

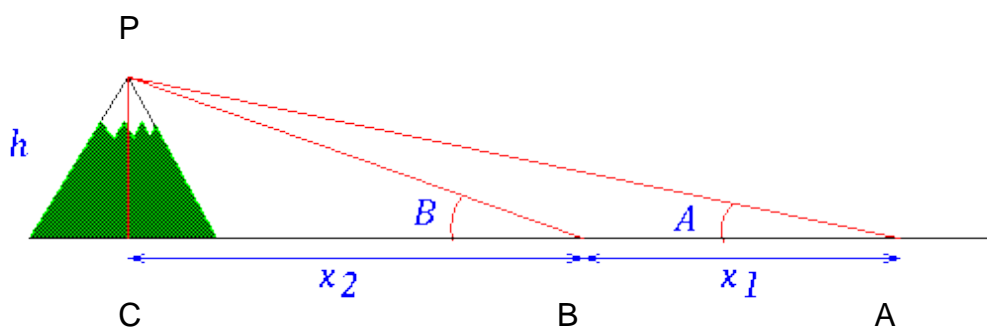
# MATHEMATICS ANSWERS

## Level Two

### Heights and Sights

You set up a Theodolite and sight to a mountain from two places. The first angle (A) is measured at  $13^\circ$  and the second angle (B) is measured at  $16.5^\circ$  1km closer to the mountain.

- 1) What is the height of the mountain?



From your measurements, you know AB. You want to determine  $h$  and  $x_2$ .

The angles should be:

$$A = 13^\circ$$

$$B = 16.5^\circ$$

Now to calculate  $x_2$ ...

$$\text{The angle } PBA = 180^\circ - 16.5^\circ = 163.5^\circ.$$

$$\text{Also, the angle } BPA = 180^\circ - 163.5^\circ - 13^\circ = 3.5^\circ$$

$$\text{Using the Sin Rule we have } 1000/\sin 3.5^\circ = PB/\sin 13^\circ$$

$$\text{Thus } PB = 3684.79 \text{ m}$$

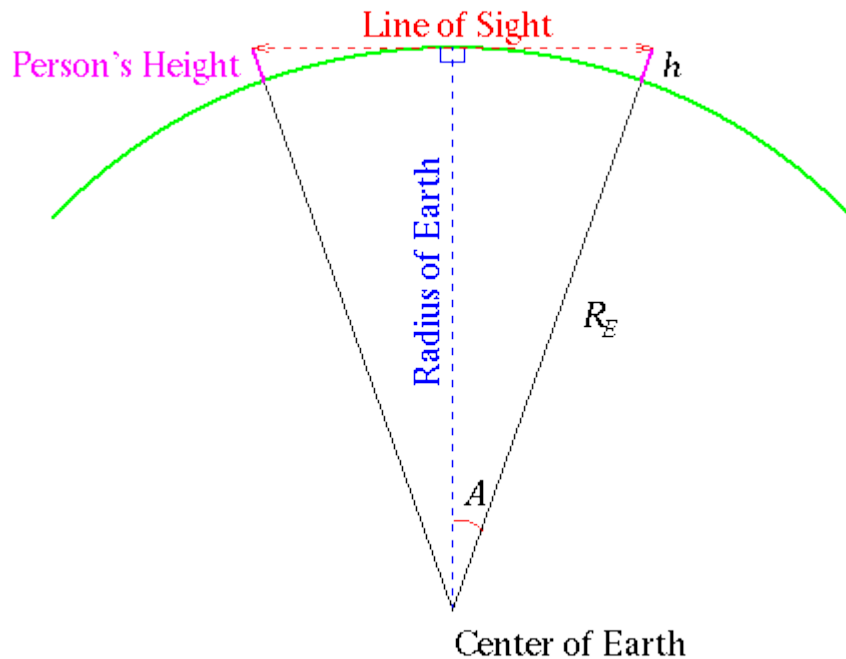
$$\text{Hence } h = 3684.79 \sin 16.5^\circ = 1046.54 \text{ m, and}$$

$$CB = 3684.79 \cos 16.5^\circ = 3533.04 \text{ m}$$

### Question B

Assuming nothing else blocks their view, how far can two 2.2m tall people walk from each other until they can no longer see each other due to the curvature of the Earth?

### Solution



The last point they would be able to see each other is when the line of sight is tangent to the Earth's surface as in the picture. The distance we want is the length of the arc between the two people. (Not the line of sight)

The first step is to find out the angle  $A$ .. (Actually we will need to know what  $2A$  is...)

$$\cos A = \frac{R_E}{R_E + h}$$

$$A = \cos^{-1} \left( \frac{R_E}{R_E + h} \right)$$

$$= \cos^{-1} \frac{6374000}{6374000 + 2.2}$$

$$= \cos^{-1} (0.9999997)$$

$$= 0.0047604 = 0^\circ 02' 51''$$

$$2A = 0.0095208 = 0^\circ 05' 42''$$

So now we know the angle, need to use this to find the arc's length ( $l$ ):

$$l = \theta_{radians} r$$

Converting degrees to radians is analogous to a unit conversion:

$$\theta_{radians} = \frac{2\pi}{360} \theta_{degrees}$$

$$\theta_{radians} = \frac{2\pi}{360} .095208$$

$$= 1.661693 \times 10^{-3} radians$$

So...

$$l = \theta_{radians} r$$

$$l = \theta_{radians} R_E$$

$$(1.661693 \times 10^{-3} rads)(6374000)$$

$$= 10592m$$