of Sports Economics*, 3(4), 367-373.
- Van Scyoc, L., & McGee, M. K. (2016). Testing for competitive balance. *Empirical Economics*, 50(3), 1029–1043.
- York, K. M., & Miree, C. (2015). A longitudinal exploration of strategic isomorphism: The case of the National Football League. *American Journal of Business and Management*, 4(2), 61-70.