The Power Law: Hallmark Of A Complex System

Or

Playing With Data Can Be Dangerous For Your Mental Health

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Acknowledgements

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Some of this material is recycled from publications by:

- John Doyle
- Per Bak

Outline

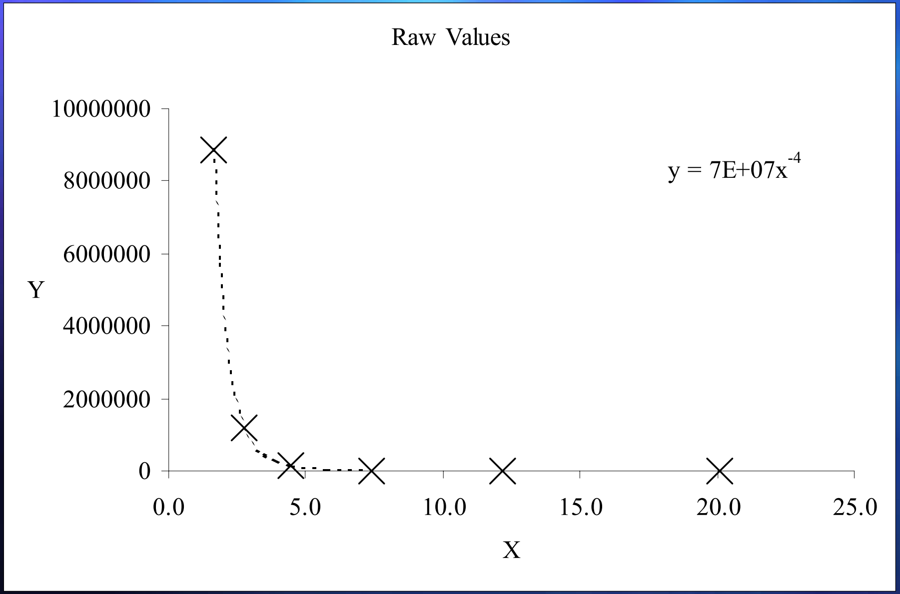
- Power law relationships, what they look like and where you see them.
- Power laws observed in health systems.
- Design or self organisation: more detail about power laws and how to analyse them.
- Wild speculation about health systems, primary care and possibilities for research.

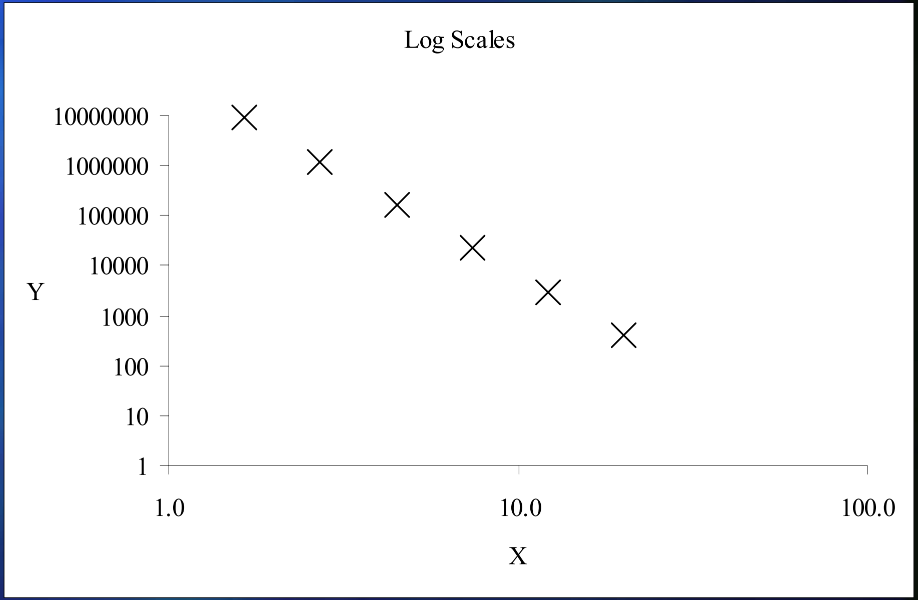
Power Laws

- Power law scaling denotes a relationship between two variables which can be described by the equation: $y=cx^a$
- When you look at a graph of a power relationship on an ordinary linear scale, it looks rather like an exponential relationship, although there is a fatter 'tail' of rare events then in the exponential case.
- Power laws are scale invariant
- Plot on a log-log scale, and look for a straight line.

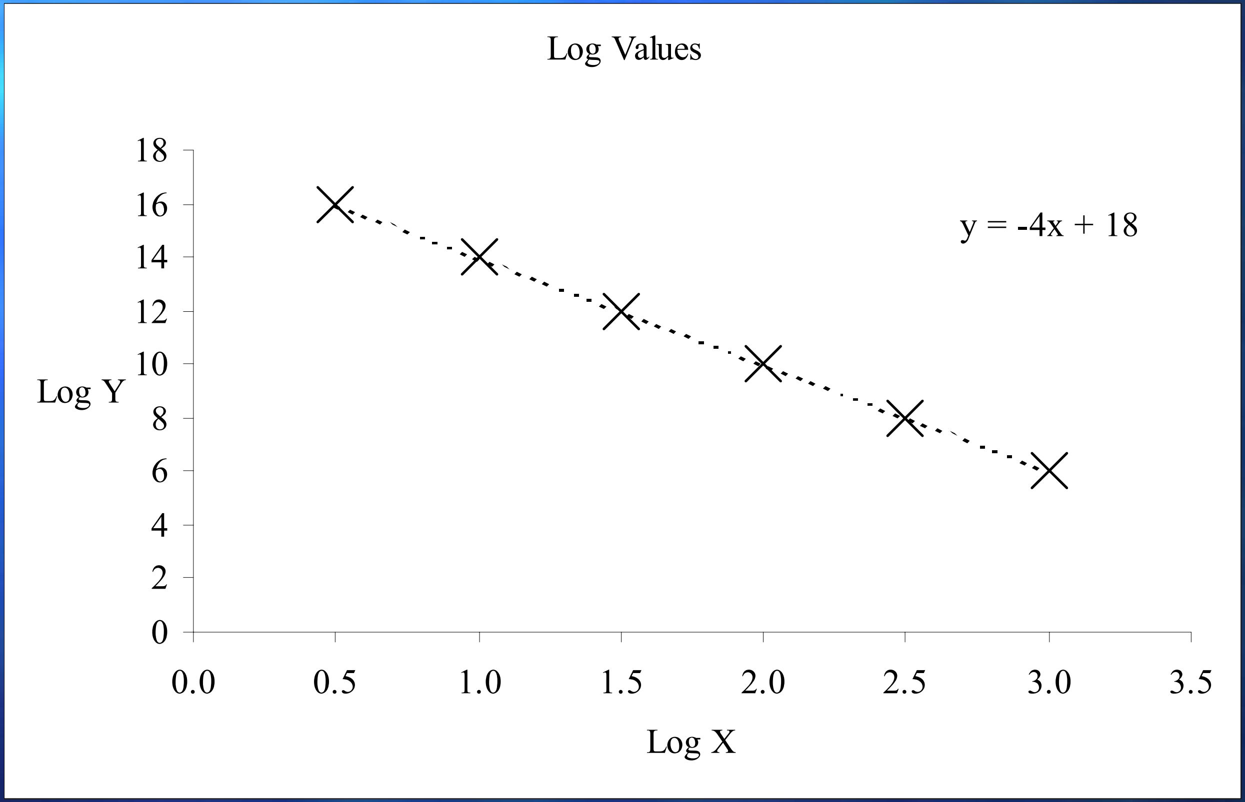
Power Laws Look Like This:

X	Y	Log X	Log Y
1.6	8886111	0.5	16
2.7	1202604	1.0	14
4.5	162755	1.5	12
7.4	22026	2.0	10
12.2	2981	2.5	8
20.1	403	3.0	6





Log Scales



Log Values
18
16
14
12
10
8
6
4
2
0
Log Y
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Log X
y = -4x + 18

Why Is This Important?

- Per Bak
- Physicist at Brookhaven National Laboratory
- With various colleagues, discovered the importance of sandpiles

Sandpiles And Avalanches

- When you add grains of sand to a pile you get avalanches
- A few big avalanches, many little avalanches
- The distribution of avalanche sizes shows a power law relationship.
- Why do we see this pattern?
- Something fundamental, an emergent property of a pile of sand
- Can be replicated in the laboratory or simulated on a computer

Sandpiles

Sandpile Models

- Typically proceed by defining a matrix and randomly adding grains of sand.
- When sand reaches a given height on any point of the matrix, an avalanche occurs.
- Measure the frequency and size of avalanches.

Sandpile Matrix

	1		1	1
1		1		1
1	1	1	1	
1		1		
1	1	1		1

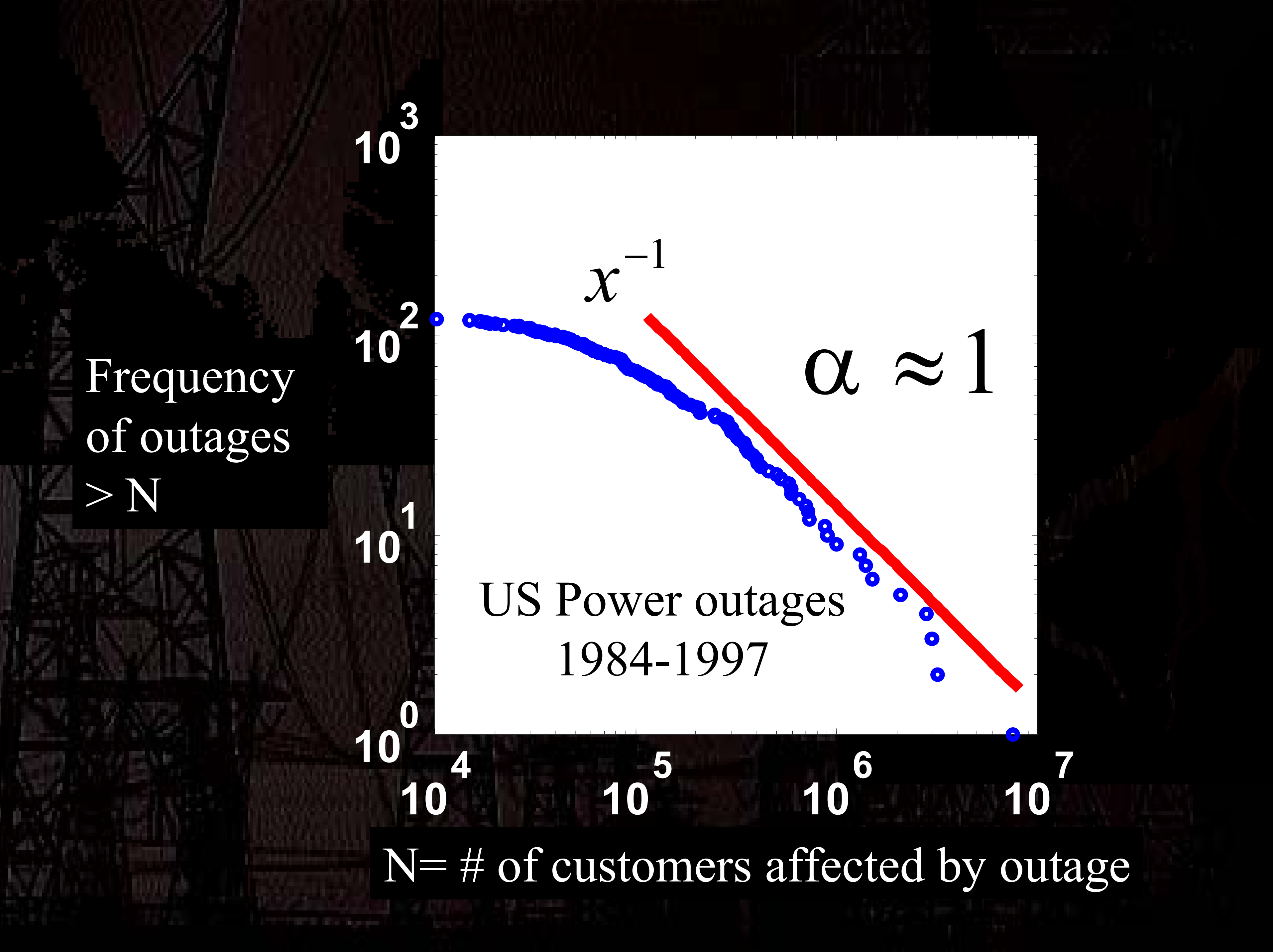
1	1		1	**2**
1		1		1
1	**2**	1	**2**	
2		1		**1**
1	1	**2**	1	1

1	1		1	2
1		1		1
1	**3**	1	2	
2		1		1
1	1	2	1	1

1	1		1	2
2		1		1
2	**0**	1	2	
3		1		1
1	1	2	1	1

There's A Lot Of It About

- Earthquakes
- Traffic jams
- Mutations and evolution
- Share prices
- Forest Fires
- Power cuts



Frequency of outages > N
10^3
10^2
10^1
10^0
x^−1
α ≈ 1
US Power outages 1984-1997
10^4
10^5
10^6
10^7
N= # of customers affected by outage

Properties

Systems which produce power law relationships tend, in physics, to:

- Be in some sense critical systems which have a special yield, efficiency or other parameter.
- Be robust to external influences, which have only a moderate impact upon the system.
- Have developed this special power law state spontaneously.

Acronyms

- CAS: Complex Adaptive Systems.
  - Emergent complexity via self-organisation of elements within a system.
- SOC: Self Organised Criticality
  - A system which is at a stable yet critical point.

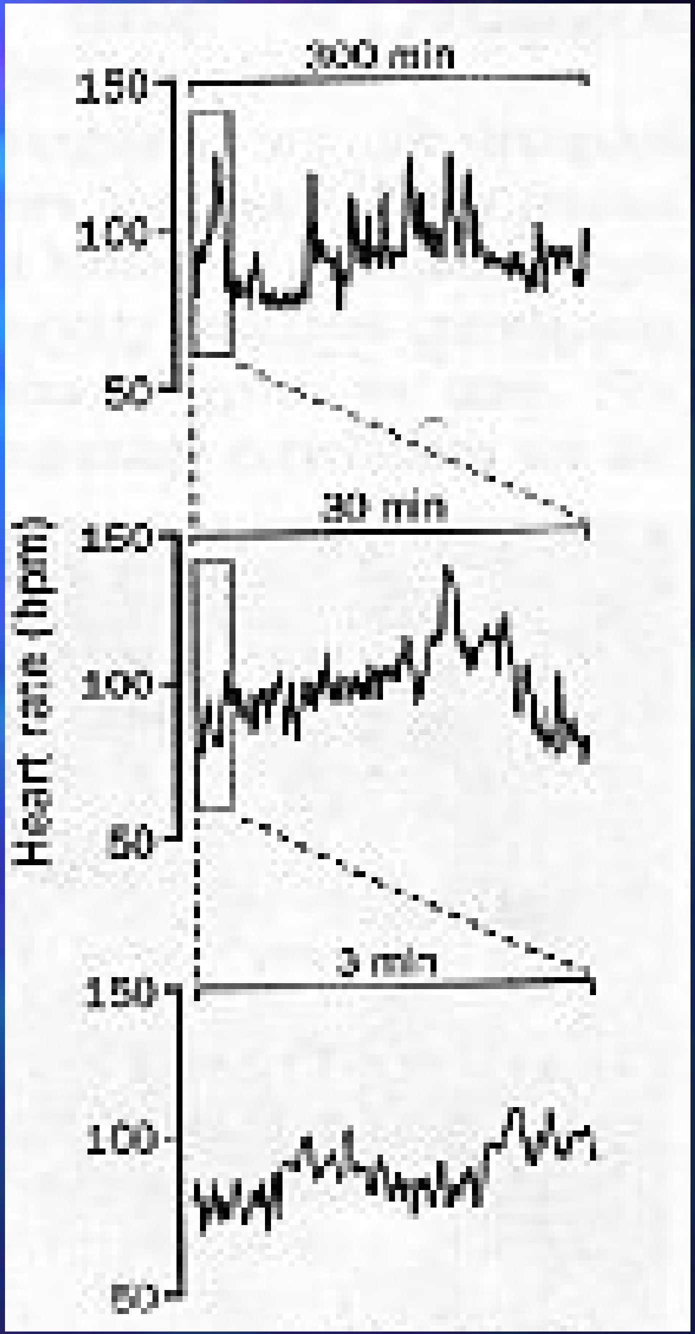
Self-Organized Criticality

- Under a stable flux of external events, a system adapts itself in an optimal way.
- There is scale invariance in the distribution of events, a perturbation can induce an event at any scale.

Complexity In Health

- Goldberger A.L. 'Non-linear dynamics for clinicans: chaos theory,fractals, and complexity at the bedside'. Lancet. 1996; 347: 1312-1314
- Complex fluctuations with self-similar fractal properties observed in heart rate variability, respiration, systemic blood pressure, white blood cell counts.

Fluctuations In Heart Rates

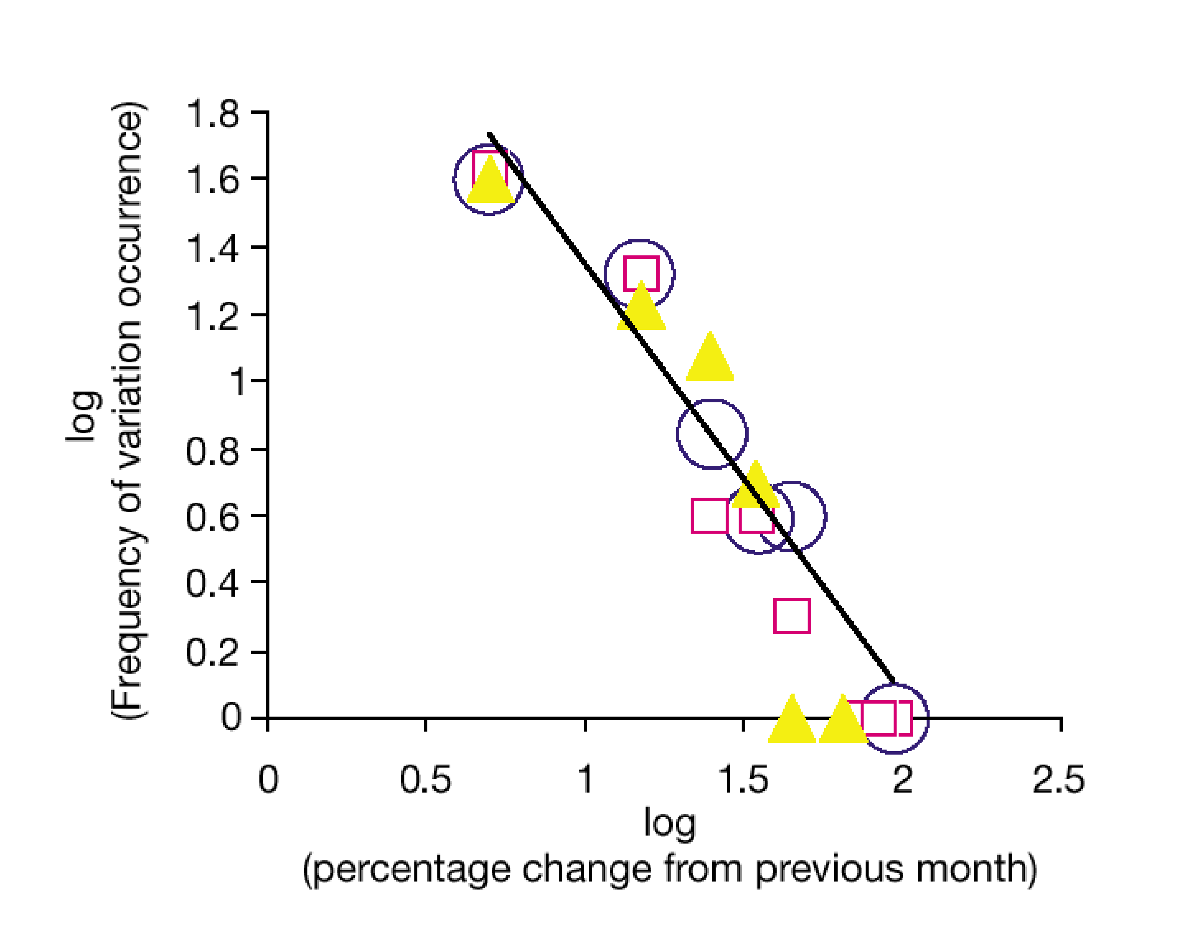


Power Laws In Health Systems

Waiting lists:

- Smethurst D, Williams H. Are hospital waiting lists self-regulating? *Nature* 2001; 410(6829):652-3.
- Papadopoulos M et al. Is the National Health Service at the edge of chaos? *J R Soc Med* 2001; 94:613-6

Waiting Lists



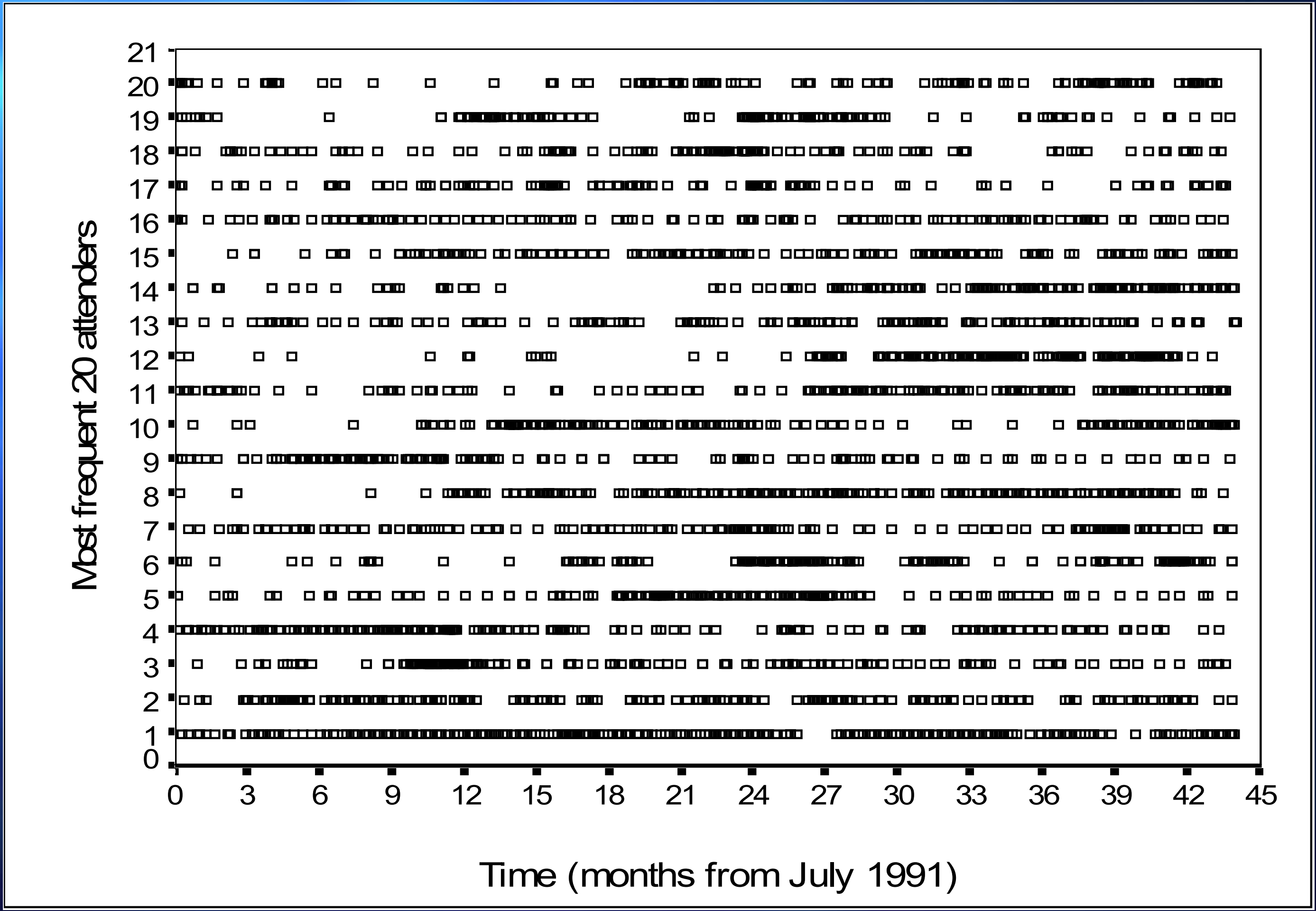
Interpretation

- Waiting lists may be inherent, a self organizing feature of a health system which is not going to go away.
- Waiting lists could represent a critical state, perhaps an efficient state of rationing a limited resource
- Waiting lists aren't a very good way of measuring the health system.

Frequent Consultations

- Observed in General and Hospital Practice
- 3% of patients account for 15% of overall workload
- Assumptions abound about heartsink patients, characteristics of frequent consulters.

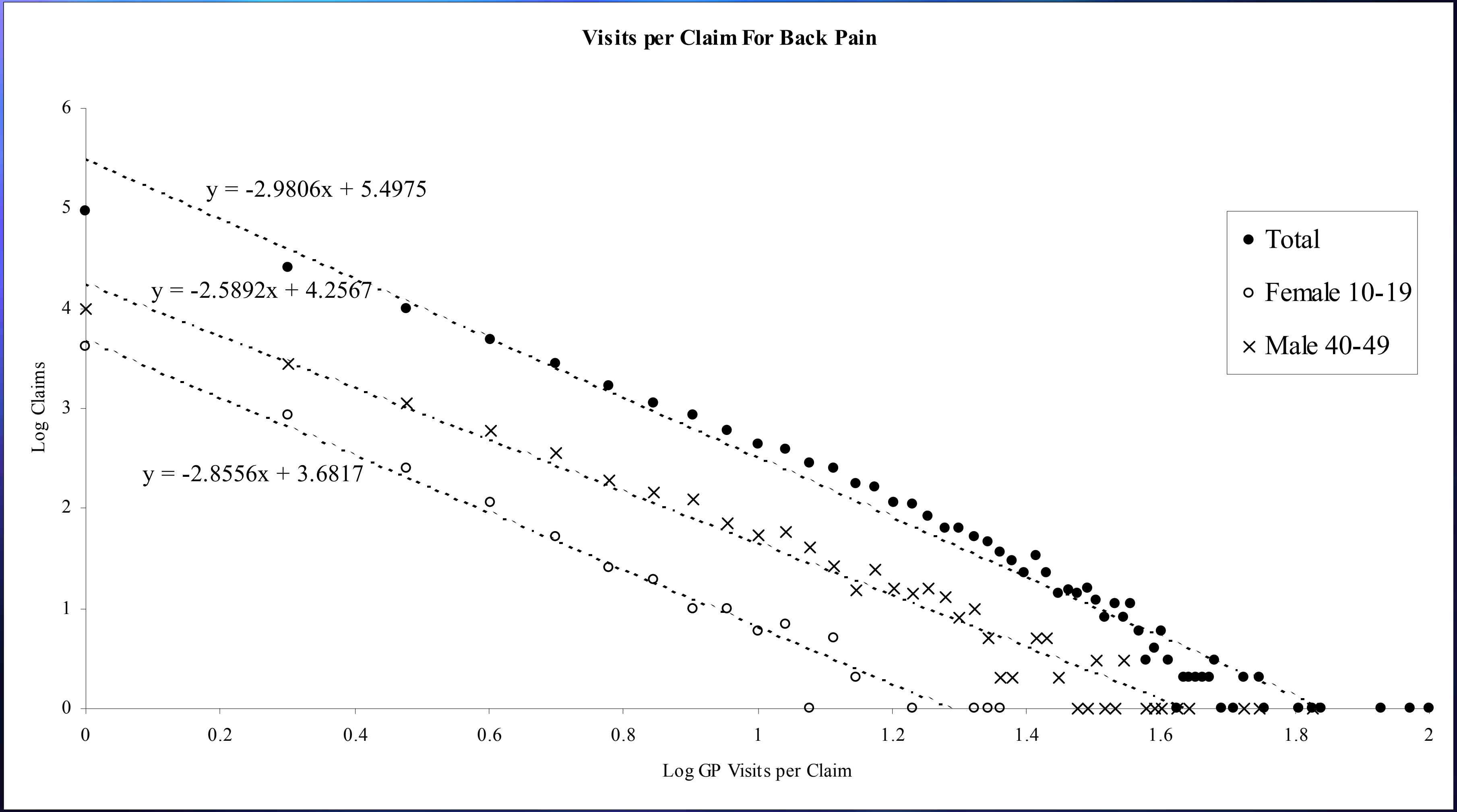
Patterns of frequent consulting in general practice



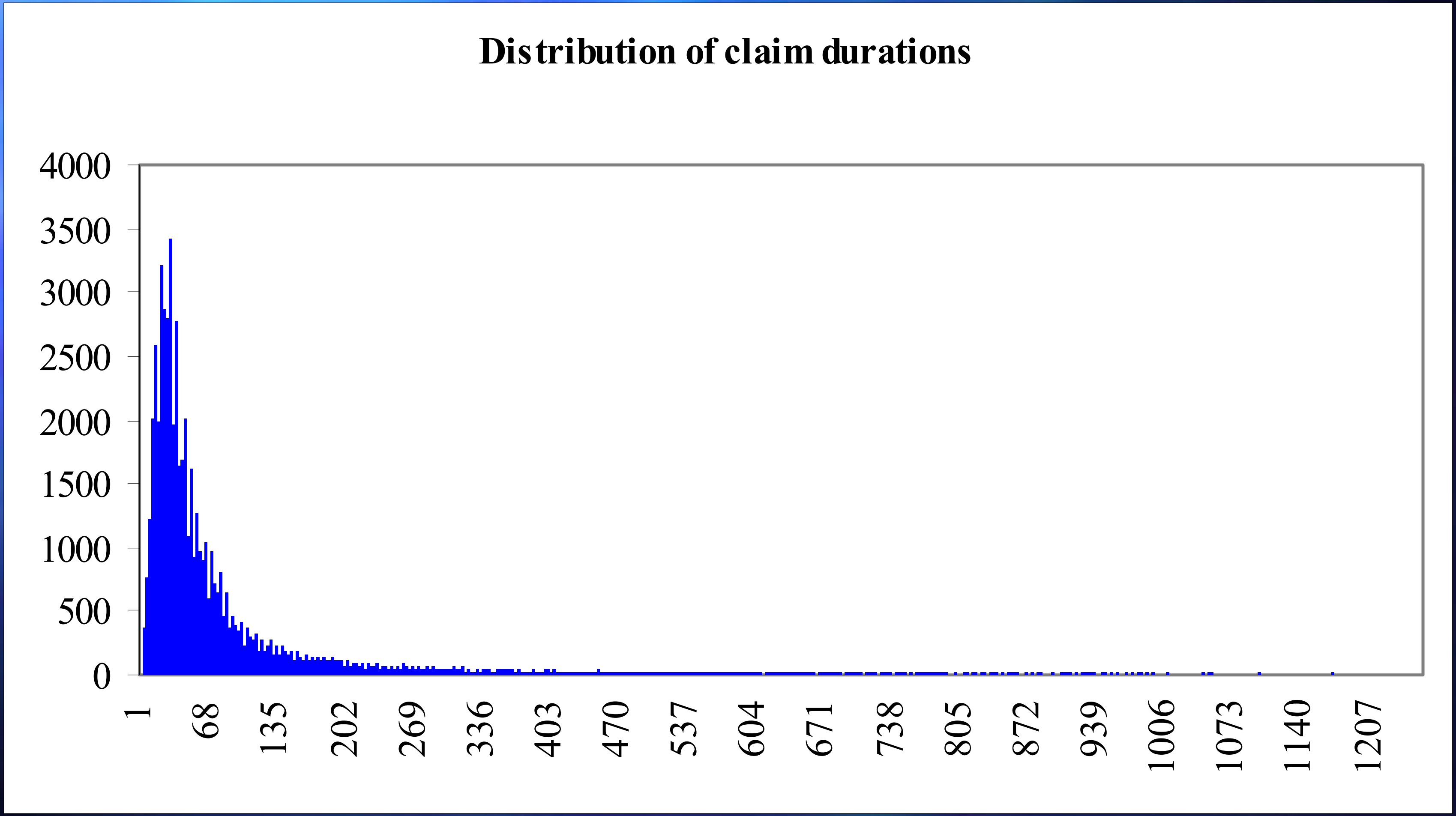
ACC Data

- Claims for back pain initiated in the 1998 calendar year
- ~150,000 claims
- Number of GP consultations in each claim
- Plot on a log-log scale

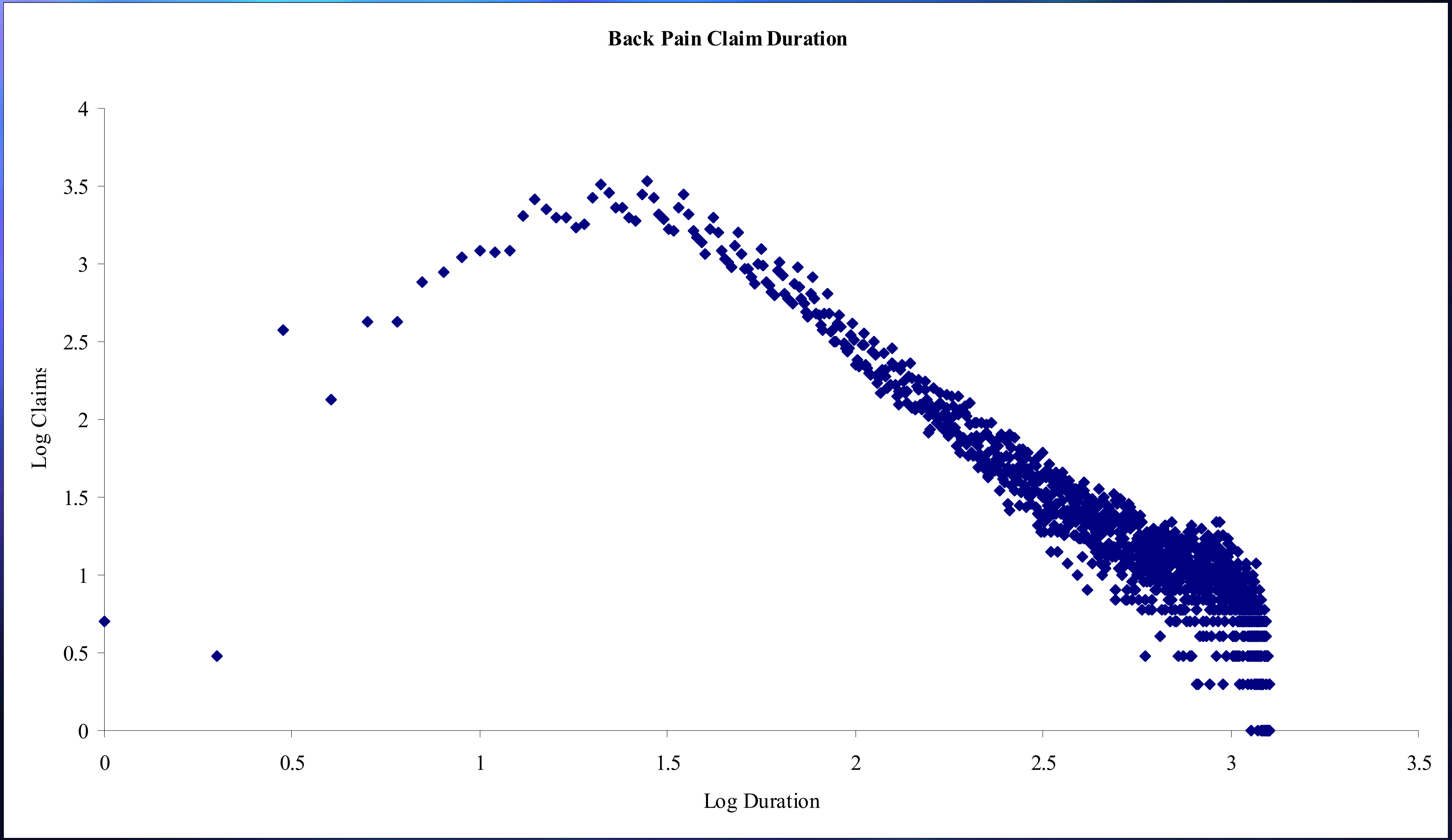
Frequent Consulting



Claim Duration



Claim Duration Again



Observations

- With consultations, the probability that there are more consultations to come gets you a power law curve.
- With duration, the probability that the condition hasn't resolved yet (the count of people who have not yet completed) is not a power law curve, but a heavily skewed gaussian distribution

Interpretation

- Duration of claim does not show a power law distribution. This is an individual level phenomenon: how long your back pain lasts.
- Attendance shows a power law distribution. This pattern is generated by the interaction of individuals with a system, and consequently with a wide variety of system factors which influence attendance. The existence of power law scaling implies that the system is self-organizing to some critical state.

Questions

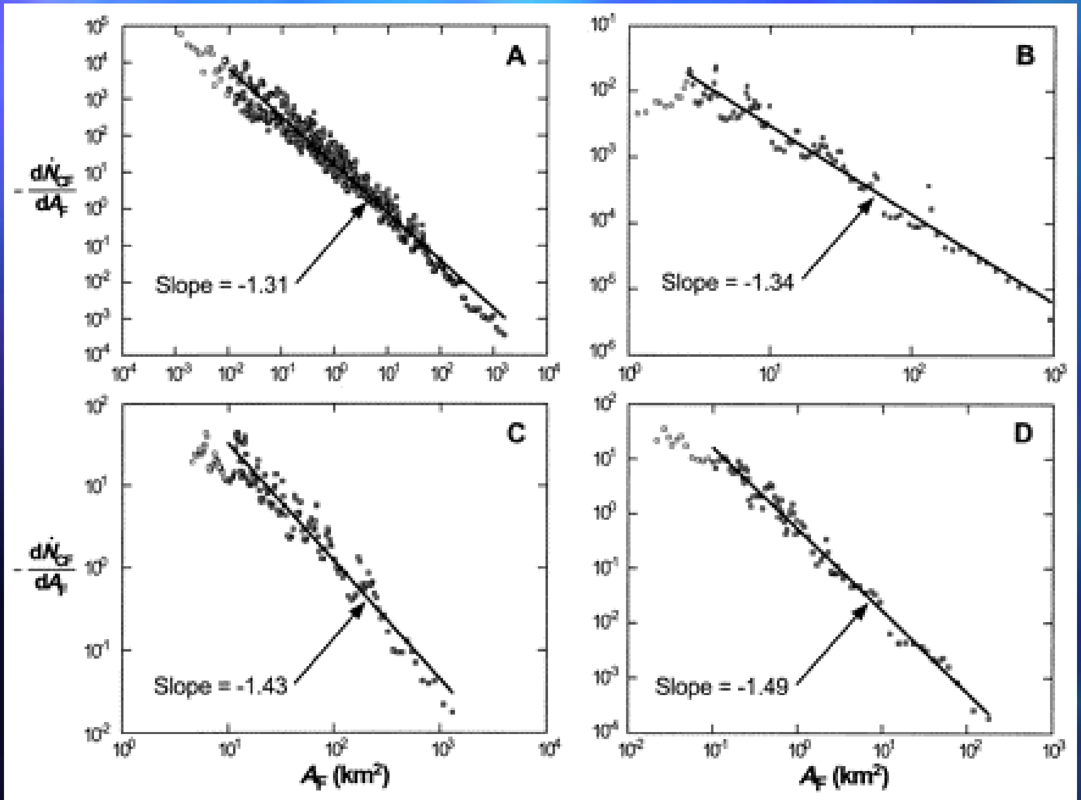
- Is the existence of frequent consulters a self emergent phenomenon, when a given resource is spread over a population?
- Is the emergence of frequent consulters in some sense efficient: a ‘critical state’?
- Can this sort of relationship be used to predict how many frequent consulters can be expected in a population?
- Can frequent consulting be modeled in the same way that sand piles and other physical phenomena are modeled?

More Details About Power Laws

- Forest fires
- Analysed in detail by Doyle and others
- An example of power laws observed in systems where there are very concrete applications

Forest Fires : An Example of Self-Organized Critical Behavior

In the natural state the distribution of small and large forest fires follows a power law



Malamud et al. Science; 1998

Managing Forest Fires

- Forest owners try to alter the distribution in order to minimize destruction by fires: put out fires as quickly as possible.
- The system doesn't always act predictably, and can actually produce worse results.
- Big fires may be even more frequent and destructive
- The power law distribution of forest fires seems to be an emergent and stable phenomenon

Self Organising To An Efficient State?

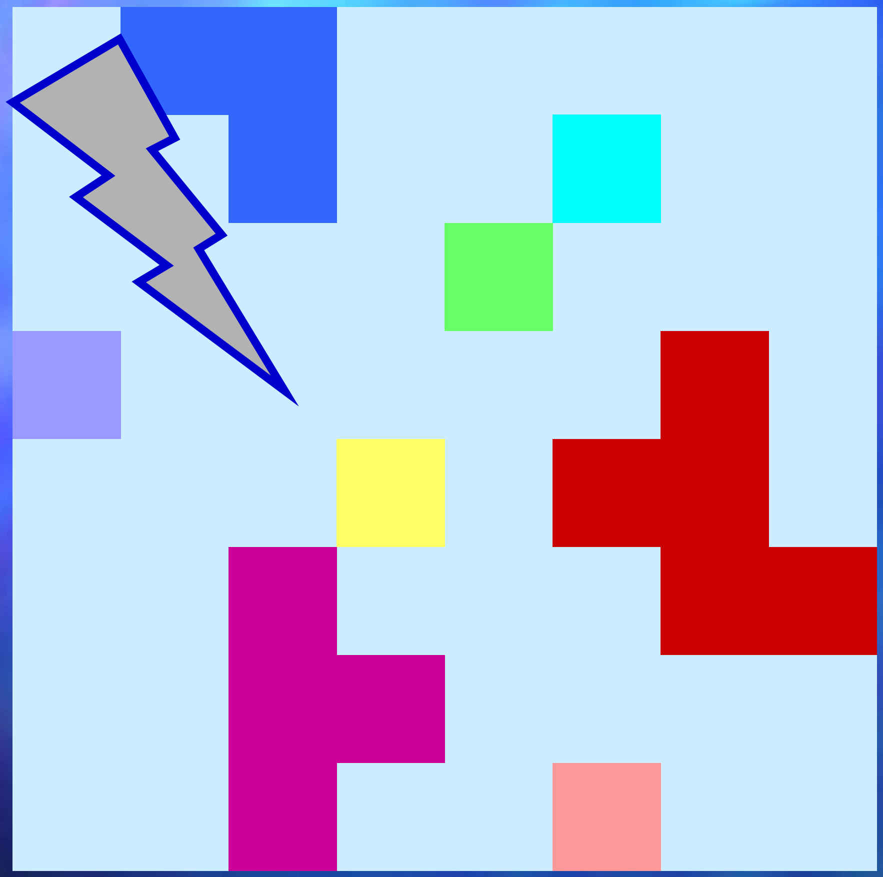
- Complex systems in biology, ecology, technology, sociology, economics, …
- Driven by design or evolution to high-performance states which are also tolerant to uncertainty in the environment and components.
- This leads to specialized, modular, hierarchical structures, often with enormous “hidden” complexity
- with new sensitivities to unknown or neglected perturbations and design flaws.
- Robust, yet fragile

Highly Optimised Tolerance (HOT)

- A system with many hundreds of sub components.
- Optimised for speed, load and fuel efficiency
- Robust to effects considered in design
- Fragile to unpredicted effects

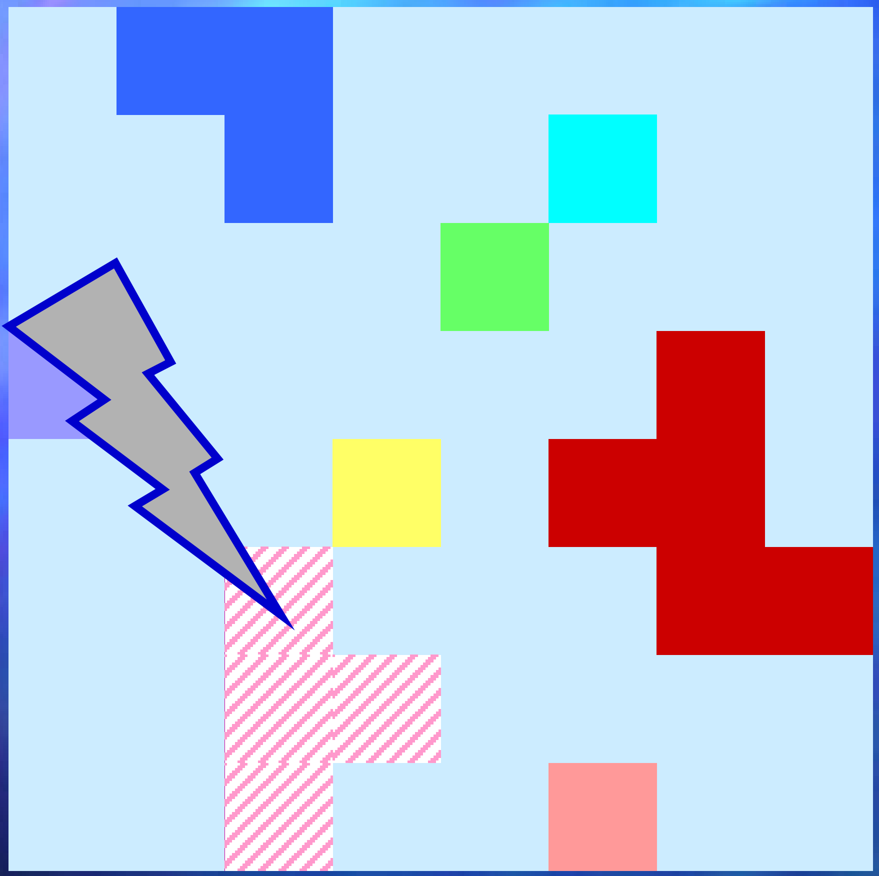
Designing Forests

- Is it possible, then to go beyond the natural power law state of a forest, and to design a more efficient one?
- Think of the yield from a forest: the number of trees in the forest, less those lost in a fire event.



Assume one "spark" hits the lattice at a single site.

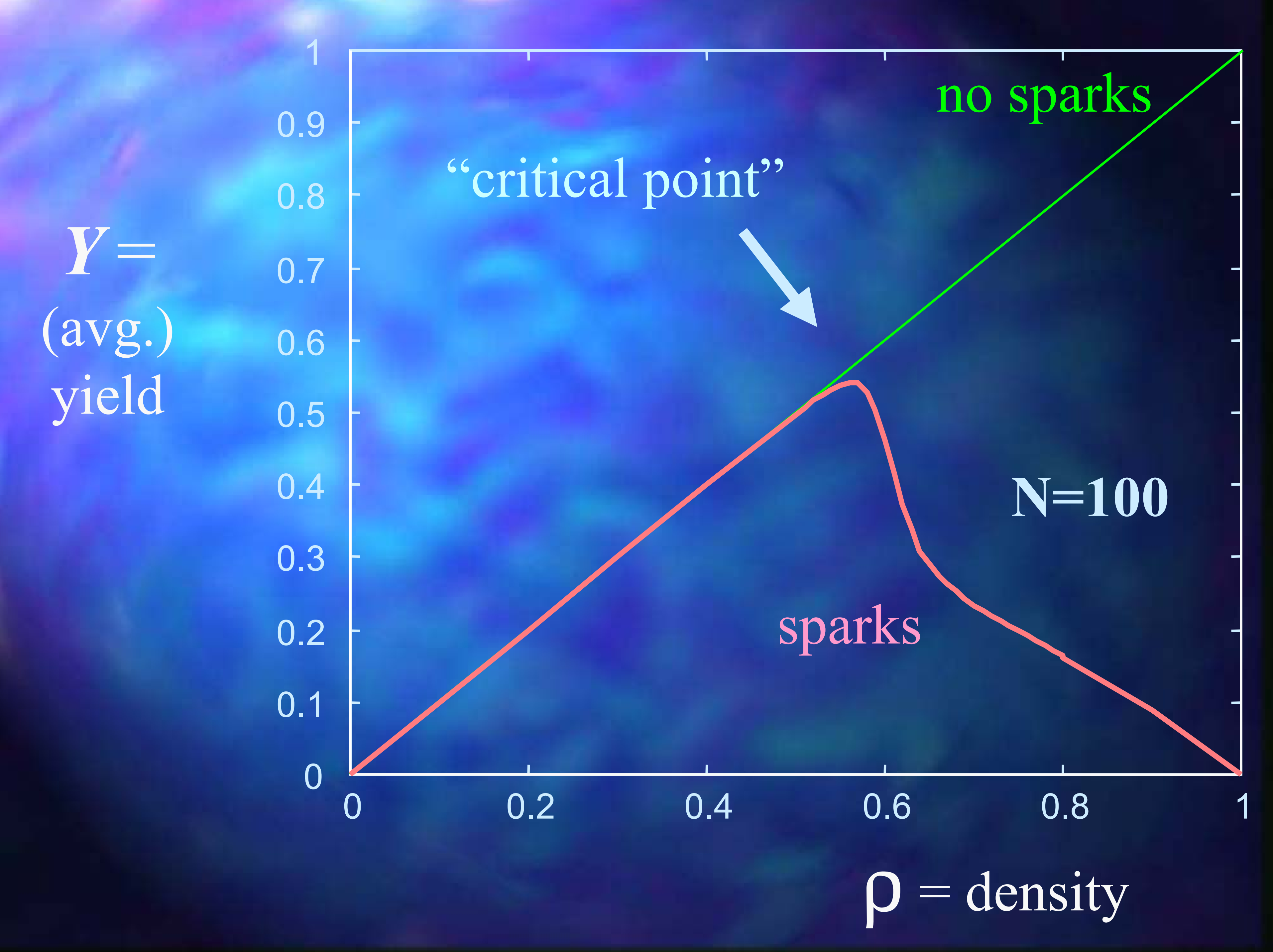
A "spark" that hits an empty site does nothing.



A “spark” that hits a cluster of trees causes loss of that cluster.

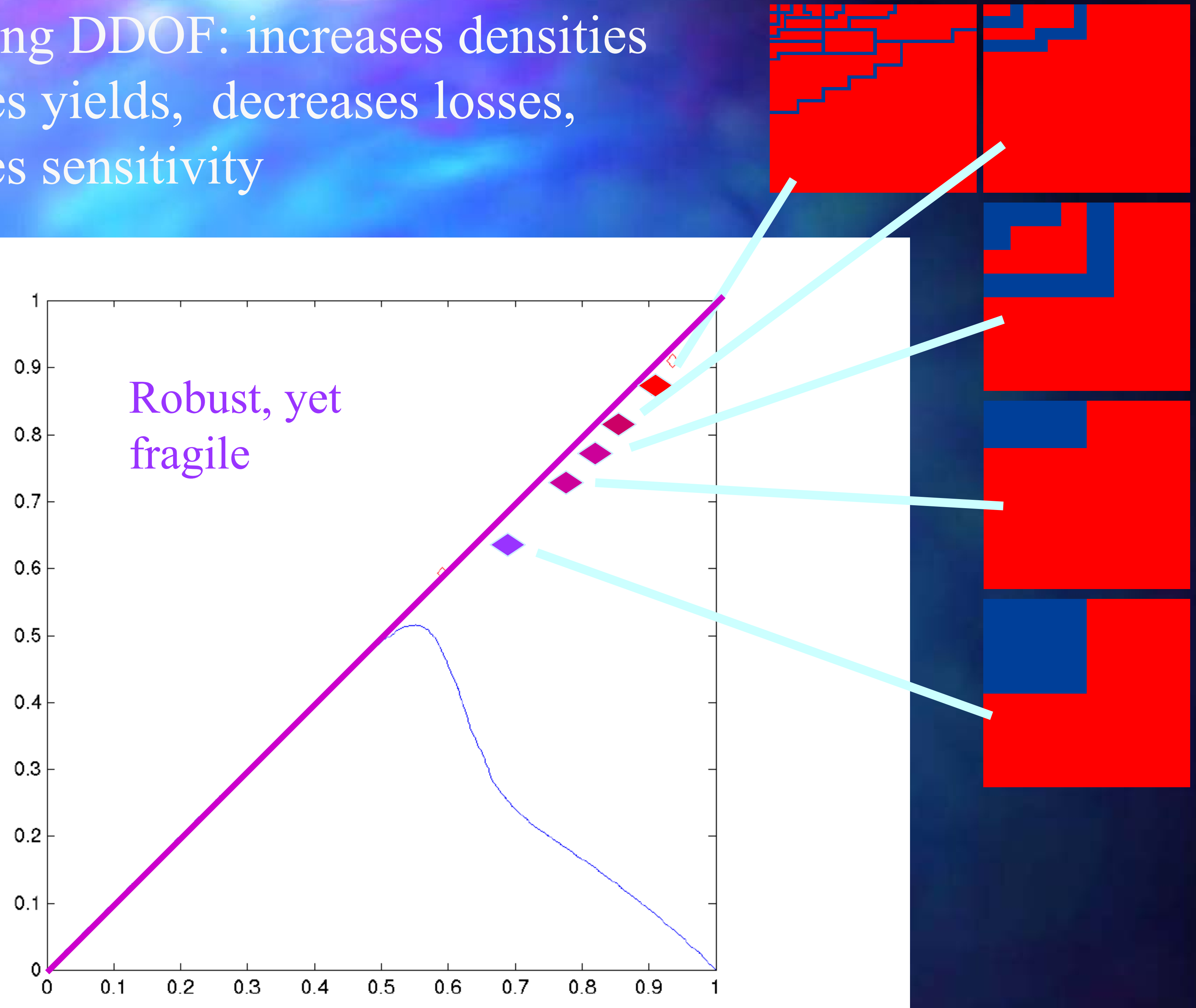
Yield = the density *after* one spark

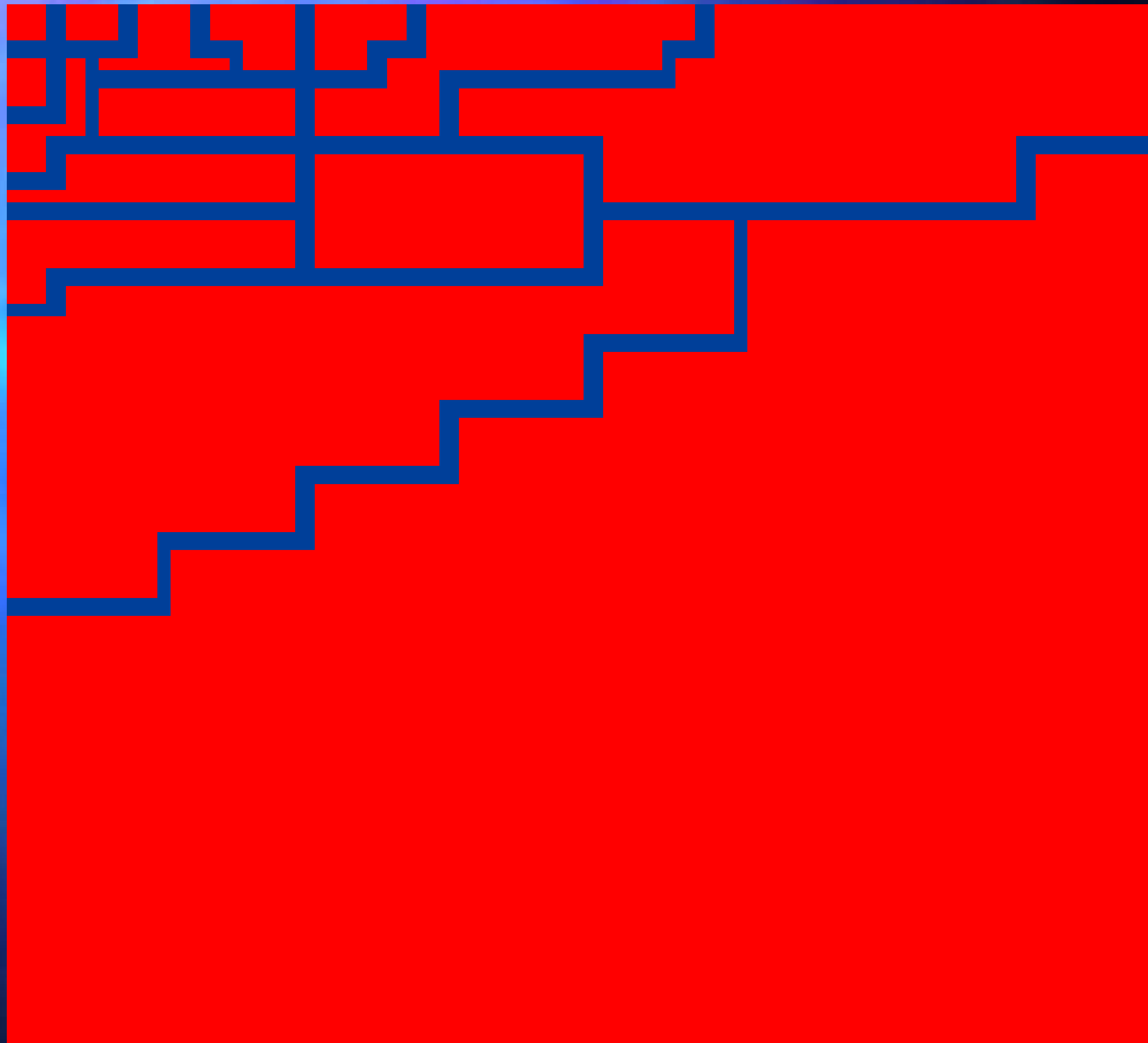
$$\boxed{\text{yield}} = \boxed{\text{density}} - \boxed{\text{loss}}$$

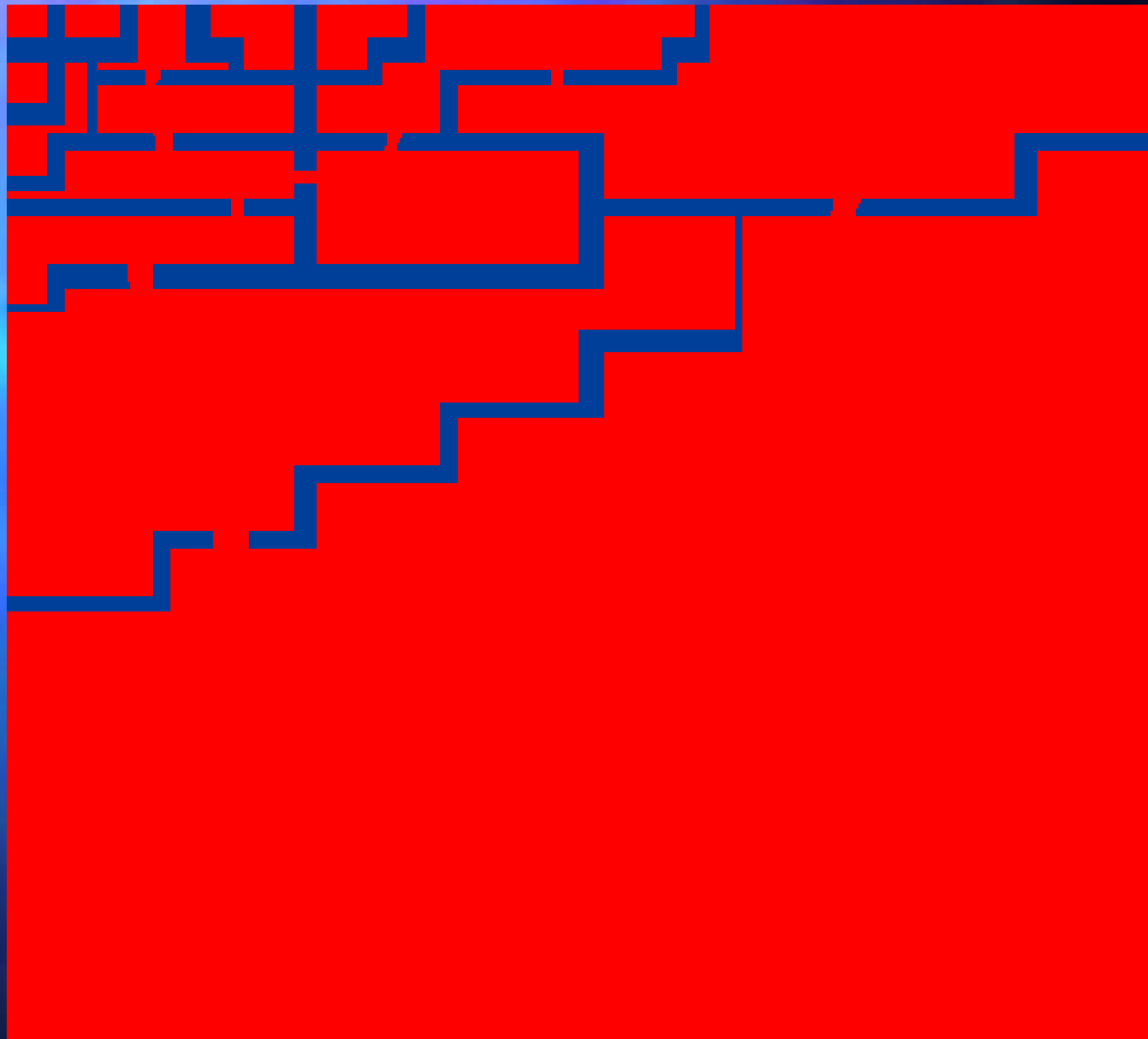


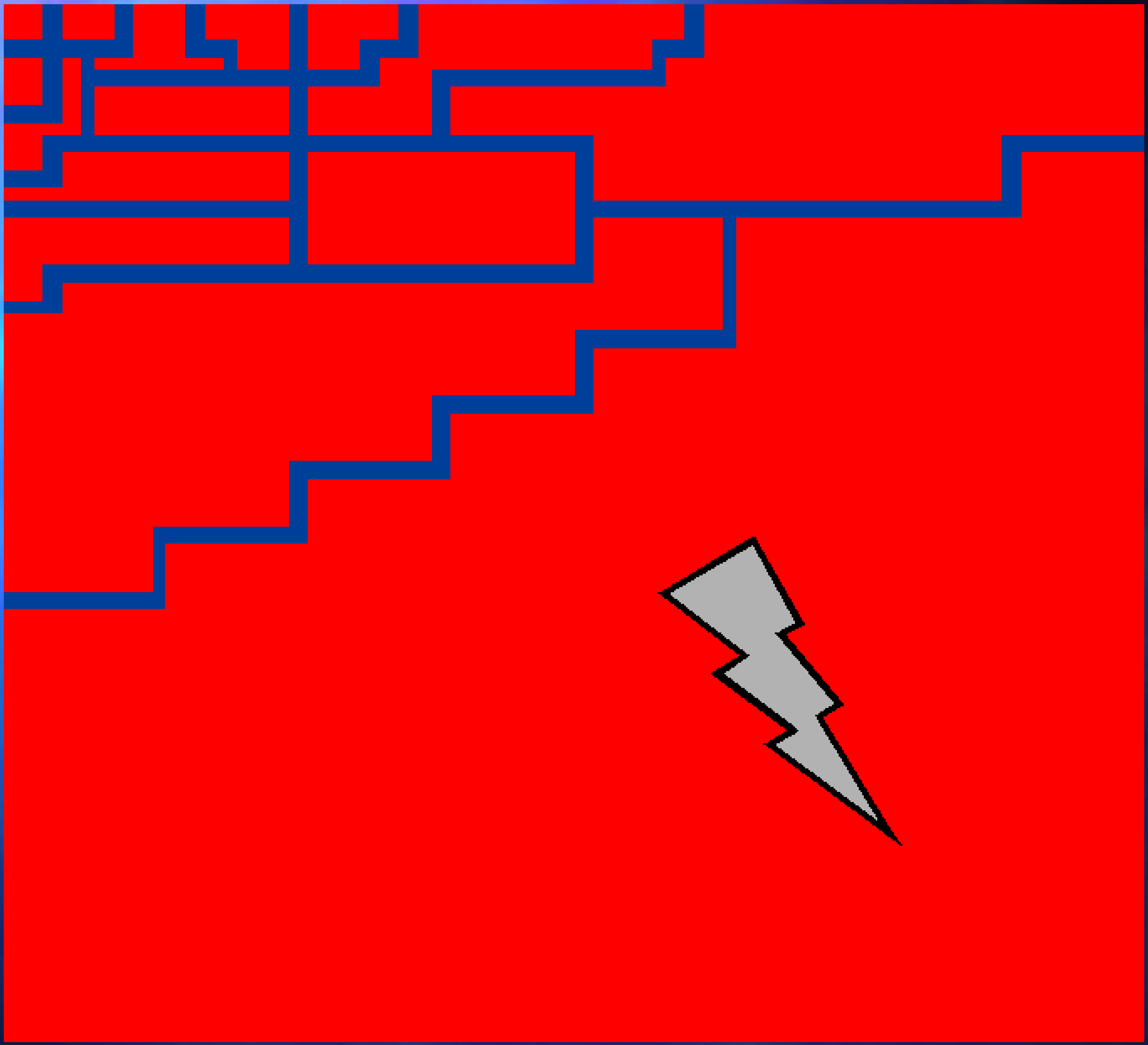
Y = (avg.) yield
"critical point"
no sparks
N=100
sparks
ρ = density
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

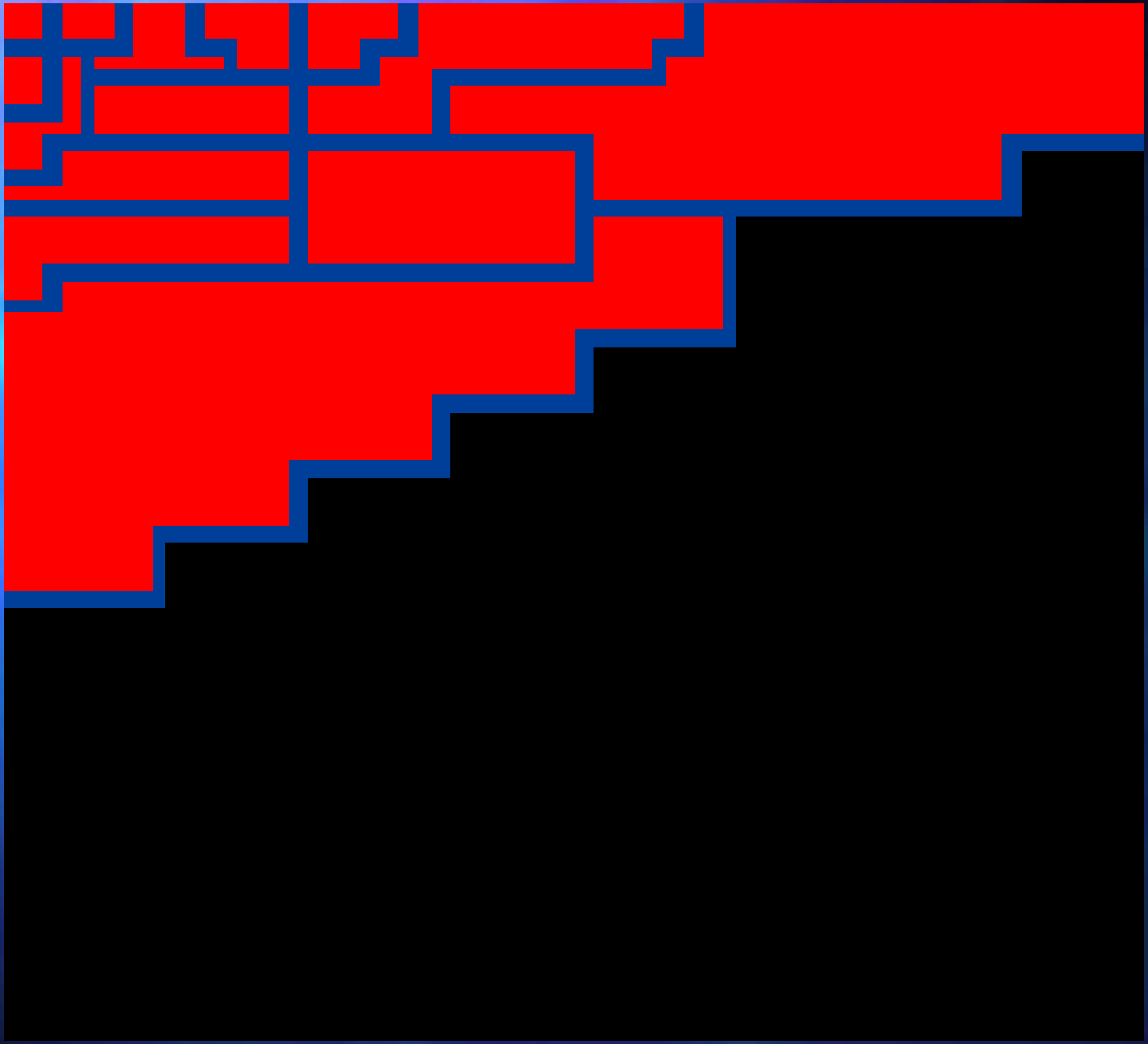
Increasing DDOF: increases densities increases yields, decreases losses, increases sensitivity











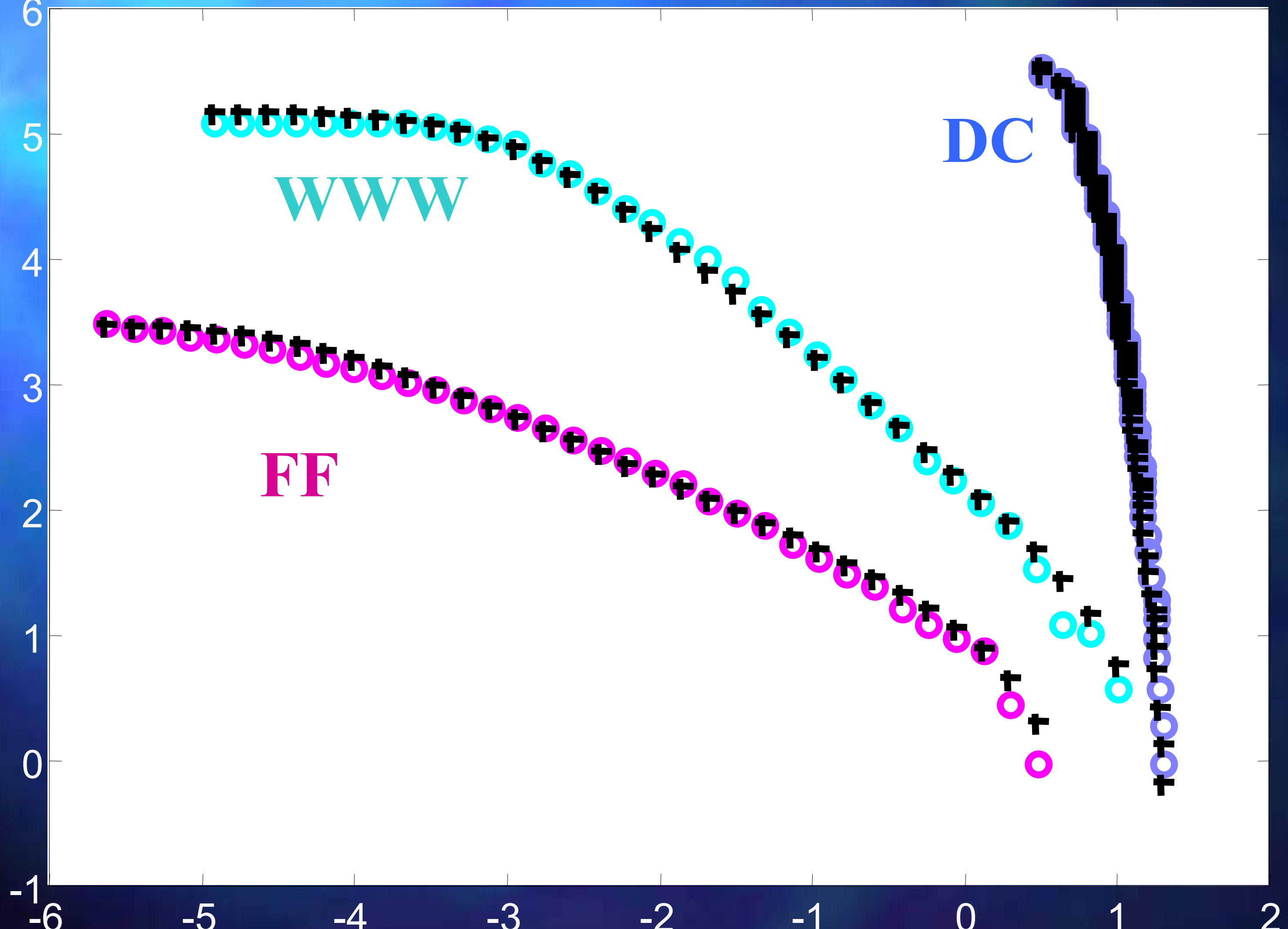
Modelling

- This approach is susceptible to analytical modelling (if you know enough maths):
  - P: uncertain events with probabilities p_i
  - R: limited resources r_i to minimize…
  - L: loss l_i due to event i

$$J = \left\{ \sum p_i l_i \;\middle|\; \sum r_i \le R \right\}$$

$$l(r) = \frac{c}{\beta}\left(r^{-\beta} - 1\right)$$

Doyle's Results Fit The Data Closely



Highly Optimised Tolerance

Interesting because:

- Example of analysis of a physical interpretation for phenomena which show power laws.
- Informs interpretations of self organising criticality.
- Demonstrates aspects of design in complex systems.
- Informs the tradeoff between robustness and yield.

What Does This Mean For Us?

- Discipline to help us develop rigorous interpretations of the observations we make about complexity.
- Consider the design of the systems we work in, and how ‘yield’ and optimisation approaches might be used to improve them.

Where Do We Stand In Primary Care?

- Have good reason to think that many of the systems we work with are complex, and could self organise to critical states.
- A few quantitative observations, but very little quantitative analysis, at least in health systems (as opposed to physiological systems).
- Great need for modelling work, either numerical or analytical.

What Applications Might There Be?

- Frequent Consulting
  - Predicting patterns of utilisation within populations
  - Helping people to organise care in an efficient fashion, optimising patterns of care (or at least understanding whether there is room for optimisation).

Applications...

- Medical Error
  - Yield and optimisation, patterns of error
  - Understanding what the fragility trade-off is when we optimise patterns of medical error. By analogy with the forest fires, perhaps there is a need for care in making otherwise intuitive changes.

Applications...

- Patterns of medical decision making
  - Often an expectation of homogeneous practice, embodied in indicators etc.
  - May be able to consider systems of diverse practice, eg. high and low prescribers and referrers.

Overall

- Analysing power laws in complex systems holds a lot of promise for developing new directions in health services research.
- Analysing power laws in complex systems holds a lot of promise for addressing some old problems in health services research.
- Primary care presents some particularly interesting questions which could perhaps be analysed this way.

Resources

- John Doyle's Website: www.cds.caltech.edu/~doyle/CmplxNets/
- Per Bak: *How Nature Works*
- Bielefeld: www.physik.uni-bielefeld.de/complexity
- Complexity in ecology: http://algodones.unm.edu/~ehdecker/complexity